# Development of Probabilistic Analysis Tools for Optimal Selection of Power Quality Mitigation Hardware

*Technical Report*

# Development of Probabilistic Analysis Tools for Optimal Selection of Power Quality Mitigation Hardware

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# REPORT SUMMARY

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Probabilistic analysis of power quality data can assist in selecting the optimal power quality mitigation hardware system. This report describes how to use probabilistic analysis techniques to calculate the net present value of a power quality mitigation hardware system and compares this approach to a deterministic analysis technique.

## Background

Power quality concerns continue to be an important factor in the electric supply. New energy-storage technologies and power electronics conversion topologies are providing new alternatives for power quality mitigation that may be increasingly attractive from an economic perspective. However, these devices need to be evaluated not solely on their technical merits, but also on their total life-cycle costs. By using the techniques discussed in this report, utilities will have the advantage of helping to provide economic cost analysis for their customers thereby continuing to provide value-added service. To assist in selecting the most cost-effective power quality mitigation system, or the least expensive life-cycle cost, industry increasingly is using quantitative risk analysis techniques to make such optimal economic decisions. This report reviews traditional approaches for analyzing uncertain cash flow projections and describes common techniques for evaluating the risk associated with different aspects of an economic decision-making process. A relatively new technique to economic decision making, Monte Carlo simulation, is examined in an attempt to improve upon the approaches currently adopted by managers who are responsible for the allocation of capital resources in economic decisions. An example of an economic decision analysis is described using a traditional approach called the deterministic method along with a newer technique called the probabilistic method.

## Objective

To demonstrate how to conduct a life-cycle cost analysis for a power quality mitigation project at a commercial customer site and incorporate statistical information in the decision-making processes.

## Approach

The project team reviewed the traditional deterministic method for analyzing the economic costs and benefits associated with the selection of a power quality mitigation hardware system, including the total life-cycle cost of power quality mitigation hardware solutions for both embedded and system-wide implementation. They then discussed how statistical information can be incorporated into the economic decision process and used as part of the capital budgeting process.

## Results

Many power quality mitigation systems have been planned, designed, produced, and operated with very little concern for their cumulative life-cycle cost. Experience shows that a large portion of the total cost of a power quality mitigation system is the direct result of activities associated with the operation and support of these systems; but the commitment of these costs is based on decisions made early in the system life cycle in the research, design, and procurement stages.

At the inception of a new power quality mitigation project, the total system life-cycle cost is uncertain, particularly those costs associated with system operation and support. However, the completion of a life-cycle cost analysis serves as an aid in the decision-making process. By better understanding the statistics associated with the number and duration of power quality events, the optimal selection of hardware systems used to mitigate power quality disruptions can be analyzed more effectively and their life-cycle costs evaluated.

This report begins by reviewing statistical analysis techniques followed by discussing traditional approaches for analyzing the economic benefits associated with the selection of a power quality mitigation hardware system, methods collectively referred to as deterministic analysis. The report then discusses probabilistic analysis techniques that incorporate statistical information into the economic decision process. An example shows how selecting the wrong statistical distribution representing the time between power quality events leads to the wrong economic conclusions and compares the results of probabalistic analysis to those of deterministic analysis.

## EPRI Perspective

By providing a clear roadmap to determine the total life-cycle cost for a power quality mitigation hardware system, EPRI is making utilities better able to provide higher value-added services to their customer base. This report provides a comprehensive methodology for performing an economic analysis of the life-cycle cost for power quality mitigation hardware solutions. In addition, this report reviews both a deterministic and statistical method for selecting the optimal solution based on power quality events. This report will assist today's utilities to better understand the decision processes their customers go through in selecting power quality mitigation hardware and how a life-cycle cost analysis is performed to select the optimal solution that meets the customer's needs.

## Keywords

Power Quality
Power Conditioning
End-Use Mitigation Systems
Life-Cycle Costing
Time Value of Money
Capital Budgeting
Net Present Value
Project Balance Method

# GLOSSARY

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Below is a list of terms with their definitions used in this report.

*Accelerated Cost Recovery System (ACRS) (Modified).* The Tax Reform Act of 1986 established the modified ACRS tax appreciation system prescribing depreciation methods for each ACRS class in lieu of statutory tables. Equipment is assigned among 3-, 5-, 7-, 10-, 15-, or 20-year classes depending on their useful life.

*Annuity.* A series of equal payments, at equal intervals.

*Budget.* A description, in monetary terms, of the organization's plans.

*Compound Amount.* The future value of money invested or loaned at compound interest.

*Cost Model. A*n approach based on technical and programmatic parameters for computing concerned costs.

*Cost.* The amount of money paid or payable for the acquirement of services, materials, or property.

*Deterministic Expense*. An expense for which a budget can be set objectively and about which one can say "the lower the better."

*Discretionary Expense*. An expense for which a budget can be set only with artful judgment and one that typically has more long-term than short-term benefits to the company.

*Downtime.* The total time during which the product (or item) is not in a condition to carry out its stated function or mission.

*Economic Life (Useful Life).* The period of time during which an asset will have economic value and be usable.

*Failure Rate.* The number of failures of a product per unit measure of life (e.g., hours).

*Failure.* The termination of the ability of an item to carry out its stated mission or function.

*Hurdle Rate*. The internal interest rate that is required by a firm to justify investment in a project.

*Life-Cycle Cost.* The sum of all costs incurred during the lifetime of an item, i.e., the total of procurement and ownership costs.

*Maintainability.* The probability that a failed item will be restored to its satisfactory operational state within a specified total downtime when maintenance action is started according to stated conditions.

*Maintenance Cost.* The labor and materials expense required to maintain item(s) in suitable use condition.

*Maintenance.* All scheduled and unscheduled actions appropriate for keeping an item in a serviceable condition or restoring it to serviceability. It includes inspection, repair, modification, servicing, remove and replace, etc.

*Managerial Accounting*. The use of accounting data to assist management in making operating decisions.

*Manufacturing Cost.* The sum of fixed and variable costs chargeable to the production of a specified item.

*Mean Time to Repair.* The mean time needed to repair a product (or an item).

*Nonrecurring Cost.* The cost that is not repeated.

*Present Value.* The current equivalent of payments or a stream of payments to be received at various times in the future. The present value will vary with the discount interest factor applied to future payments.

*Pro Forma Financial Statements*. Financial statements that are projected, or estimated in advance. An operating budget is a pro forma income statement.

*Probability Distributions*. Probability distribution of a discrete random variable is a list of probabilities associated with each of its possible values. It is also sometimes called the probability function or the probability mass function. More formally, the probability distribution of a random variable X is a function which gives the probability $p(x_i)$ that the random variable equals $x_i$, for each value $x_i$.

*Probability*. A probability provides a quantitative description of the likely occurrence of a particular event. Probability is conventionally expressed on a scale from 0 to 1; a rare event has a probability close to 0, a very common event has a probability close to 1.

*Procurement Cost.* The total of investment or acquisition costs (recurring and non-recurring).

*Random Sample*. A sample of subjects that is randomly selected from a group and is therefore assumed to be representative of that group.

*Random Variable*. A variable that can have any of a range of values that occur randomly but can be described probabilistically.

*Recurring Cost.* The cost that recurs periodically during the life of a project.

*Regression Model*. Describes the relationship between a response variable and one or more explanatory variables in a linear or non-linear equation.

*Reliability.* The probability that an item will carry out its mission satisfactorily for the desired period when used according to specified conditions.

*Repair Cost.* The cost of restoring an item or a facility to its original condition or performance.

*Statistics*. Branch of mathematics that deals with the collection, organization, and analysis of numerical data and with such problems as experiment design and decision making.

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# 1

# EXECUTIVE SUMMARY

How does one select the most economical power quality mitigation hardware system? Many of these power quality mitigation systems have been planned, designed, produced, and operated with very little concern for their cumulative life-cycle cost. Although different facets of costs have been considered in the development of these electrical systems, the costs often have been viewed in a fragmented manner. The costs associated with research, design, testing, construction, production, operations, maintenance, and support have been considered as independent costs and typically addressed only at their specific life-cycle stage, not viewed on an integrated basis. Experience has indicated that a large portion of the total cost associated with a power quality mitigation system is the direct result of activities associated with the operation and support of these systems, while the commitment of these costs is based on decisions made in the early stages of the system life cycle, i.e., research, design, and procurement. Total system life-cycle cost often is not visible, particularly those costs associated with system operation and support.

The future outcomes of any power quality mitigation system decision and their life-cycle costs are never certain. Likewise, the economic evaluation of a power quality mitigation system will never be exact. However, by better understanding the statistical nature of the number and duration of power quality events, the optimal selection of hardware systems used to mitigate these power quality disruptions can be analyzed more effectively and their economic options evaluated.

If the number and duration of power quality events, along with the cost and benefit of mitigating the power quality events, were known accurately, there would be no need for statistical analysis. Then, the selection of the most feasible power quality mitigation solution and its life-cycle costs could be analyzed using the appropriate economic evaluation criteria. However, in reality, future economic benefits are only estimates of what *may* happen. As a result, economic decision analysis must incorporate techniques to evaluate the uncertainty associated with future economic benefits related to the expected number and frequency of power quality events and the power quality hardware system required to mitigate such electrical disruptions.

This report begins by reviewing statistical analysis techniques followed by discussing traditional approaches for analyzing the economic benefits associated with the selection of a power quality mitigation hardware system referred to in this report as deterministic analysis method. The report then discusses probabilistic analysis method by incorporating statistical information into the economic decision process. Finally, statistical techniques are discussed as part of a capital budgeting process.

While the future economic benefits of a power quality mitigation system is never certain, the uncertainty can be better quantified by understanding the statistical nature of power quality events that the mitigation system is designed to prevent. This report describes one technique to analyze the expected number of future power quality events and relates this statistical variable or random variable to the expected future benefit derived from the power quality mitigation system. The expected benefit is then incorporated into the system life-cycle cost to calculate the net present value or project balance of the future economic benefit derived from the power quality mitigation system.

The two methods used to calculate the future economic benefits are the deterministic method and the probabilistic method. The deterministic method assumes that all future power quality events and the benefit derived from a power quality mitigation system are known with certainty. The probabilistic method assumes that all future power quality events and the benefits derived from a power quality mitigation system are not known with certainty and, therefore, can be described by a random variable with a statistical distribution.

The additional information content contained in the probabilistic method is analyzed to provide a better understanding of the expected range of economic outcomes associated with the use of a power quality mitigation system. The economic outcomes using the probabilistic method are compared to the deterministic method and the results are discussed.

The conclusion is that selecting the appropriate statistical distribution that describes the number of power quality events, therefore affecting the economic benefits derived from using a power quality mitigation system, can affect significantly the life-cycle cost. This additional information can than be included in the capital budgeting process to provide management a better understanding of the range of potential economic outcomes and the probability associated with each outcome.

# 2
# STATISTICAL ANALYSIS OF POWER QUALITY DATA

## Introduction

The future outcomes of any financial decision are never certain. Likewise, the economic evaluation of a power quality mitigation system will never be exact. However, by better understanding the statistical nature of the number and duration of power quality events, the optimal selection of hardware systems used to mitigate power quality disruptions can be more effectively analyzed and their economic options evaluated.

If the number and duration of power quality events along with the cost and benefit of mitigating the power quality events were known accurately, there would be no need for statistical analysis. Then, the selection of the most feasible power quality mitigation solution could be determined using the appropriate economic evaluation criteria. However, in reality, future economic benefits are only estimates of what *may* happen. As a result, economic decision analysis must incorporate techniques to evaluate the uncertainty associated with future economic benefits related to the expected number and frequency of power quality events and the power quality hardware system required to mitigate such electrical disruptions.

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## Statistical Analysis

Statistical analysis is a tool that allows the decision maker to make rational decisions under uncertain conditions. For example, suppose a coin is tossed twenty times, and it comes up heads fifteen times. Is the coin fair? Instead, suppose 99 out of 100 hospital back-up power generators start. Is the system reliable? From the point of view of statistics, these two questions are identical. Statistical analysis is a set of tools used to answer this question.

Statistics are used everywhere in the modern world. Polls influence policy decisions in government. Clinical trials determine whether to approve drugs for human use. DNA matching based on statistical tests is used in courts with increasing frequency. Predictions are made based on linear regression models in all fields of science. Samples are taken in countless fields to learn about larger populations.

One important use of statistics is to summarize a collection of data in a clear and understandable way. For example, assume a utility engineer is measuring the number of power quality events on a distribution feeder. How might these measurements be summarized? There are two basic methods: numerical and graphical. Using the numerical approach, one might compute statistics such as the mean and standard deviation. These statistics convey information about the average number of power quality events and the frequency to which these events occur. Using the graphical approach, one might create a stem and leaf display and a box plot. These plots contain detailed information about the distribution of power quality events.

Graphical methods are better suited than numerical methods for identifying patterns in the data. Numerical approaches are more precise and objective.

Inferential statistics are used to draw inferences about a population from a sample. Consider an experiment in which a power quality monitor collected the number and duration of power quality events during a 4-month period, recorded 10 events, then the same monitor collected the number of events during a different 4-month period, and recorded 15 events. Is the difference real or could it be due to chance? How much larger could the real difference be than the five events difference found in the sample? These are the types of questions answered by inferential statistics.

There are two main methods used in inferential statistics: estimation and hypothesis testing. In estimation, the sample is used to estimate a parameter and a confidence interval about the estimate is constructed.

In the most common use of hypothesis testing, a "straw man" null hypothesis is put forward and it is determined whether the data are strong enough to reject it. For the power quality study, the null hypothesis might be “there is no difference in the number of power quality events between the two different four-month periods.”

The core value of statistical methodology is its ability to assist one in making inferences about a large group (a population) based on observations of a smaller subset of that group (a sample). For this to work correctly, a couple of things have to be true: the sample must be similar to the target population in all relevant aspects, and certain aspects of the measured variables must conform to assumptions that underlie the statistical procedures to be applied.

Representative sampling is one of the most fundamental tenets of inferential statistics: the observed sample must be representative of the target population for inferences to be valid. A simple random sample design is the establishment of samples by randomly selecting members of the population, with each member having an equal probability of being selected for the sample. A stratified random sample design ensures that the samples ‘parallel’ the population with respect to certain key characteristics that are thought to be important to the investigation at hand.

## Statistical Distributions

Statistical analysis of power quality data requires consideration of the distribution of the data. Selecting the correct statistical distribution is one of the most important aspect of any decision process. Statistical distribution is a function that represents the probability of occurrence for each random variable outcome. In this case, the random variable is either the number of power quality events during a specified time period or the duration between power quality events.

The most common statistical distribution is the normal distribution. However, for most applications, the normal distribution is only an approximation of the “true” distribution. If the normal distribution is used to represent the “true” distribution when the “actual” distribution is an exponential, Weibull or one of several other types of distributions, the final analysis could be significantly distorted, thereby allowing the decision maker to arrive at an incorrect economic decision. This section is a basic introduction to statistical distributions. The next section is an introduction to Weibull distribution and the rationale behind why it may be the preferred distribution for power quality data.

## *Popular Distributions and Their Typical Applications*

### Binomial Distribution

Application: Gives probability of successes in $n$ independent trials, when probability of success $p$ on single trial is a constant. Used frequently in quality control, reliability, survey sampling, and other industrial problems.

Example: What is the probability of 7 or more "heads" in 10 tosses of a fair coin?

Comments: Can sometimes be approximated by normal or by Poisson distribution.

### Geometric Distribution

Application: Gives probability of requiring exactly $x$ binomial trials before the first success is achieved. Used in quality control, reliability, and other industrial situations.

Example: Determination of probability of receiving exactly five power quality disruptions before first actual disruption is achieved.

### Pascal Distribution

Application: Gives probability of exactly $x$ power quality failures preceding the $s^{\text{th}}$ success.

Example: What is the probability that the 3rd success takes place on the 10th trial?

### Poisson Distribution

Application: Gives probability of exactly $x$ independent occurrences during a given period of time if events take place independently and at a constant rate. Used frequently in quality control, reliability, queuing theory, and so on.

Example: Used to represent distribution of number of defects in a piece of material, customer arrivals, insurance claims, incoming telephone calls, alpha particles emitted, and so on.

Comments: Frequently used as approximation to binomial distribution.

### Normal Distribution

Application: A basic distribution of statistics. Many applications arise from central limit theorem (average of values of *n* observations approaches normal distribution, irrespective of form of original distribution under quite general conditions); consequently, appropriate model for many, but not all, physical phenomena.

Example: Distribution of physical measurements on living organisms, intelligence test scores, product dimensions, average temperatures, and so on.

Comments: Many methods of statistical analysis presume normal distribution.

### Gamma Distribution

Application: A basic distribution of statistics for variables bounded at one side--for example *x* greater than or equal to zero. Gives distribution of time required for exactly *k* independent events to occur, assuming events take place at a constant rate. Used frequently in queuing theory, reliability, and other industrial applications.

Example: Distribution of time between recalibrations of instrument that needs recalibration after *k* uses; time between inventory restocking; time-to-failure for a system with standby components.

### Exponential Distribution

Application: Gives distribution of time between independent events occurring at a constant rate. Equivalently, probability distribution of life, presuming constant conditional failure (or hazard) rate, consequently, applicable in many, but not all reliability situations.

Example: Distribution of time between arrival of particles at a counter. Also life distribution of complex non-redundant systems, and usage life of some components--in particular, when these are exposed to initial burn-in, and preventive maintenance eliminates parts before wear-out.

Comments: Special case of both Weibull and gamma distributions.

### Beta Distribution

Application: A basic distribution of statistics for variables bounded at both sides--for example *x* between 0 and 1. Useful for both theoretical and applied problems in many areas.

Example: Distribution of proportion of population located between lowest and highest value in sample; distribution of daily percent yield in a manufacturing process; description of elapsed times to task completion (PERT).

### Uniform Distribution

Application: Gives probability that observation will occur within a particular interval when probability of occurrence within that interval is directly proportional to interval length.

Example: Used to generate random values.

Comments: Special case of beta distribution.

### Rayleigh Distribution

Application: Gives distribution of radial error when the errors in two mutually perpendicular axes are independent and normally distributed around zero with equal variances.

Example: Bomb-sighting problems; amplitude of noise envelope when a linear detector is used.

Comments: Special case of Weibull distribution.

### Weibull Distribution

Application: General time-to-failure distribution due to wide diversity of hazard-rate curves, and extreme-value distribution for minimum of N values from distribution bounded at left.

The Weibull distribution is often used to model "time until failure." In this manner, it is applied in actuarial science and in engineering work. It is also an appropriate distribution for describing data corresponding to resonance behavior, such as the variation with energy of the cross section of a nuclear reaction or the variation with velocity of the absorption of radiation in the Mossbauer effect.

Example: Life distribution for some capacitors, ball bearings, relays, and so on.

Comments: Rayleigh and exponential distribution are special cases.

## *Data Summary Measurement*

How do you describe the “average” or “typical” power quality data set? The measurement of the “average” or “typical” power quality data set is important in defining the type of statistical distribution. Different procedures are used to summarize the most representative information depending of the type of question asked and the nature of the data being summarized.

Measures of “central tendency” within a power quality data set include the mean, the median, and the mode. For example, if the mean were located in the center of the data with equal area on both sides of the distribution, the data would be normally distributed.

The arithmetic mean (or the average or simple mean) is computed by summing all numbers in an array of numbers ($x_i$) and then dividing by the number of observations (n) as shown in the formula below.

$$\overline{x} = \sum_{i=1}^{n} \frac{x_i}{n}$$

The mean uses all of the power quality observations, and each observation affects the mean. Even though the mean is sensitive to extreme values, i.e., extremely large or small data can cause the mean to be pulled toward the extreme data, it is still the most widely used measure of location. This is because the mean has valuable mathematical properties that make it convenient for use with inferential statistical analysis. For example, the sum of the deviations of the numbers in a set of power quality data from the mean is zero, and the sum of the squared deviations of the numbers in a set of power quality data from the mean is the minimum value.

The median is the middle value in an ordered array of power quality observations. If there is an even number of observations in the array, the median is the average of the two middle numbers. If there is an odd number of data in the array, the median is the middle number. The median is often used to summarize the distribution of an outcome and in helping to quickly select the best statistical distribution.

Generally, the median provides a better measure of central location than the mean when there are some extremely large or small observations, i.e., when the data are skewed to the right or to the left. For this reason, median income is used as the measure of location for the U.S. household income. Note that if the median is less than the mean, the data set is skewed to the right. If the median is greater than the mean, the data set is skewed to the left.

The mode is the most frequently occurring value in a set of power quality observations. Why use the mode? The classic example is the shirt/shoe manufacturer who wants to decide what sizes to introduce. Data may have two modes. In this case, we say the data are bimodal, and sets of observations with more than two modes are referred to as multimodal. Note that the mode does not have important mathematical properties for future use. In addition, the mode is not a helpful measure of location, because there can be more than one mode or even no mode.

### *Selecting Among the Mean, Median, and Mode Index*

It is a common mistake to specify the wrong index for central tendency. Figure 2-1 below is a simple flow chart used in selecting the appropriate index.

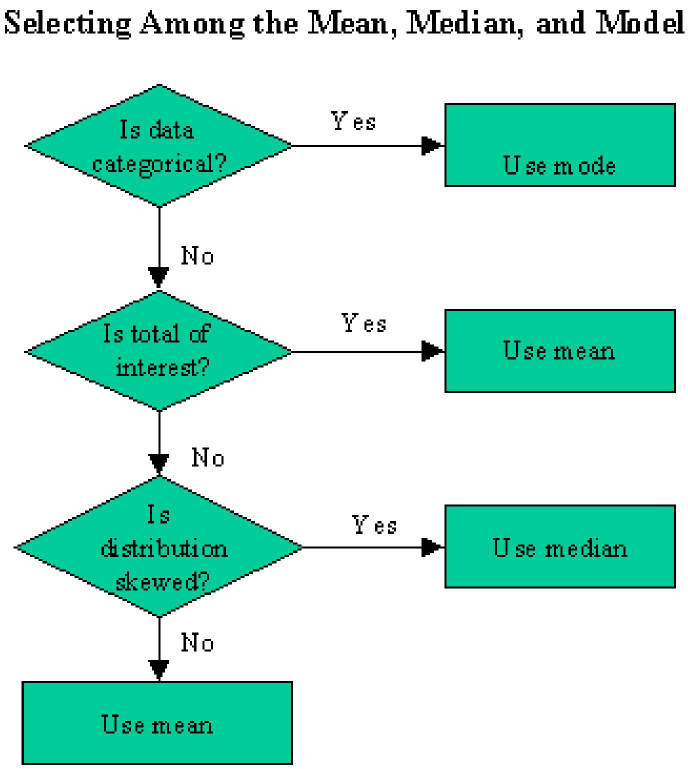


**Figure 2-1**
**Flow Chart for Selecting Appropriate Index**

The first consideration is the type of data, if the variable is categorical; the mode is the single measure that best describes that data.

The second consideration in selecting the index is to ask whether the total of all observations is of any interest. If the answer is yes, then the mean is the proper index of central tendency.

If the total is of no interest, then depending on whether the histogram is symmetric or skewed, one must use either mean or median, respectively.

### *Quality of a Sample: Measures of Dispersion*

Average by itself is not a good indication of quality. You need to know the variance to make any educated assessment. As an example, consider the dilemma of the six-foot tall statistician who drowned in a stream that had an average depth of three feet. While the average depth was three feet, there were several areas in the stream where the depth was much greater than six feet. There are statistical procedures for describing the nature and extent of differences among the information in the distribution. A measure of variability is generally reported with a measure of central tendency.

Statistical measures of variation are numerical values that indicate the variability inherent in a set of power quality data measurements. Note that a small value for a measure of dispersion indicates that the data are concentrated around the mean; therefore, the mean is a good representative of the data set. On the other hand, a large measure of dispersion indicates that the mean is not a good representative of the data set. In addition, measures of dispersion can be used when you want to compare the distributions of two or more sets of data. Quality of a data set is measured by its variability: larger variability indicates lower quality. That is why high variation makes a plant production manager very worried. A statistician's job is to measure the variation and, if it is too high and unacceptable, then it is the job of the technical staff to fix the process.

The decision situations with flat uncertainty have the largest risk. For simplicity, consider the case when there are only two outcomes, one with probability of $p$. Then, the variation in the outcomes is $p(1-p)$. This variation is the largest if we set $p = 50\%$, that is, equal chance for each outcome. In such a case, the quality of information is at its lowest level. Remember, quality of information and variation is inversely related. The larger the variation in the data, the lower is the quality of the data (i.e., information).

The four most common measures of variation are the range, variance, standard deviation, and coefficient of variation.

The range of a set of power quality observations is the absolute value of the difference between the largest and smallest values in the data set. It measures the size of the smallest contiguous interval of real numbers that encompasses all of the data values. It is not useful when extreme values are present. It is based solely on two values, not on the entire data set. In addition, it cannot be defined for open-ended distributions such as normal distribution.

An important measure of variability is variance. Variance is the average of the squared deviations of each observation in the set from the arithmetic mean of all observations.

$$\text{Variance} = (x_i - \bar{x})^2/(n - 1),\ n \geqslant 2.$$

The variance is a measure of spread or dispersion among values in a data set. Therefore, the greater the variance, the lower the quality of information.

The variance is not expressed in the same units as the observations. In other words, the variance is hard to understand because the deviations from the mean are squared, making it too large for logical explanation. This problem can be solved by working with the square root of the variance, which is called the standard deviation.

Both variance and standard deviation provide the same information, one can always be obtained from the other. In other words, the process of computing a standard deviation always involves computing a variance. Since standard deviation (S) is the square root of the variance, it is always expressed in the same units as the raw data:

$$S = \sqrt{\sum (x_i - \bar{x})^2 / (n-1)}$$

For large data sets (more than 30), approximately 68% of the data will fall within one standard deviation of the mean, 95% will fall within two standard deviations, and 97.7% (or almost 100%) will fall within three standard deviations from the mean.

Standard error is a statistic indicating the accuracy of an estimate. That is, how different the estimate (such as $\bar{X}$) is from the population parameter (such as m). It is, therefore, the standard deviation of a sampling distribution of the estimator such as $\bar{X}$'s.

Coefficient of Variation (CV) is the relative deviation with respect to size $\bar{X}$:

$$CV = S / \overline{X}$$

CV is independent of the unit of measurement. In estimation of a parameter when CV is less than 10%, the estimate is assumed acceptable. The inverse of CV, namely 1/CV, is called the Signal-to-noise Ratio. The coefficient of variation is used to represent the relationship of the standard deviation to the mean, telling how much representative the mean is of the numbers from which it came. It expresses the standard deviation as a percentage of the mean, i.e., it reflects the variation in a distribution relative to the mean.

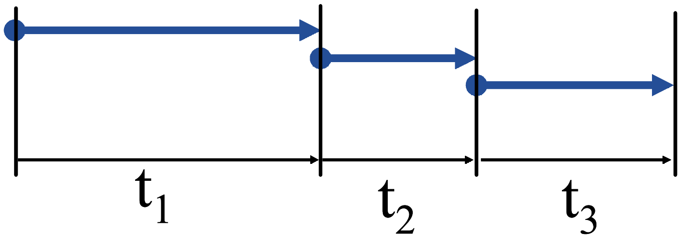
### *Frequency Distribution*

One of the most common ways to describe a single variable is with a frequency distribution. Depending on the particular variable, all of the data values may be represented, or you may group the values into categories first (e.g., with age); it would usually not be sensible to determine the frequencies for each value. Rather, the values are grouped into ranges and the frequency determined.

Frequency distributions can be depicted in two ways, as a table or as a graph. (This type of graph is often referred to as a histogram or bar chart.) Grouped data is derived from raw data and it consists of frequencies (counts of raw values) tabulated with the classes in which they occur. The Class Limits represent the largest (Upper) and lowest (Lower) values that the class will contain.

## Time-to-Failure and Power Quality Data

For this report, a power quality event is broadly defined as any electrical disturbance, whether a voltage sag or surge, that causes a piece of electrical equipment to not function as intended by the end user. The power quality data is then defined as the raw data that is collected from one of many types of power quality monitoring systems and stored in a computer database for further analysis. This report discusses one method for analyzing power quality data using a statistical analysis technique based on the Weibull distribution. For this analysis, the power quality event of interest is not the voltage sag or surge, but the time between events. It is assumed that the magnitude of power quality event, for example, 70 percent voltage sag, is irrelevant to the discussion. The power quality event of interest is the time between power quality events and not the severity of each event. If a power quality event occurs that causes the electrical equipment to malfunction, the event is by definition a power quality event. The statistical analysis discussion in this report will therefore be limited to the time between power quality events or time-to-failure between power quality events. Statisticians sometimes refer to time-to-failure as “life data.” Figure 2-2 shows graphically the time-to-failure for three power quality events. For example, in Figure 2-2, it shows that a power quality event occurred 3.4 weeks from the start of the study, the second event occurred 1.2 weeks after the first event, and the final event occurred 2.1 weeks after the second event.



| Period | Time (weeks) |
|---|---|
| 1 | 3.4 |
| 2 | 1.2 |
| 3 | 2.1 |

**Figure 2-2**
**Time-to-Failure for Power Quality Events**

For multiple power quality events, the time-to-failure or time-to-event can be displayed on a frequency chart resulting in a frequency distribution as shown in Figure 2-3. The data in Figure 2-3 is actual data collected from a power quality monitor over a several month time period. The number of events is displayed on the vertical axis and the time-to-event is the horizontal axis.

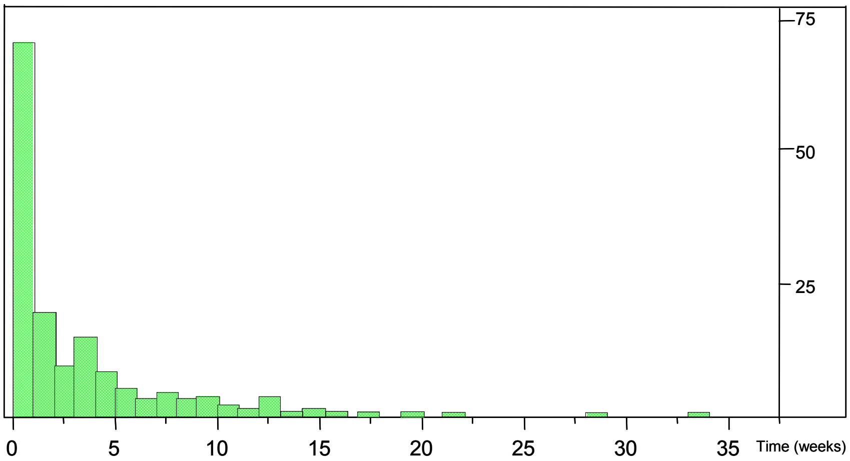


**Figure 2-3**
**Time-to-Failure for Power Quality Events**

## Weibull Distribution

The Weibull distribution is often used to describe the time-to-failure for mechanical and electrical equipment. These can be light bulbs, capacitors, disk drives, ball bearings, etc. When a number of parts are put on test, they do not all fail at the same time. (If they do, you might wonder if something went wrong.) Usually, there is some spread in the failure times.

While the normal distribution is a very handy tool in describing all sorts of different data, it does not work well in describing the statistical distribution associated with the time-to-failure. One reason for this is that the normal distribution allows some observations to be negative. When you life test something, you know it did not fail before time t = zero. Therefore, the normal distribution will not work.

If parts fail according to a Weibull distribution, the probability that any single part will fail at a particular time t is

$$F(t) = 1 - \exp[-(t/a)b]$$

where a is called the scale parameter, b is called the shape parameter, and F is called the cumulative distribution function. If we knew a and b, then we could plug them into the above formula and calculate F (the probability of failure) at any time t.

The parameters a and b can be estimated from the data. An example will make this clear. Suppose some light bulbs are life tested and fail at the following times: 270, 289, 290, 292, 293, 296, 310, 313, 339, and 345 hours. The equation for F(t) can be turned into a regression equation as follows:

$$F(t) = 1 - \exp[-(t/a)b]$$

$$\ln[(1 - F(t)] = -(t/a)b$$

$$\ln(-\ln[1 - F(t)]) = b\ln(t) - b\ln(a)$$

or

$$Y = mX + c$$

where: $Y = \ln(-\ln[1 - F(t)])$, $m = b$, $X = \ln(t)$, and $c = -b\ln(a)$.

"ln" means the natural log. Now we have the more familiar equation for a straight line with slope m and y-intercept c. The next step is to estimate F(t). Note that the failure times above are ranked from lowest to highest. There are 10 failure times and each one can be assigned a rank of 1, 2, 3, etc. The probability of failure at a particular time t, F(t), can be estimated roughly by the rank of the failure time divided by the sample size, in this case, 10. Table 2-1 below shows the Weibull life data for this example.

**Table 2-1**
**Weibull Life Data**

| Rank | failure time | F | Y<br>ln(-ln[1 – F(t)]) | X<br>ln(t) |
|---|---|---|---|---|
| 1 | 270 | 0.1 | -2.250 | 5.598 |
| 2 | 289 | 0.2 | -1.500 | 5.666 |
| 3 | 290 | 0.3 | -1.031 | 5.670 |
| 4 | 292 | 0.4 | -0.672 | 5.677 |
| 5 | 293 | 0.5 | -0.367 | 5.680 |
| 6 | 296 | 0.6 | -0.087 | 5.690 |
| 7 | 310 | 0.7 | 0.186 | 5.737 |
| 8 | 313 | 0.8 | 0.476 | 5.746 |
| 9 | 339 | 0.9 | 0.834 | 5.826 |
| 10 | 345 | 1 | - | - |

The last row has no data since ln(0) isn't defined. Plotting this data yields:

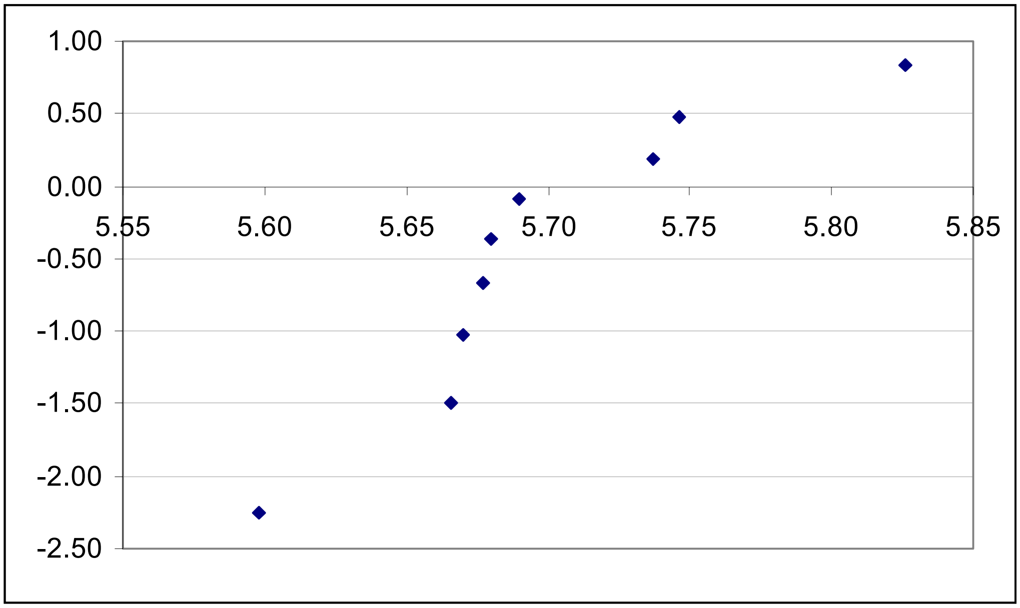


**Figure 2-4**
**Plot of Time-to-Failure Data**

Using regression software, the slope of the best-fit line in Figure 2-4 is estimated to be 14.2, and the intercept is - 81.2. From above, the slope is an estimate of *b* and *a* and is found to be 309.5. Using another technique called "maximum likelihood," the estimates of *a* and *b* are 13.6 and 314.5, respectively.

If you tested thousands of parts and made up a histogram of the life times, it would look like a slightly "lop-sided" bell curve.

Another interesting point about the Weibull distribution is the shape parameter *b* can also tell you something about the failure rate. It turns out that when $b < 1$, the failure rate is decreasing, but when $b > 1$, the failure rate increases. Suppose your life data come from a Weibull distribution and you find that $b < 1$. That means that as time goes on, the failure rate becomes smaller. This might be because you have defects in some of your parts causing high failure rates at the beginning (so-called infant mortality) and by using a power quality mitigation device, the number of power quality events is reduced per unit time compared to no mitigation device. As the defective parts die, the failure rate goes down. Alternatively, if $b > 1$, then the failure rate is increasing. As parts start to approach their maximum possible life, they will begin wearing out, causing an increased failure rate. The *a* or scale parameter is approximately equal to the mean-time-to-failure and is equal to the mean-time-to-failure when the shape is equal to one.

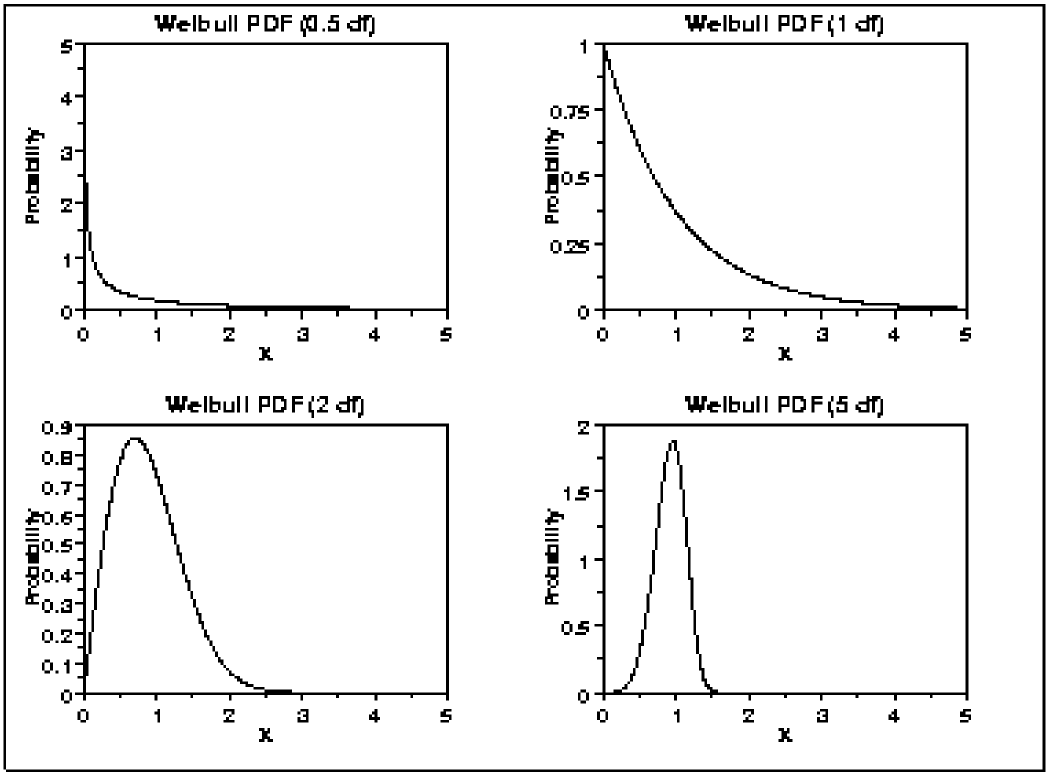


**Figure 2-5**
**Plot of the Weibull Probability Density Function (PDF). Notice the Slightly "Lop-Sided" Bell Curve Look**

Figure 2-6 is a plot of the Weibull cumulative distribution function with the same values of $\gamma$ as the PDF plots in Figure 2-5.

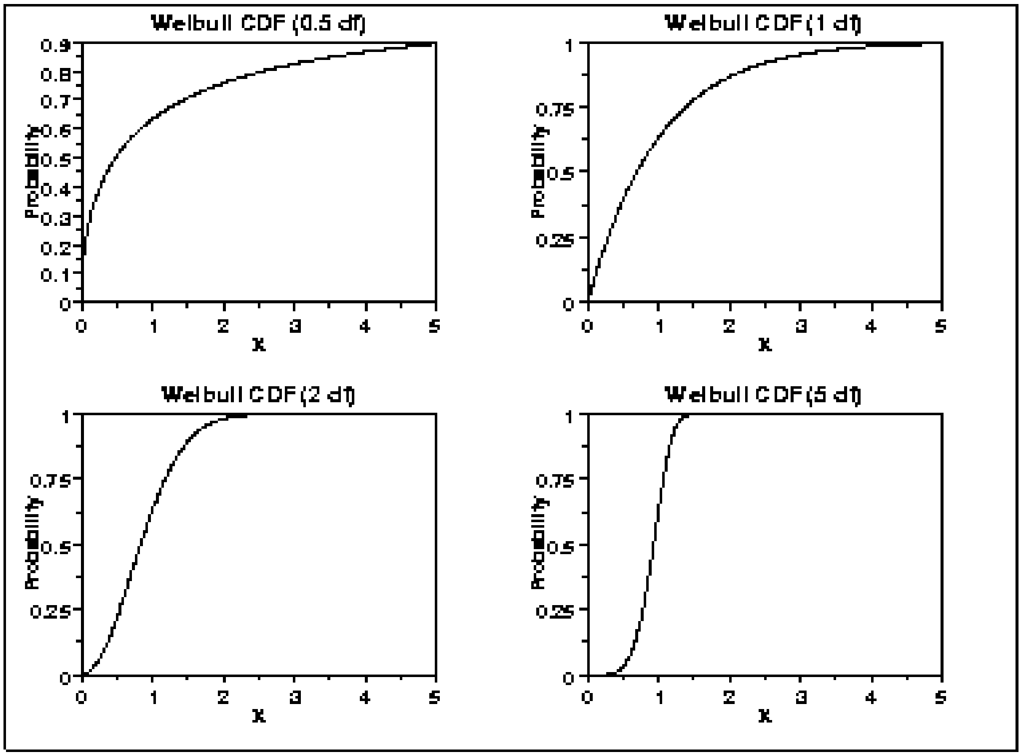


**Figure 2-6**
**Plot of the Weibull Cumulative Density Function (CDF)**

## Developing a Methodology for Statistical Analysis of Power Quality Data

Data are not information! To define statistical data analysis, one must first define statistics. Statistics is a set of methods that is used to collect, analyze, present, and interpret data. Statistical methods are used in a wide variety of occupations and help people identify, study, and solve many complex problems. In the business and economic world, these methods enable decision makers and managers to make informed and better decisions about uncertain situations.

Vast amounts of statistical information are available in today's global and economic environment because of continual improvements in computer technology. To compete globally, managers and decision makers must be able to understand the information and use it effectively. Statistical data analysis provides hands-on experience to promote the use of statistical thinking and techniques to apply to make educated decisions in the business world.

Computers play a very important role in statistical data analysis. Advanced statistical software packages offer extensive data-handling capabilities and numerous statistical analysis routines that can analyze small to very large data statistics. The computer will assist in the summarization of data, but statistical data analysis focuses on the interpretation of the output to make inferences and predictions.

Studying a power quality problem through the use of statistical data analysis involves four basic steps.

1. Defining the problem
2. Collecting the power quality data
3. Analyzing the data
4. Reporting the results

### *Defining the Problem*

An exact definition of the problem is imperative to obtain accurate data about it. It is extremely difficult to gather data without a clear definition of the problem.

### *Collecting the Data*

We live and work at a time when data collection and statistical computations have become very easy, almost to the point of triviality. Paradoxically, the design of data collection, never sufficiently emphasized in a statistical data analysis textbook, has been weakened by an apparent belief that extensive computation can make up for any deficiencies in the design of data collection. One must start with an emphasis on the importance of defining the population about which you are seeking to make inferences; all the requirements of sampling and experimental design must be met.

Designing ways to collect data is an important job in statistical data analysis. Two important aspects of a statistical study are:

- Population - a set of all the elements of interest in a study
- Sample - a subset of the population

Statistical inference refers to extending your knowledge obtained from a random sample from a population to the whole population. This is known in mathematics as an Inductive Reasoning. That is, knowledge of whole from a particular. Its main application is in hypotheses testing about a given population.

The purpose of statistical inference is to obtain information about a population from information contained in a sample. It is just not feasible to test the entire population, so a sample is the only realistic way to obtain power quality data because of the time and cost constraints. Data can be either quantitative or qualitative. Qualitative data are labels or names used to identify an attribute of each element. Quantitative data are always numeric and indicate either how much or how many.

For the purpose of statistical data analysis, distinguishing between cross-sectional and time-series data is important. Cross-sectional data are data collected at the same or approximately the same point in time. Time series data are data collected over several time-periods.

Data can be collected from existing sources or obtained through observation and experimental studies designed to obtain new data. In an experimental study, the variable of interest is identified. Then one or more factors in the study are controlled so that data can be obtained about how the factors influence the variables. In observational studies, no attempt is made to control or influence the variables of interest. A survey is perhaps the most common type of observational study.

### Analyzing the Data

Statistical data analysis divides the methods for analyzing data into two categories: exploratory methods and confirmatory methods. Exploratory methods are used to discover what the power quality data seems to be saying by using simple arithmetic and easy-to-draw pictures to summarize data. Confirmatory methods use ideas from probability theory in the attempt to answer specific questions. Probability is important in decision making because it provides a mechanism for measuring, expressing, and analyzing the uncertainties associated with future power quality events.

### Reporting the Results

Through inferences, an estimate or test claims about the characteristics of a population can be obtained from a sample. The results may be reported in the form of a table, a graph or a set of percentages. Because only a small collection (sample) is examined and not an entire population, the reported results must reflect the uncertainty through the use of probability statements and intervals of values.

To conclude, a critical aspect of managing any organization is planning for the future. Good judgment, intuition, and an awareness of the state of the economy may give a manager a rough idea or "feeling" of what is likely to happen in the future. However, converting that feeling into a number that can be used effectively is difficult. Statistical data analysis helps managers forecast and predict future aspects of a business operation. The most successful managers and decision makers are the ones who can understand the information and use it effectively.

Managers need to understand variation for two key reasons: first, so that they can lead others to apply statistical thinking in day-to-day activities and second, to apply the concept for the purpose of continuous improvement.

### Data to Knowledge

The sequence from data to knowledge is: from Data to Information, from Information to Facts, and finally, from Facts to Knowledge.

Data becomes information when it becomes relevant to the decision problem. Information becomes fact when the data can support it. Fact becomes knowledge when it is used in the successful completion of a decision process. Figure 2-7 illustrates the statistical thinking process based on data in constructing statistical models for decision making under uncertainties.

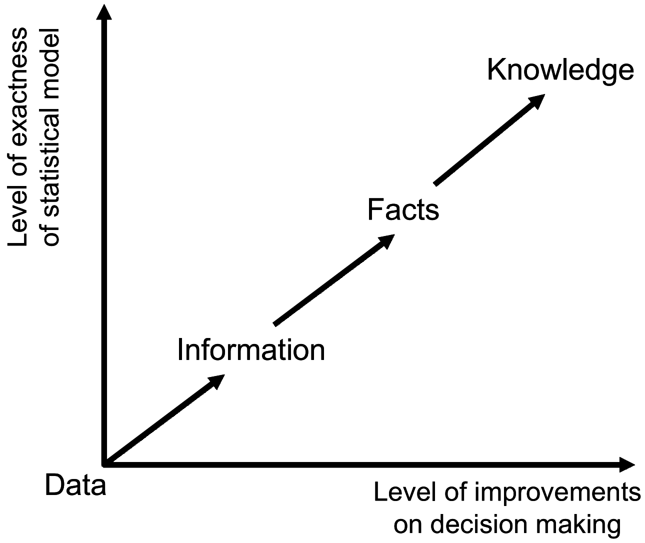


**Figure 2-7**
**Statistical Analysis Process**

Statistics arose from the need to place knowledge on a systematic evidence base. This required a study of the laws of probability, the development of measures of data properties and relationships, and so on. Knowledge is more than knowing something technical. Knowledge needs wisdom, and wisdom comes with age and experience. Wisdom is about knowing how something technical can be best used to meet the needs of the decision maker. Wisdom, for example, creates statistical software that is useful, rather than technically brilliant.

Statistics is a science assisting you to make decisions under uncertainties (based on some numerical and measurable scales). The decision-making process must be based on data, neither on personal opinion nor on belief.

Once data is collected, statistical and analytical tools can be used to make informed decisions as shown in Figure 2-8.

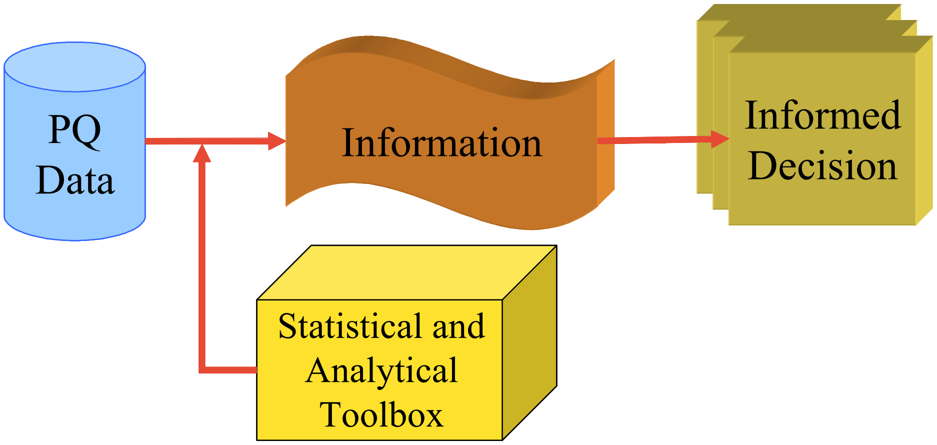


**Figure 2-8**
**Statistical Analysis Process**

## Bibliography

Walpole, Ronald E.; and Myers, Raymond H., *Probability and Statistics for Engineers and Scientists, 2nd Edition*: MacMillian Publishing Company, New York, 1978.

Myers, Raymond H., *Classical and Modern Regression with Applications*: PWS-KENT Publishing Company, Boston, 1990.

Spiegel, Murray R., Theory and Problems of Probability and Statistics: McGraw-Hill, Inc., New York, 1992.

# 3

# DETERMINISTIC LIFE-CYCLE COST ANALYSIS

The last chapter discussed statistical analysis of power quality data. This chapter discusses the deterministic analysis of the life-cycle cost for power quality mitigation hardware. A detailed example is included.

## Life-Cycle Cost Analysis and Application Methodology

This chapter describes how to conduct a life-cycle cost (LCC) analysis based on the critical aspects of a power quality mitigation hardware system. Power quality improvement alone may not be sufficient to justify investment in such a system. Additional items such as reliability improvement, productivity enhancement, and environmental benefits, should be included as part of the power quality mitigation hardware investment decisions.

Many power quality mitigation systems have been planned, designed, produced, and operated with little concern for their cumulative life-cycle cost. Although different facets of costs have been considered in the development of these electrical systems, the costs have often been viewed in a fragmented manner. The costs associated with research, design, testing, construction, production, operations, maintenance, and support have been considered as independent costs and typically addressed only at their specific life-cycle stage, not viewed on an integrated basis.

Experience has indicated that a large portion of the total cost associated with a power quality mitigation system is the direct result of activities associated with the operation and support of these systems, while the commitment of these costs is based on decisions made in the early stages of the system life cycle, i.e., research, design, and procurement. However, the various costs associated with the different phases of the power quality mitigation equipment life cycle are all interrelated. Thus, in addressing the economic aspects of a power quality mitigation system, one must look at total cost in the context of the overall life cycle, particularly during the early stages of conceptual design and advanced system planning. Life-cycle costs, when included as an important and essential parameter in the selection of a power quality mitigation system, provide the opportunity to select the optimal power quality mitigation equipment. The objective is to allow selection, from a set of feasible alternatives, of the equipment that provides the most economic and feasible solution. Throughout this chapter, the term “system” will refer to a “power quality mitigation hardware system.”

The recent combination of economic trends, unexpected cost growth experienced for many power quality mitigation hardware systems, continuing reduction in buying power, budget limitations, increased competition, and so on, has created an awareness and interest in total system cost. Not only are the acquisition costs associated with new systems rising, but also the costs of operating and maintaining systems already in use are increasing at alarming rates.

It has been noted by several cost analysts that the cost growth due to these various causes alone typically range from 5 to 10 times the rate of inflation.

At a time when considerable system cost growth is being experienced, budget allocations for many categories of systems are decreasing from year to year. The net result is that less money is available for acquiring and operating new systems and in maintaining and supporting the systems that are already in use. The available funds for projects are decreasing at a rapid rate when inflation and cost growth are considered. The current economic situation is further complicated by additional problems related to the determination of the optimal system configuration cost.

Total system cost is often not visible, particularly those costs associated with system operation and support. The cost visibility problem can be related to the "iceberg effect" illustrated in Figure 3-1 where most of the costs are “below the surface” and not easily visible from the “surface” represented by the horizontal line. One must address both system acquisition cost and other costs. In estimating cost, individual factors are often improperly applied. For instance, costs are identified and often included in the wrong category, variable costs are treated as fixed costs (and vice versa), indirect costs are treated as direct costs, and so on. Existing accounting procedures do not always permit a realistic and timely assessment of total cost. In addition, it is often difficult (if not impossible) to determine costs on a functional basis.

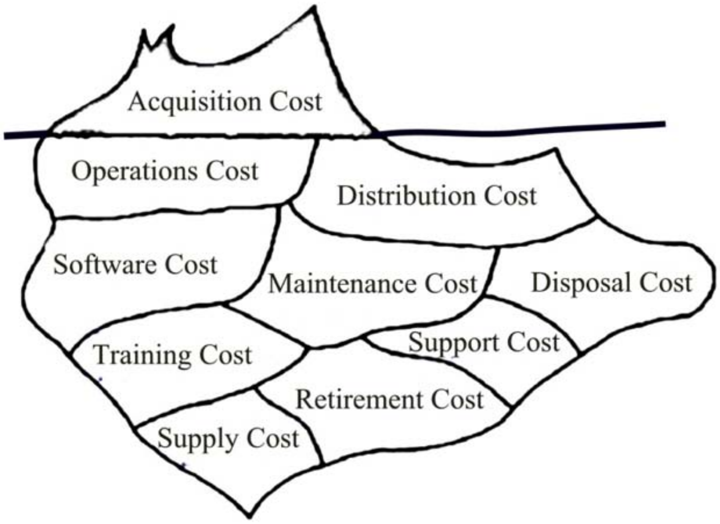


**Figure 3-1**
**Total Cost Visibility**

The current trends of inflation and cost growth, combined with these additional problems, have caused inefficiencies in the utilization of valuable resources. Further, it is anticipated that conditions will become worse unless an increased degree of cost consciousness is assumed in day-to-day activities. Design for economic feasibility must address all aspects of life-cycle cost, not just segments thereof.

Life-cycle cost refers to all costs associated with the system as applied to its life cycle. The life cycle and the major functions associated with each phase are illustrated in Figure 3-2. The life cycle, tailored to the specific system being addressed, forms the basis for life-cycle costing. Typically, life-cycle cost includes the following four cost areas:

1. *Research and development*--initial planning, market analysis, feasibility studies, product research, engineering design, design documentation, software, test and evaluation of engineering models, and associated management functions.

2. *Production and construction*--industrial engineering and operations analysis; manufacturing (fabrication, assembly, and test); facility construction; process development; production operations; quality control; and initial logistic support requirements (e.g., initial consumer support, the manufacture of spare parts, the production of test and support equipment, etc.).

3. *Operation and support*--operations of the system in the field; product distribution (marketing and sales, transportation, and traffic management); and sustaining logistic support throughout the system or product life cycle (e.g., customer service, maintenance activities, supply support, test and support equipment, transportation and handling, technical data, facilities, system modifications, etc.).

4. *Retirement and disposal*--disposal of non-repairable items throughout the life cycle, system/product retirement, material recycling, and applicable logistic support requirements.

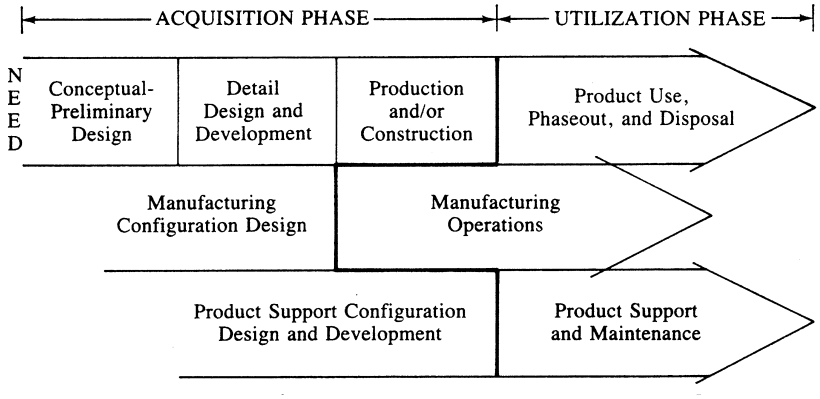


**Figure 3-2**
**Life-Cycle Phases**

Life-cycle cost is determined by identifying the applicable functions in each phase of the life cycle, costing these functions, applying the appropriate costs by function on a year-to-year schedule, and ultimately accumulating the costs for the entire span of the life cycle. The application of life-cycle costing methods in system design and development is realized through the accomplishment of life-cycle cost analyses. A life-cycle cost analysis may be defined as a systematic analytical process of evaluating various alternative courses of action with the objective of choosing the most economical allocation of scarce resources.

Life-cycle costing is employed in the evaluation of alternative system design configurations, alternative production schemes, alternative logistic support policies, and so on. The analysis constitutes a systematic approach employing life-cycle cost figures-of-merit as criteria to arrive at a cost-effective solution. The analysis process is iterative in nature and can be applied to any phase of the system or product life cycle.

## Information Needed for Life-Cycle Costing

To conduct life-cycle costing studies, the required relevant information has to be identified and collected. Before starting a life-cycle costing study, one should seek answers to questions such as:

1. Goal of the estimate.
2. Ground rules and assumptions.
3. Constraints associated with analysis.
4. Involved personnel.
5. Fund limitations.
6. Life-cycle costing time schedule.
7. Estimating procedures.
8. Treatment of uncertainties.
9. Responsibility of the cost analyst.
10. Format of life-cycle cost analysis.
11. Users of the life-cycle cost analysis.
12. Required details of the analysis.
13. Required precision and accuracy of the analysis.
14. Auditing and controlling the life-cycle costing process by the purchaser's management.
15. Auditing and controlling requirements of the life-cycle costing process.

As a minimum, the specific information required for a life-cycle cost study includes:

1. Useful operational life of the item in years.
2. Discount and escalation rates.
3. Annual maintenance cost of the item.
4. Salvage value or disposal cost of the item.

5. Procurement cost of the item.
6. Transportation (delivery) and installation costs.
7. Taxes (e.g., tax benefits from depreciation, investment tax credit).
8. Annual operating cost of the item. This includes:
   - Energy cost,
   - Cost of supplies, labor cost, and
   - Cost of materials insurance.

## Elements and Steps Associated with Life-Cycle Cost Analysis

Before the steps associated with a life-cycle cost analysis are considered, the primary activities of a life-cycle cost analysis should be:

1. Identify the cost drivers.
2. Develop the cost estimating relationships for every component in the life-cycle cost breakdown structure.
3. Develop escalated and discounted life-cycle costs.
4. Define the system’s life cycle.
5. Define activities that generate the system’s ownership costs.
6. Perform sensitivity analyses.
7. Establish constant dollar cost profiles.
8. Determine cause and effect relationships.
9. Establish an accounting cost breakdown structure.

## Data for Life-Cycle Costing

To have effective life-cycle cost estimates of power quality mitigation systems, the availability of reliable historical cost data on similar items or products is vital. The accuracy is often sacrificed when there is a lack of data sources of adequate consistency and quality, or the available data are not compatible with the estimating process. This shortcoming in life-cycle costing can be overcome only by having relevant historical data.

Life-cycle costing databases are useful for performing life-cycle cost studies. An organization involved in life-cycle costing should have a life-cycle costing database or at least have access to such a database. When a new database is being developed, attention must be paid to factors such as:

1. *Flexibility:* The database should have enough flexibility to take care of local conditions effectively.
2. *Ready accessibility:* The database should be readily accessible for retrieval, analysis, and maintenance without the help of computer support specialists.
3. *Comprehensiveness:* The database should be comprehensive enough in scope to include all life-cycle cost decision needs.
4. *Responsiveness:* The database should be dynamic and responsive to varying needs for information.
5. *Size:* The database size should be such that it is not too large to be incomprehensible or too small to be regarded as trivia.
6. *Expansion or contraction capability:* The database should possess expansion or contraction capability to accommodate users' financial resources and level of participation.
7. *Orientation:* The database should be oriented toward quantitative and qualitative factors.
8. *Uniformity:* The database should be uniform to a degree that allows aggregation of adequate samples with similar characteristics for performing reasonable analysis.

At minimum, the life-cycle costing database should include the following:

1. Cost records.
2. Procedural records: operation and maintenance.
3. User pattern records.
4. Descriptive records: hardware and site.

If such data are not available, the vendor(s) offering the power quality mitigation hardware should provide historical data from existing users.

## Development of Cost Profile

In developing a cost profile, there are different procedures that may be followed. The following steps are suggested.

1. Identify all activities throughout the life cycle that will generate costs of one type or another. This includes functions associated with planning, research and development, test

and evaluation, production, construction, product distribution, system/product operational use, logistic support, and so on.

2. Relate each activity identified in Item 1 to a specific cost category in the cost breakdown structure (CBS). All program activities should fall into one or more of the categories in the CBS.
3. Establish the appropriate cost factors in constant dollars for each activity in the CBS where constant dollars reflect the general purchasing power of the dollar at the time of decision (i.e., today). Relating costs in terms of constant dollars will allow for a direct comparison of activity levels from year to year prior to the introduction of inflationary cost factors, changes in price levels, economic effects of contractual agreements with suppliers, and so on. This will often cause some confusion in the evaluation of alternatives. Also, using constant dollars tends to assure consistency in accomplishing comparative studies.
4. Within each cost category in the CBS, the individual cost elements are next projected into the future on a year-to-year basis over the life cycle as applicable. The result should be a cost stream in constant dollars for the activities that are included.
5. For each cost category in the CBS, and for each applicable year in the life cycle, introduce next the appropriate inflationary factors, economic effects of learning curves, changes in price levels, and so on. The modified values constitute a new cost stream and reflect realistic costs as they are anticipated for each year of the life cycle (i.e., expected 2000 costs in 2001, 2002 costs in 2003, etc.). These costs may be used directly in the preparation of future budget requests since they reflect the actual dollar needs anticipated for each year in the life cycle.
6. Next, summarize the individual cost streams by major categories in the CBS and develop a top-level cost profile as shown in Figure 3-3 and Figure 3-4.

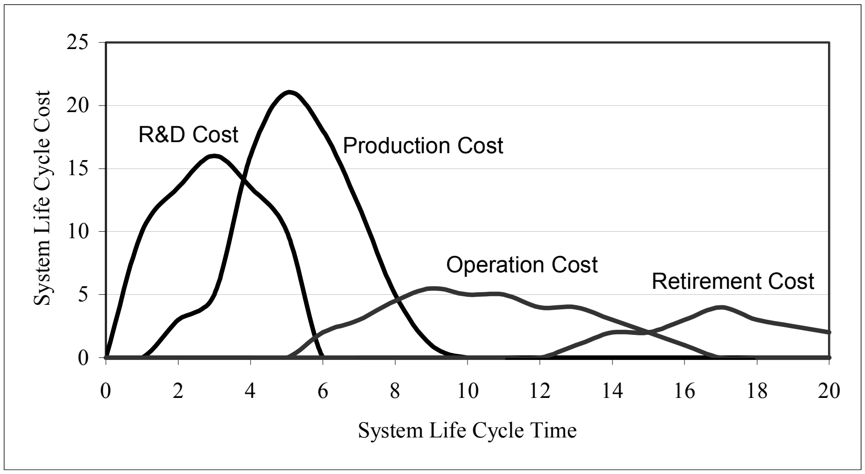


**Figure 3-3**
**Top-Level Life-Cycle Cost Profile**

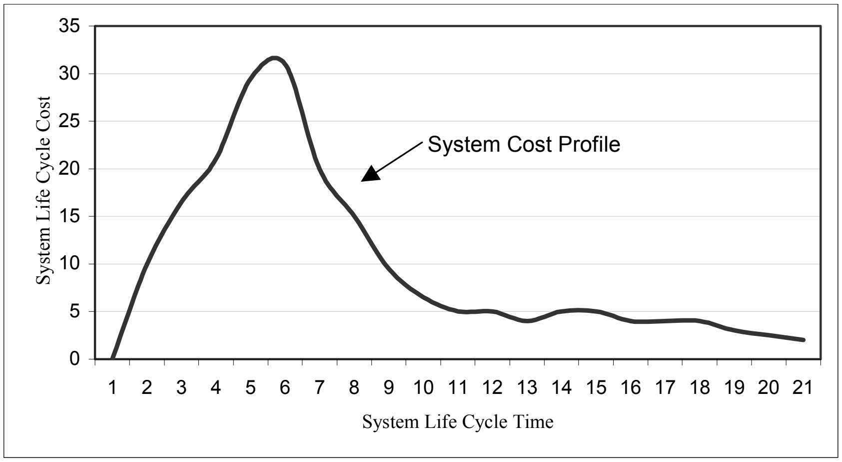


**Figure 3-4**
**Development of Cost Profiles**

As the six steps outlined above demonstrate, it is possible and often beneficial to evaluate the cost stream for individual activities of the life cycle such as research and development, production, operation and support, and so on. Second, these individual cost streams may be shown in the context of the total cost spectrum. Finally, the total cost profile may be viewed from the standpoint of the logical flow of activities and the proper level and timely expenditure of dollars.

When dealing with two or more alternative system configurations, each will include different levels of activity, different design approaches, different logistic support requirements, and so on. No two alternatives will be identical. Thus, individual profiles will be developed for each alternative and ultimately compared on an equivalent basis utilizing the economic analyses techniques discussed later.

## Time Value of Money

The development of a given system requires many decisions to be made. Such decisions evolve from the evaluation of alternative proposals of one type or another. Each proposal considered in the evaluation process represents a potential investment and should be viewed from the standpoint of anticipated revenues (i.e., benefits) and costs that will occur over the designated life cycle.

Since revenues and costs are related to different activities at different points in time over the life cycle, a common point of reference must be assumed so that all alternatives can be compared on an equivalent basis. The flow of revenues and costs, having time value, for each alternative being considered, must be equated to a common reference point. This reference point is generally in the present time (or now) when decisions that have a significant impact on the future are made; thus, all future revenues and costs for each year in the life cycle may be discounted to their present equivalent amounts. For non-revenue generating systems, the cost savings or benefit derived from the system is used.

In the evaluation of alternatives, such as those illustrated by the cost profiles in Figure 3-5, all costs must be converted to a common point in time to view these alternatives on an equivalent basis.

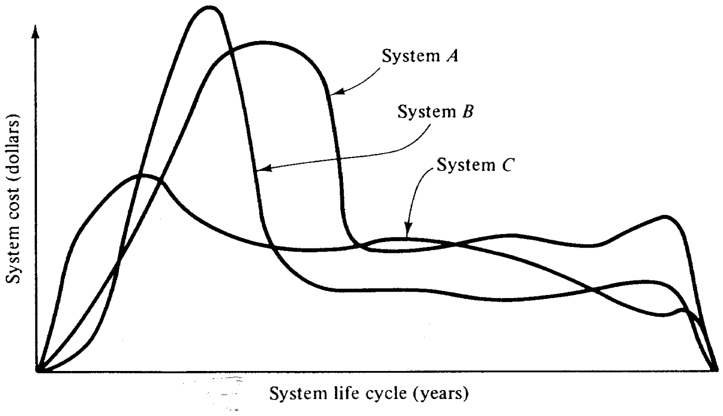


**Figure 3-5**
**Life-Cycle Cost Profiles of Alternatives**

### *Example: Summarization of Costs*

Table 3-1 reflects a summary listing of costs and a breakdown showing the percent contribution of each major category to the total. The categories listed are those indicated in the cost breakdown structure. These costs constitute the summation of individual costs for each category and for each year in the life cycle, and are discounted to the present value. The application of discounting here assumes that the configuration reflected by the cost summary is being compared with other configurations. If the purpose is to view system cost from a budgetary standpoint (representing only this configuration), then undiscounted costs could be presented.

**Table 3-1**
**Life-Cycle Cost Summary**

| Cost Category | Cost($) | Contribution(%) |
|---|---|---|
| 1. Research and development ($C_{R}$) | $ 130,579 | 10.3 |
| a. System/product management ($C_{RM}$) | 19,016 | 1.5 |
| b. Product planning ($C_{RP}$) | 2,536 | 0.2 |
| c. Product research ($C_{RR}$) | 6,339 | 0.5 |
| d. Engineering design ($C_{RE}$) | 68,459 | 5.4 |
| e. Design documentation ($C_{RU}$) | 10,142 | 0.8 |
| f. System/product software ($C_{RS}$) | 8,874 | 0.7 |
| g. System/product test and evaluation ($C_{RT}$) | 15,213 | 1.2 |
| 2. Production and construction cost ($C_{P}$) | 574,296 | 45.3 |
| a. Production/construction management ($C_{PA}$) | 10,021 | 0.8 |
| b. Industrial engineering and operations analysis ($C_{P}$,) | 13,945 | 1.1 |
| c. Manufacturing ($C_{PM}$) | 448,908 | 35.4 |
| d. Construction (Cpc) | 67,191 | 5.3 |
| e. Quality control (Cpc) | 11,411 | 0.9 |
| f. Initial logistics support ($C_{PL}$) | 22,820 | 1.8 |
| 3. Operation and maintenance support cost (Co) | 505,836 | 39.9 |
| a. System/product life-cycle management ($C_{OA}$) | 19,016 | 1.5 |
| b. System/product operations ($C_{OO}$) | 40,568 | 3.2 |
| c. System/product distribution ($C_{OO}$) | 111,563 | 8.8 |
| d. System/product maintenance ($C_{OM}$) | 192,699 | 15.2 |
| e. Inventory spares and material support ($C_{OL}$) | 59,585 | 4.7 |
| f. Operator and maintenance training ($C_{OT}$) | 70,995 | 5.6 |
| g. Technical data (Cop) | 11,410 | 0.9 |
| h. System/product modifications ($C_{OR}$) | - | - |
| 4. Retirement and disposal cost ($C_{D}$) | 57,049 | 4.5 |
| Grand total (C) | $1,267,760 | 100.0% |

Referring to Table 3-1, categories where the percent contribution is relatively high should be broken down into the different subcategories included therein, and the high-cost areas should be investigated further to determine the cause(s). The breakout of costs in this fashion not only allows for a comparison of different activities for a given system or product configuration, but also facilitates the direct comparison with other systems where costs are presented in a like manner.

## Present Worth Formula for Life-Cycle Cost Analysis

This section presents several formulas for performing life-cycle cost analysis. The present worth of a future sum of money is $PW = PA = \frac{FW}{(1+i)^m}$ where *PW* is the present worth, *FW* is the future worth, *i* is the compounded interest rate, and *m* is the study period for converting *FW* to *PW*.

Example 3.1

Assume that the total cost of equipment at the end of its seven-year operation will be \$80,000. The estimated annual compound interest rate is 10 percent. Compute the present worth of the \$80,000.

Using the equation above for $i$ = 10%, $m$ = 7 years, and $FW$ = \$80,000, we get $PW = \frac{\$80{,}000}{(1+0.1)^7}$ = \$41,053. Thus, the present value of the total cost is \$41,053.

## Depreciation Methods

Depreciation means the decline in value. There are several causes for the decrease in value. These are as follows:

1. Functional depreciation.
2. Technological depreciation.
3. Physical depreciation.
4. Monetary depreciation

In the case of functional depreciation, a change in demand or service expected from a product during its useful life makes it less worthy from the point of view of its owner, even though it can still carry out its original mission effectively. Technological depreciation is the result of the development of better approaches of carrying out a function, which makes earlier equipment designs uneconomical. In the case of physical depreciation, normal wear and tear of operations slowly reduce the equipment's capability to carry out its specified function. Finally, monetary depreciation is the result of a change in the buying power of money.

To take into consideration the change in the values of the engineering products, the depreciation charges are made during the useful life of such products. When computing taxable income, the depreciation charges can be deducted as operating expense. Three methods for determining depreciation are straight-line method, declining-balance method, and MACRS. Straight-line method is presented below.

The straight-line method assumes the linear decrease in the value of product over time. During the service life of the product, an equal amount of money is charged for depreciation each year.

The yearly depreciation charge is given by $CH_{DY} = \frac{AC - SV}{SL}$ where $AC$ is the acquisition cost of the property, $SV$ is the salvage value of property at the end of its service life, $SL$ is the service life of the property in years, $CH_{DY}$ is the yearly depreciation charge.

The book value of the property at the end of year (*k*) may be obtained from the following equation:

$BVP_k = AC - k(CH_{DY})$ where $BVP_k$ is the book value of the property at the end of year *k*, *k* is the number of years in actual service. Substituting the two equations above yields

$$BVP_k = AC - k\left[\frac{AC - SV}{SL}\right].$$

Example 3.2

Assume that the acquisition cost and the salvage value of power quality mitigation equipment are \$250,000 and \$20,000, respectively. If the equipment's useful life is 10 years and its annual depreciation is constant, calculate the yearly depreciation charge for the equipment.

Substituting the given data in the above equations yields

$$\mathrm{CH_{DY}} = \frac{(\$250{,}000 - \$20{,}000)}{10} = \$23{,}000$$

Thus, the yearly depreciation charge for the equipment is \$23,000.

## Deterministic After-Tax Cash Flow and Life-Cycle Costing

Up to this point, there has been no consideration of income taxes in our discussion of life-cycle costing. However, income taxes do affect the choice among alternatives; therefore, an after-tax study is essential to properly select the optimal system.

Some of the basic differences between before-tax and after-tax studies can be explained in terms of the flow of capital within a typical firm. Funds are generated so that the firm can continue as a going concern from year to year. New debt and equity capital are obtained initially from external sources so that investments in buildings, land, equipment, and working capital can be made and supported. As gross revenues are produced through the sale of goods and/or services, they are reduced by operating expenses in arriving at operating income. Then, operating income has depreciation allowances and interest on debt capital subtracted from it to yield *net income before income taxes,* which is often referred to as *profit before income taxes.* Income taxes are imposed on net income before taxes. Below is an overview of the calculation used to determine the Net Income After Tax (NIAT):

R = gross revenues or cost savings.

E = operating expenses (these are current and tend to be proportional to the extent of business activity, thus can be controlled to a degree) plus interest paid for the use of borrowed capital (which is a financial cost due to the use of borrowed money; this cost does not exist if 100 percent equity capital is employed by a firm).

$d$ = depreciation which is the recognition of cost due to loss in the value of assets such as property, buildings, or equipment, as was explained earlier.

T = income taxes; these are costs to a firm that are obviously not ordinary costs because they depend on profits remaining after expenses are paid or accounted for.

$t$ = effective income tax rate used for computing income taxes.

Two classes of profit are of concern in life-cycle cost studies. The first is profit before income taxes, the other is profit after income taxes. The relationship between net income before taxes (NIBT) and net income after taxes (NIAT) is shown below:

$\text{NIBT} = R - (E + d)$ thus, $\text{NIAT} = \text{NIBT} - T$ or

$\text{NIAT} = R - (E + d) - t[R - (E + d)] = (1 - t)[R - (E + d)]$.

Because the interest paid for the use of borrowed capital is included as an expense in the above equation, the profits belong to the owners of equity capital (i.e., stockholders). Thus, profits after income taxes (NIAT) are distributed as cash dividends on preferred stock and common stock, with the remainder going to retained earnings. Once the firm is profitable, funds for further investment and increased working capital are accumulated from retained earnings, depreciation credits, deferred income taxes, and new equity-debt capital.

Investment capital is thus *transformed* into goods and services that the company hopes will result in after-tax profits. However, after-tax profits are usually different in amount from the after-tax cash flows that are produced by a project. If depreciation deductions are added back to net income after taxes, a project's after-tax cash flow (ATCF) in year $k$ can be estimated by $\text{ATCF}_k = \text{NIAT}_k + d_k$.

Note: NIAT (i.e., after-tax profits) is not usually the same as ATCF.

In evaluating the financial performance of the investment, life-cycle cost studies should consider the after-tax *cash flows* that a project produces. The ATCF represents the amount of money that a project or venture contributes to (or drains from) the treasury of a firm. Generally, ATCF is regarded as a better indicator of profitability than NIAT. This is because a firm can go bankrupt while reporting profits, but it will remain solvent as long as its cash (i.e., ATCF) and liquidity positions are strong.

Another difference between before-tax and after-tax studies is related to the interest rate (i.e., the Minimum Alternative Rate of Return [MARR]) that is used in performing time value of money calculations. An approximation of the before-tax MARR requirement, which includes the effect of income taxes, for studies involving only before-tax cash flows can be obtained from the following relationship:

(Before-tax MARR)[(1 - effective income tax rate)] = after tax MARR

In practice, it is essential to make after-tax analyses in any income tax-paying organization. After-tax analyses can be performed by exactly the same methods (PW, IRR, etc.) as before-tax analyses. The only difference is that after-tax cash flows must be used in place of before-tax cash flows, and the calculation of a measure of merit is based on an after-tax MARR.

The mystery behind the sometimes complex computation of income taxes is reduced when one recognizes that income taxes paid are just another type of expense, while income taxes saved (through business deductions, expenses, or direct tax credits) are identical to other kinds of reduced expenses (e.g., savings).

The basic concepts underlying federal and state income tax regulations that apply to most economic analyses of capital investments generally can be understood and applied without difficulty.

## Taxable Income of Business Firms

At the end of each tax year, a corporation must calculate its net (i.e., taxable) before-tax income or loss. Several steps are involved in this process, beginning with the calculation of gross income. Gross income represents the gross profits from operations (revenues from sales minus the cost of goods sold) plus income from dividends, interest, rent, royalties, and gains (or losses) on the exchange of capital assets. The corporation may deduct from gross income all ordinary and necessary operating expenses (including interest) except capital expenditures. Deductions for depreciation are permitted each tax period as a means of consistently and systematically recovering capital. Consequently, allowable expenses and depreciation deductions may be used to determine taxable income as shown below:

Taxable income = gross income - all expenses except capital expenditures - depreciation deductions

Taxable income is often referred to as net income before taxes and, when income taxes are subtracted from it, the remainder is called the net income after taxes. There are two types of income for tax computation purposes: ordinary income (and losses) and capital gains (and losses).

A net operating loss (NOL) results when a corporation's deductions and expenses exceed its gross revenues. The existence of NOLs carries important ramifications for tax planning. An NOL may be carried back 3 years and forward 15 years to offset taxable income in other years. Utilizing an NOL may either offset the current year's tax liability or result in a tax refund. Generally, the carryback is first charged to the third preceding year, then to the second preceding year, and, if applicable, to the previous year. The corporation may elect to forego the carryback and only carry forward for 15 years. The election to forego the carryback proves valuable when, for example, future income tax rates exceed the prior year's rates.

Example 3.3

In 2000, a company generates $1,500,000 of gross income and incurs operating expenses of $800,000. Interest payments on borrowed capital amount to $48,000. The total depreciation deductions in 2000 equal $114,000. (a) What is the taxable income (NIBT) of this firm? (b) If interest expenses had been $590,000, what would the NOL have been?

Solution

(a) Based on the above equation, this company's taxable income in 2000 would be

$$\$1,500,000 - \$800,000 - \$48,000 - \$114,000 = \$538,000$$

(b) NOL = $1,500,000 - $800,000 - $590,000 - $114,000 = $4,000

## General Procedures for Making Deterministic After-Tax, Life-Cycle Analyses

After-tax economic analyses can be performed by exactly the same methods as before-tax analyses. The only difference is that ATCFs are used in place of before-tax cash flows (BTCFs) by including expenses (or savings) due to income taxes, then making equivalent worth calculations using an after-tax MARR. The tax rates and governing regulations may be complex and subject to changes, but once those rates and regulations have been translated into their effect on ATCFs, the remainder of the after-tax analysis is relatively straightforward.

To formalize the procedure described in previous sections for determining NIBT, NIAT, and ATCF, the following notation and equations are restated. For any given year *k* of the study period, $k = 0, 1, 2, \ldots, N$, let

$R_k$ = revenues from the project; this is the positive cash flow from the project during period *k*.

$E_k$ = cash outflows during year *k* for deductible expenses and interest.

$d_k$ = for all non-cash, or book, costs during year *k*, such as depreciation and depletion.

t = effective income tax rate on ordinary income (federal, state, and other); t is assumed to remain constant during the study period.

$T_k$ = income taxes paid during year *k*.

$ATCF_k$ = ATCF from the project during year *k*.

Because the NIBT (i.e., taxable income) is $(R_k - E_k - d_k)$, the ordinary income tax liability when $R_k > (E_k + d_k)$ is computed using $T_k = -t(R_k - E_k - d_k)$.

The NIAT is then simply taxable income (i.e., net income before taxes) minus the tax liability amount determined by:

$$NIAT_k = \underbrace{R_k - E_k - d_k}_{taxable\ income} - \underbrace{t(R_k - E_k - d_k)}_{income\ taxes}$$

or

$NIAT_k = (R_k - E_k - d_k)(1 - t)$

The ATCF associated with a project equals the NIAT plus non-cash items such as depreciation:

$\mathrm{ATCF}_k = \mathrm{NIAT}_k + d_k \quad = (R_k - E_k - d_k)(1 - t) + d_k$

or

$\mathrm{ATCF}_k = (1 - t)(R_k - E_k) + td_k$

Tabular headings to facilitate the computation of after-tax cash flow is shown below:

| *Year* | *(A) BTCF* | *(B) Depreciation* | *(C) = (A) – (B) Taxable Income* | *(D) = -t(C) Cash Flow for Income Taxes* | *(E) = (A) + (D) ATCF* |
|---|---|---|---|---|---|
| k | $R_k - E_k$ | $d_k$ | $R_k - E_k - d_k$ | $-t(R_k - E_k - d_k)$ | $(1 - t)(R_k - E_k) + td_k$ |

Column A consists of the same information used in before-tax analyses, namely, the cash revenues (or savings) less the deductible expenses. Column B contains depreciation that can be claimed for tax purposes. Column C is the taxable income, or amount subject to income taxes. Column D contains the income taxes paid (or saved). Finally, Column E shows the ATCFs to be used directly in after-tax economic analyses.

Example 3.4

In 2000, if the revenue from a project is \$10,000, out-of-pocket expenses are \$4,000, and depreciation claimed for income tax purposes is \$2,000, what is the ATCF when $t = 0.40$? What is the NIAT?

$\mathrm{ATCF}_{2000} = (1 - 0.4)(\$10{,}000 - \$4{,}000 - \$2{,}000) + \$2{,}000 = \$4{,}400$

The depreciation contributes a credit of \$2,000 to the after-tax cash flow in 2000. The NIAT is \$4,400 - \$2,000 = \$2,400.

The ATCF attributable to depreciation (a tax savings) is $td_k$ in year $k$. After income taxes, an expense becomes $(1 - t)E_k$ .

The net present value (NPV) is considered the standard economic investment measure used to equate different points in time. Other common economic investment criteria include: future value, annual equivalent, internal rate of return, Solomon's average rate of return, modified internal rate of return, aggregate benefit/cost (B/C) ratio, netted B/C ratio, Lorie-Savage ratio, and project balance (PB). This paper presents an example of the PB method as the preferred method in evaluating life-cycle cost analysis investment criteria for power quality mitigation equipment.

The NPV is calculated by computing the present value of the cash flow projections based on a rate of interest. The NPV expression is:

$$NPV(i,n) = \sum_{n=0}^{N} F_n (1+i)^{-n}$$

Where $i$ is the MARR per period, $n$ represents time and is measured in discrete compounding periods. $F_n$ is the project cash flow projection, and N is the project evaluation period.

If the NPV is greater than \$0, the project generates a surplus of funds and should be accepted. Thus, the firm should accept any project for which the present value NPV($i, n$) is positive, and reject any project for which the NPV($i, n$) is negative. The net present value can also be viewed as the cumulative sum of all cash flows generated by the project in excess of the investment and discounted at the firm's MARR.

The PB is defined as the net equivalent amount of investment remaining during the life of the project. The PB is calculated as $PB(i)_n = (1 + i)PB(i)_{n-1} + F_n$, where $PB(i)_0 = F_0$. If the PB is greater then \$0, the project recovers the initial investment plus any interest owed and has a profit at the end of the project life. Therefore, if the PB is greater then \$0, the project should be accepted. In addition, the PB provides quantitative information about four important characteristics associated with investment decisions. The project balance shows the future value, discounted payback period, area of negative balance, and the area of positive balance.

## Example – Deterministic Project Balance Method

As an example of life-cycle cost analysis using the project balance method, consider a firm that is evaluating an investment in new power quality mitigation equipment. The problem identified by the firm is frequent voltage sags causing excessive down time, scrap, rework, and general loss of production. After reviewing the firm's annual production report and the anticipated demand for its product in the next few years, the firm's management team has decided to evaluate the investment in a new power quality mitigation device that will help reduce the number of voltage sags. This new power quality mitigation equipment is anticipated to eliminate voltage sags from disrupting the sensitive electronic manufacturing equipment, thereby adversely affecting production operations. The analysis will require a one-year needs assessment and vendor selection. The decision objective is to determine if the investment in a new power quality mitigation system is feasible and profitable. Details of the project benefits and costs are shown below.

- *Needs Analysis Costs:* The engineering staff estimates that the cost to perform the initial needs analysis is \$28,000 which includes labor to review operational performance associated with lost production due to frequent voltage sags that cause the firm's sensitive electronic equipment to malfunction. These costs will be expensed in the year they occur.

- *Benefit:* The benefit of the power quality mitigation equipment includes the reduction of scrap and downtime and increased production. The benefit derived by the installation of the equipment is shown in Table 3-2 for years one through six.
- *System Planning Costs:* System planning costs are estimated to be $20,000. This involves initial planning on how the new power quality mitigation system will be integrated into the manufacturing electrical system and its impact on overall electrical and mechanical infrastructure.
- *System Procurement Costs:* The engineering and procurement staff expenses associated with developing the procurement specification, reviewing the alternative solution offering, and negotiating the final selection is estimated to be a $20,000, one-time expense.
- *Construction Costs:* The construction costs associated with the new power quality mitigation equipment is estimated to be $65,000.
- *Operations and Support Costs:* The recurring operations support costs are shown in Table 3-2. These estimated costs include such items as maintenance, repair, staff training, supplies, storage, rent, overhead charges, and other contingencies.
- *Retirement and Disposal Costs:* The retirement and disposal costs for the power quality mitigation equipment is estimated to be 8.8 percent of the initial capital investment at the end of year six or $49,500.
- *Capital Investment:* The initial capital investment required to purchase the new power quality mitigation equipment is $560,000 and will be depreciated for seven years.
- *Marginal Tax Rate:* 40 percent.
- *Minimum Attractive Rate of Return (MARR):* 15 percent.

The company's six-year benefit and cost statement is shown in Table3-2. The net present value of the investment is $443,944. Since the net present value is positive, the project should be considered. The project balance is shown graphically in Figure 3-6. Again, since the sixth year project balance is positive, the project should be considered.

**Table 3-2**
**Benefit and Expenses for Project Balance Example**

| Year | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Benefit | $0 | $295,000 | $412,000 | $495,000 | $535,000 | $589,000 | $550,000 |
| Needs Analysis | $28,000 | $0 | $0 | $0 | $0 | $0 | $0 |
| System Planning R&D | $20,000 | $0 | $0 | $0 | $0 | $0 | $0 |
| Procurement | $20,000 | $0 | $0 | $0 | $0 | $0 | $0 |
| Construction | $65,000 | $0 | $0 | $0 | $0 | $0 | $0 |
| Operational | $0 | $25,000 | $35,000 | $95,000 | $53,000 | $28,000 | $36,000 |
| Total Expenses | $133,000 | $25,000 | $35,000 | $95,000 | $53,000 | $28,000 | $36,000 |
| Depreciation | $0 | $89,286 | $89,286 | $89,286 | $89,286 | $89,286 | $89,286 |
| NIBT | ($133,000) | $180,714 | $287,714 | $310,714 | $392,714 | $471,714 | $424,714 |
| Income Tax (40%) | $53,200 | ($72,286) | ($115,086) | ($124,286) | ($157,086) | ($188,686) | ($169,886) |
| NET INCOME | ($79,800) | $108,429 | $172,629 | $186,429 | $235,629 | $283,029 | $254,829 |
| | | | | | | | |
| CASH FLOW | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| NET INCOME | ($79,800) | $108,429 | $172,629 | $186,429 | $235,629 | $283,029 | $254,829 |
| Depreciation | $0 | $89,286 | $89,286 | $89,286 | $89,286 | $89,286 | $89,286 |
| Initial Investment | ($560,000) | $0 | $0 | $0 | $0 | $0 | $0 |
| Salvage Value | $0 | $0 | $0 | $0 | $0 | $0 | $49,500 |
| Gains Tax | $0 | $0 | $0 | $0 | $0 | $0 | ($19,800) |
| AFTER TAX CASH FLOW | ($639,800) | $197,714 | $261,914 | $275,714 | $324,914 | $372,314 | $373,814 |

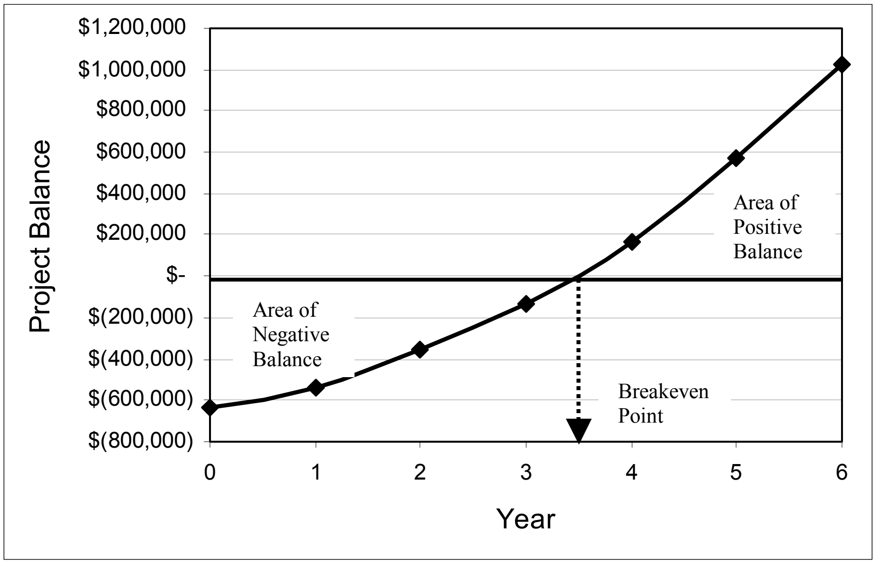


**Figure 3-6**
**Project Balance**

The project balance describes the net equivalent amount of dollars tied up in or committed to the project at each point in time over the life of the project. The project balance is denoted as $PB(i)_n$ where $i$ is the opportunity cost rate (MARR) and $n$ is the period computing the PB.

If PB($i$)$_N$>0, the firm recovers the initial investment plus any interest owed, with a profit at the end of the project. If PB($i$)$_N$=0, the firm recovers only the initial investment plus interest owed and breaks even. If PB($i$)$_N$<0, the firm ends up with a loss by not being able to recover even the initial investment and interest owed. Naturally, the firm should accept a project only if PB($i$)$_N$>0. The net present equivalent value of this project is simply $NPV(i,N) = PB(i)_{Nn}(1+i)^{-N}$ and the future value is the PB at period N. The discounted breakeven point for the project is the point where the PB($i$)$_n$=0. Thus, using the PB method, one can easily derive several important economic interpretations about the project such as net present value, future value, and breakeven point. In addition, one can visually see the amount of capital at risk indicated by the negative area in Figure 3-6.

The project balance shown in Figure 3-6 indicates that the breakeven point is 3.5 years. Initially, the firm has $639,000 of capital exposed to risk. However, in each future year, the benefit is greater than the expense thus reducing the exposed risk. Beginning in year 3.5, the firm recognizes a gain on its capital assuming a 15 percent MARR. A greater MARR would result in extending the breakeven point and exposing more capital to risk. A greater benefit or less expenses in earlier years would have the reverse effect, that is, reducing the amount of exposed capital, thus reducing the breakeven point.

## Bibliography

DeGarmo, E.; Sullivan, W. G.; and Bontadelli, James A., *Engineering Economy*, 9th Edition: MacMillian Publishing Company, New York, 1993.

Dhillon, B. S., *Life Cycle Costing*: Gordon and Breach Science Publishers, New York, 1989.

Graedel, Thomas E., *Streamlined Life-Cycle Assessment*: Prentice-Hall, Inc., New York, 1998.

# 4

# PROBABILISTIC LIFE-CYCLE COST ANALYSIS

## Introduction to Probabilistic Life-Cycle Cost Analysis

One of the most important characteristics of a successful business is the ability to make the right decision when confronted with insufficient information. Traditionally, the decision-making process was based on intuition, habit, and guess. However, with the continued expansion of the global economy, increased competition, and customer demand for quality, the era of intuitive decision making is over. Today's business environment requires a disciplined approach to decision making.

A structured and systematic decision-making process has two major roles. First, it offers a disciplined approach to the decision-making process, and second, it provides a set of techniques for evaluating the worth of alternative decisions. Thus, a structured decision-making process involves decomposing and structuring the problem, assessing the uncertainties and values of possible outcomes, and determining the optimal strategy.

Specifically, the decision process consists of the following steps:

1. Define the problem
2. Specify criteria
3. List decision alternatives
4. Analyze alternatives
5. Select optimal alternative
6. Evaluate post-optimal analysis
7. Implement decision

Step 4, analyze alternatives, carefully studies each alternative to determine the option's ability to attain the criteria or objective listed in Step 2. This involves acquiring and analyzing all relevant information to determine the likelihood of meeting the desired outcome. For example, a capital investment decision, such as the expansion of a manufacturing facility, the introduction of a new product, or an investment in a new company, requires evaluating expected cash flow of the investment throughout its expected life. These future cash flow projections depend on information that is typically unknown with any degree of precision.

The elements of an integrated approach to analyzing alternatives, Step 4, include four basic components:

1. Identify all relevant cost and revenue components for each alternative. This includes, but is not limited to, identifying: sales projections; material, labor, and overhead costs; research and development (R&D) expenditures; sales and marketing costs; general and administrative costs; working capital requirements; initial capital expenses; company tax rate; depreciation method; study period; and the minimum attractive rate of return (MARR).
2. Identify all uncertain cost and revenue components per study period. This includes making projections of all uncertain cost and revenue components including the probability and/or possibility distribution for each element.
3. Develop the financial model using the data compiled from the elements listed above to generate an income and cash flow statement.
4. Use one or more economic investment criteria and analyze the alternatives.

Additional uncertain parameters estimated in an economic decision may include market size, product development estimates, market growth, cost and life of equipment, labor rates, and inflation. Many of these factors include substantial uncertainty.

Classical approaches to analyzing future unknown events include sensitivity analysis, break-even analysis, and scenario analysis. However, if the probabilities and/or the possibilities of future events are available, stochastic methods and fuzzy mathematics can help refine the decision-analysis process. This paper will consider stochastic and possibility techniques useful in economic decision analysis.

Using the classical definition, risk refers to situations in which the outcome of an event is not known with certainty, but information is available to define the probability of the event. Uncertainty, however, refers to situations in which the outcome of an event is not known, and information is not available to assist in defining the probability of the event. In the case of uncertainty, all events have equal probability. Recently, the term "uncertainty" was generalized to refer to any situation where the degree of prediction of a future event is not known with certainty. In this report, the terms risk and uncertainty will be used interchangeably to refer to situations in which the outcome of an event is not known with certainty regardless of the availability of probabilistic information.

## Traditional Approaches to Economic Decision Analysis

Cash flow projections used in economic analysis are not known with certainty. They are subject to inaccuracies containing various degrees of uncertainty. Usually, the decision maker has some knowledge of future cash flow projections by using purely subjective methods made available from individual experts. In addition, the decision maker may employ user expectation methods to obtain new data from knowledgeable experts outside the firm or use statistical methods that extrapolate past data to produce forecasts of future cash flow data. Finally, there are modeling methods that develop mathematical models to describe a given condition, i.e., market estimates, and then make projections of future cash flow estimates.

Stochastic methods are the classical approach to risk analysis. For example, the decision maker receives information about a future cash flow event in the form of a probability distribution and, using established financial criterion, selects the optimal strategy. The probability distribution is described in terms of mean and variance. Stochastic methods include many types of continuous and discrete probability distributions, then use these distributions to represent an uncertain cash flow. The economic investment criteria are then calculated to determine the investment's expected value and variance.

## Stochastic Methods

None of the data used in economic decision analysis is known with certainty. However, using one of the forecasting methods discussed previously, the decision maker typically can define the forecast's boundaries. For example, the decision maker may estimate an expected value of $50, an expected low of $40, and an expected high of $60. Furthermore, based on experience, the decision maker may estimate the projection variance will be $5.

Expected value or mean and variance are the most widely used concepts of risk. For example, the more a future, anticipated cash flow varies about the expected value, the larger the variance of the expected cash flows.

Given a probability distribution of cash flow projections, joint probability distributions may be combined in the form of sums, differences, products, or quotients to arrive at the probability distribution of the combined cash flows. Two classical methods, probability techniques and Monte Carlo simulation, are used to estimate uncertain cash flow.

## Probability Technique

The classical technique of describing a future cash flow projection uses the mean of a probability distribution. That is, a plot of relative frequency versus the value for a data item. There are a number of types of probability distributions including both continuous and discrete distributions. Common probability distributions include normal, binomial, beta, Poisson, geometric, exponential, rectangular, triangular, trapezoidal, and Weibull.

In 1963, F. S. Hillier first developed the probability technique used to evaluate uncertainty of future cash flows. The project cash flow, $F_n$, is a random variable with a mean $E(F_n)$ and a variance $VAR(F_n)$ for each period, n. The total cash flow is the sum of all cash flows over the period, n = 1, 2, . . . M source. That is:

$$\text{Total Cash Flow} = \sum_{n=1}^{M} E(F_n).$$

The net present value (NPV) is then the sum of all cash flows discounted to the present using the firm's MARR.

When the uncertain components, which constitute the cash flow, are considered independent, the mean and variance are added to calculate the combined mean and variance of the cash flow. However, when the cash flow parameters are dependent (for example, as labor cost increases, sales decrease), the coefficient of correlation must be known to calculate correctly the variance of the net present value. For dependent cash flows, coefficient of correlation must be calculated for each cash flow element per time period. Consequently, for just a few dependent variables considered over different periods of time, the calculation of the combined variance can become substantial. When cash flows are statistically correlated, F. S. Hillier developed a model, based on some reasonable assumptions, that reduces the computational complexity. However, as a first-order approximation, most decision makers assume statistical independence.

The probability technique's advantage is derived by estimating the cash flow mean, variance, and correlation per time period, then estimating the probability distribution of the NPV. The disadvantage, however, is that the estimate of the correlation between random variables is difficult to calculate along with the variance of the cash flow.

As a first-order approximation, the triangular distribution functions can be calculated easily by knowing the interval within which the random variable is contained, then identifying the most probable value within the interval. These three points, most pessimistic (Lo), most probable (Mo), and most optimistic (Ho), are then used to calculate the mean or expected value and variance of the triangular distribution. The expected value E(X) and variance VAR(X) are computed by:

$$E(X) = (Lo + Mo + Ho) / 3$$

$$VAR(X) = 1/18\ (Lo^2 + Mo^2 + Ho^2 - LoHo - MoHo - MoLo).$$

Other common first-order approximation distribution functions include the beta distribution and uniform distribution.

## Monte Carlo Simulation

Scientists first applied Monte Carlo simulation to a class of mathematical methods while developing nuclear weapons in Los Alamos during the 1940s. Its use in capital budgeting, however, is associated principally with David Hertz and McKinsey and Company.

The Monte Carlo method simulates a future event that involves probabilistic distributions. This technique is most often used when the situation is too complex to combine several probability distributions as discussed above. The Monte Carlo simulation method is based on an estimate of the distribution and the expected value can be obtained by evaluating, at random, probable values. This process is then repeated 100 to 10,000 times. Each calculation generates a new random value that represents the cash flow. The result can be expressed as a frequency distribution from which the expected value of the project cash flow can be calculated.

The next chapter discusses how Monte Carlo simulation can be used to determine the project balance for the example in the previous chapter.

## Bibliography

Kelton, David W.; Sadowski, Randall P.; and Sadowski, Deborah A., *Simulation with Arena*: McGraw-Hill, New York, 1998.

Winston, Wayne L.; *Operations Research Applications and Algorithms, 3rd Edition*: Duxbury Press, Belmont, California, 1994.

# 5

# USING PROBABILISTIC TECHNIQUES TO DETERMINE LIFE-CYCLE COST OF EMBEDDED AND SYSTEM SOLUTIONS

## Levels of Power Conditioning

Power conditioning can be applied at several different levels, each of which has its own life-cycle cost advantages and disadvantages. Graphically, this is shown in Figure 5-1 below.

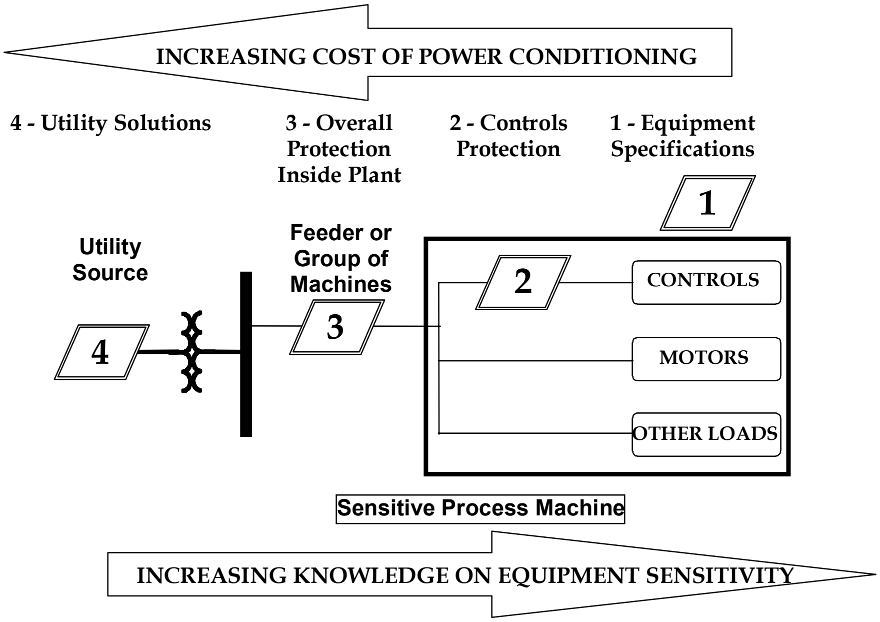


**Figure 5-1**
**Power Conditioning - Cost versus Knowledge**

As the power conditioning technology is moved from inside the facility toward the service entrance, the cost increases. In contrast, to move the point of application farther inside the plant, more knowledge of the process equipment is necessary. As an example, if one knows nothing about the particulars of the process equipment, but has unlimited budget, then power conditioning can be approached at the utility level with very large-scale devices or expensive system modifications. On the other hand, if one has the time to invest in evaluating the performance of the equipment inside the facility, then the solution may be applied at a lower voltage level and at smaller subcomponents of the process. This would lead to significant equipment cost savings.

Naturally, the least expensive option for process ride-through is to include the ride-through requirements in the specifications for the equipment. This is point #1 in Figure 5-1. If ride-through standards exist, this merely requires that the specification list the standard to be met. Unfortunately, most industries do not have such standards in place.

At point #2, the system must be studied; but there is usually some correlation between process shutdown and the control circuits used in the system. Therefore, power conditioning can be applied to the control circuit or control devices responsible for shutdown of the system. While this may not remove all the sensitivity of the system to voltage variations, it will, in all likelihood, take care of 80 percent of that sensitivity. At this level, the power conditioners are the least costly on the market.

Point #3 is at the panel level feeding the process line or lines. At this level, the power conditioning devices must be larger scale, on the order of tens to hundreds of kW. They will also carry a higher price. In this case, the knowledge of the process equipment is not required in as much detail, and the improvement in system performance may be even better than the 80 percent mentioned previously. The drawback is that power conditioning may be applied for loads that do not actually need it. In addition, some engineering will be necessary to install such a device, which increases the final cost.

Point #4 is at the service entrance to the plant where the power conditioning device(s) must support the entire plant load. This could be anywhere from one to 50 MW. As such, this is the most costly approach. Hopefully, the technology chosen for this level is such that 100 percent of the process sensitivity is removed. However, as is true at point #3, it is possible that the majority of equipment protected does not require protection.

Before selecting the optimum level of power conditioning, the power quality data should me analyzed and the appropriate economic criteria applied. The example below discusses one technique that can be used to analyze power quality data. The project balance and the net present value method are used to select the optimum level of power conditioning.

## Example 1 – Time Between Power Quality Events

Before discussing the probabilistic project balance method, let us assume the time-to-failure or the time between power quality events is that shown in the frequency distribution in Figure 5-2.

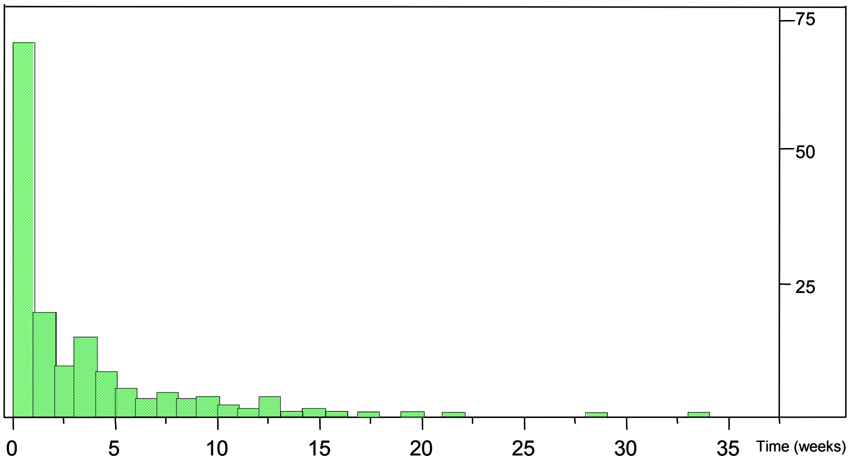


**Figure 5-2**
**Time-to-Failure for Power Quality Events**

Figure 5-2 indicates the frequency distribution for the time-to-failure for power quality events during a several-month period. Most of the power quality events occurred less than one week while few events occur more than 20 weeks.

If you assume the time between power quality events is normally distributed, the mean would be 5.05 weeks with a standard deviation of 7.60 weeks. The best-fit normal distribution to the power quality data is shown in Figure 5-3.

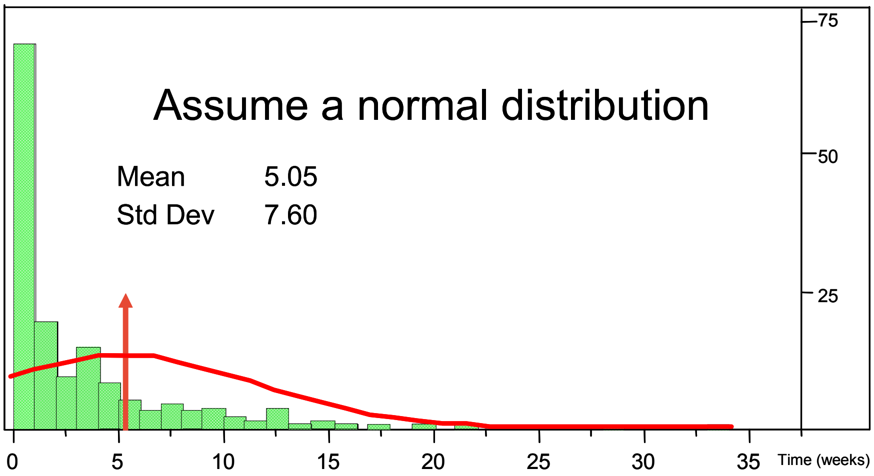


**Figure 5-3**
**Normal Distribution of the Time-to-Failure for Power Quality Events**

However, if you assume the time between power quality events for the data shown in Figure 5-2 is a Weibull distribution, the mean would be 2.08 weeks with a standard deviation of 4.72 weeks. The best-fit Weibull distribution to the power quality data is shown in Figure 5-4.

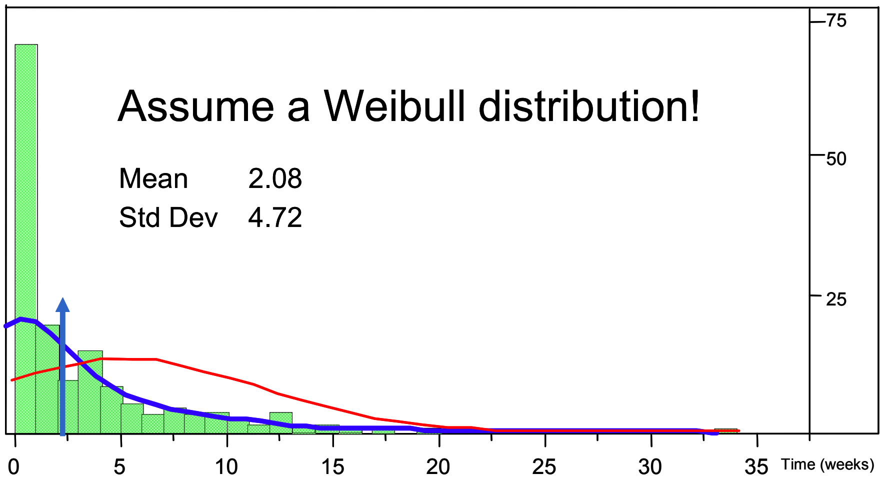


**Figure 5-4**
**Weibull Distribution of the Time-to-Failure for Power Quality Events**

Figure 5-5 shows both the normal distribution and the Weibull distribution on the same frequency distribution graph. Notice the significant difference between the mean time between power quality events assuming the Weibull distribution compared to the normal distribution. Using both statistical techniques and visual inspection, one can easily see that the Weibull distribution is a much better fit to the data as compared to the normal distribution.

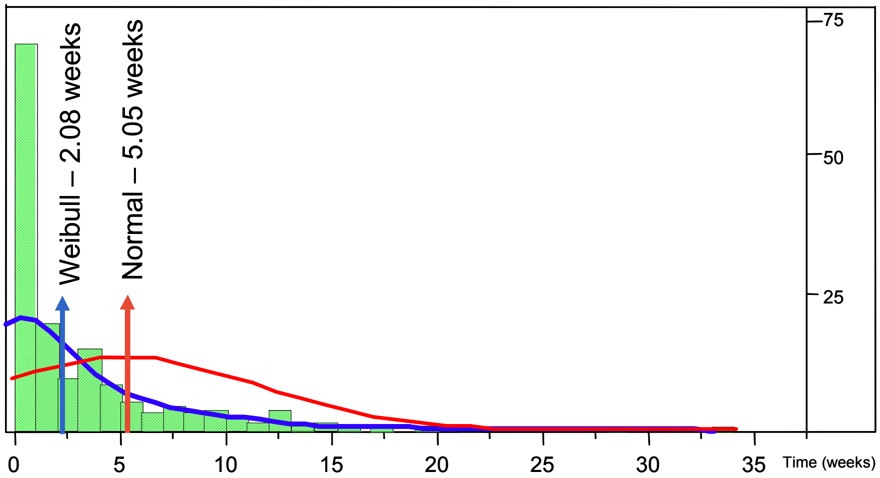


**Figure 5-5**
**Weibull Distribution of the Time-to-Failure for Power Quality Events**

As a simple NPV example, let us assume that each power quality event results in an end-use loss of \$20,000 with equipment required to mitigate the power quality event costing \$250,000. Assume a MARR of 15%. The NPV is calculated using the following formula as discussed previously:

$$NPV(i,n) = \sum_{n=0}^{N} F_n (1+i)^{-n}$$

The benefit derived from purchasing the power quality mitigation system is eliminating the end-use loss of $20,000 per power quality event. Assuming a normal distribution, the end-user is expected to incur a power quality event every 5.05 weeks with a standard deviation of 7.60 weeks. Assuming a Weibull distribution, the end-user is expected to incur a power quality event every 2.08 weeks with a standard deviation of 4.72 weeks.

Using the deterministic method to calculate the NPV with 5.05 weeks between events, the NPV for one year is a negative $221,869. Using 2.08 weeks between events, the NPV for one year is a positive $239,590. Assuming a normal distribution and using the Monte Carlo simulation to calculate the NPV, the NPV is a negative $221,869. The significant difference between the NPV using the deterministic and the probabilistic method is due to the large standard deviation between power quality events. Assuming a Weibull distribution, the NPV is a positive $1,378,646. Table 5-1 summarizes the results above. Which value is correct?

**Table 5-1**
**Summary of NPV for Deterministic and Probabilistic Calculations**

| **Method** | **NPV** |
|---|---|
| Deterministic - 2.08 weeks | $239,590 |
| Deterministic - 5.05 weeks | ($221,869) |
| Probabilistic - Weibull | $1,378,646 |

Clearly, from Table 5-1 there is a significant difference between the deterministic method and the probabilistic method based on a Weibull distribution. The difference in NPV is due to the different distributions and their associated mean value (the mean value represents the expected number of weeks between power quality events) used to calculate the expected number of weeks between power quality events. By including this additional information in the calculation and using Monte Carlo simulation, the NPV is increased significantly compared to the deterministic method. The reason is that the actual power quality data collected better fits a Weibull distribution with a mean of 2.08 weeks between power quality events versus a mean of 5.05 weeks assuming a normal distribution. The result is a significantly more number of power quality events per year that the power quality mitigation hardware system is able to prevent from occurring with a resulting benefit per event of $20,000. However, using the normal distributions, there are several fewer events expected to occur per year.

Again, which method is correct? Visual observation of the frequency distribution shown in Figure 5-5 clearly indicates the Weibull distribution is a much better fit to the data. By using the Weibull distribution, the actual number of events saved by implementing the power quality mitigation device is a much better indication of the expected future behavior of the system versus the normal distribution and deterministic calculation of the NPV.

## Example 2 - Probabilistic After-Tax, Life-Cycle Analysis using Project Balance Method and the Weibull Distribution

How can the information presented in the previous section be used to calculate the project balance?

As an example of a probabilistic after-tax, life-cycle cost analysis using the project balance method, consider the example discussed in Chapter 3 where a firm is evaluating an investment in new power quality mitigation equipment. The problem identified by the firm is frequent voltage sags causing excessive down time, scrap, rework, and general loss of production. After reviewing the firm's annual production report and the anticipated demand for its product in the next few years, the firm's management team has decided to evaluate the investment in a new power quality mitigation device that will help reduce the number of voltage sags. The new power quality mitigation equipment is anticipated to eliminate voltage sags from disrupting the sensitive electronic manufacturing equipment, thereby adversely affecting production operations. The analysis will require a one-year needs assessment and vendor selection. The decision objective is to determine if the investment in a new power quality mitigation system is feasible and profitable. Additional details of the project benefits and costs are shown below.

- *Needs Analysis Costs:* The engineering staff estimates that the cost to perform the initial needs analysis is $28,000, which includes labor to review operational performance associated with lost production due to frequent voltage sags that cause the firm's sensitive electronic equipment to malfunction. These costs will be expensed in the year they occur.
- *Benefit:* The benefit of the power quality mitigation equipment includes the reduction of scrap and downtime and increased production. The benefit derived by the installation of the equipment is shown in Table 5-2 for years one through six. The benefit is a function of the number of power quality events that will be eliminated due to the addition of a power quality mitigation device. For example, assume that the expected number of power quality events is 1 per month or 12 per year with a mean time between events of 30 days. If the power quality mitigation device eliminates 6 power quality events per year, the mean time between events increases to 60 days. For purposes of this example, an increased benefit is analogous to an increase in the mean time between power quality events.
- *System Planning Costs:* System planning costs are estimated to be $20,000. This involves initial planning on how the new power quality mitigation system will be integrated into the manufacturing electrical system and its impact on overall electrical and mechanical infrastructure.
- *System Procurement Costs:* The engineering and procurement staff expenses associated with developing the procurement specification, reviewing the alternative solution offering, and negotiating the final selection is estimated to be a $20,000, one-time expense.
- *Construction Costs:* The construction costs associated with the new power quality mitigation equipment is estimated to be $65,000.

- *Operations and Support Costs:* The recurring operations support costs are shown in Tale 5-2. These estimated costs include such items as maintenance, repair, staff training, supplies, storage, rent, overhead charges, and other contingencies.
- *Retirement and Disposal Costs:* The retirement and disposal costs for the power quality mitigation equipment is estimated to be 8.8 percent of the initial capital investment at the end of year six or $49,500.
- *Capital Investment:* The initial capital investment required to purchase the new power quality mitigation equipment is $560,000 and will be depreciated for seven years.
- *Marginal Tax Rate:* 40 percent.
- *Minimum Attractive Rate of Return (MARR):* 15 percent.

The company's six-year benefit and cost statement is shown in Table 5-2. The net present value of the investment using the deterministic method is $443,944. Since the net present value is positive, the project should be considered.

How is the net present value affected if the expected number of events, e.g., the time between power quality events, is changed? The following analysis will incorporate the Weibull distribution analysis and Monte Carlo simulation to discuss the importance of selecting the appropriate distribution for the mean time between power quality events as part of the project balance method.

**Table 5-2**
**Benefit and Expenses for Project Balance Example**

| Year | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Benefit | $0 | $295,000 | $412,000 | $495,000 | $535,000 | $589,000 | $550,000 |
| Needs Analysis | $28,000 | $0 | $0 | $0 | $0 | $0 | $0 |
| System Planning R&D | $20,000 | $0 | $0 | $0 | $0 | $0 | $0 |
| Procurement | $20,000 | $0 | $0 | $0 | $0 | $0 | $0 |
| Construction | $65,000 | $0 | $0 | $0 | $0 | $0 | $0 |
| Operational | $0 | $25,000 | $35,000 | $95,000 | $53,000 | $28,000 | $36,000 |
| Total Expenses | $133,000 | $25,000 | $35,000 | $95,000 | $53,000 | $28,000 | $36,000 |
| Depreciation | $0 | $89,286 | $89,286 | $89,286 | $89,286 | $89,286 | $89,286 |
| NIBT | ($133,000) | $180,714 | $287,714 | $310,714 | $392,714 | $471,714 | $424,714 |
| Income Tax (40%) | $53,200 | ($72,286) | ($115,086) | ($124,286) | ($157,086) | ($188,686) | ($169,886) |
| NET INCOME | ($79,800) | $108,429 | $172,629 | $186,429 | $235,629 | $283,029 | $254,829 |
| CASH FLOW | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| NET INCOME | ($79,800) | $108,429 | $172,629 | $186,429 | $235,629 | $283,029 | $254,829 |
| Depreciation | $0 | $89,286 | $89,286 | $89,286 | $89,286 | $89,286 | $89,286 |
| Initial Investment | ($560,000) | $0 | $0 | $0 | $0 | $0 | $0 |
| Salvage Value | $0 | $0 | $0 | $0 | $0 | $0 | $49,500 |
| Gains Tax | $0 | $0 | $0 | $0 | $0 | $0 | ($19,800) |
| AFTER TAX CASH FLOW | ($639,800) | $197,714 | $261,914 | $275,714 | $324,914 | $372,314 | $373,814 |

As previously stated, the Weibull distribution is the preferred distribution for defining the time between power quality events.

The Weibull distribution is used often to describe the time-to-failure for mechanical and electrical equipment. These can be light bulbs, capacitors, disk drives, ball bearings, etc. When a number of parts are put on test, they don't all fail at the same time (if they do, you might wonder if something went wrong). Usually, there is some spread in the failure times.

While the normal distribution is a very handy tool in describing all sorts of different data, it does not work well in describing the statistical distribution associated with the time-to-failure. One reason for this is that the normal distribution allows some observations to be negative. When you life test something, you know it did not fail before time $t = 0$. Therefore, the normal distribution will not work.

If parts fail according to a Weibull distribution, the probability that any single part will fail at a particular time, $t$ is $F(t) = 1 - \exp[-(t/a)^b]$ where $a$ is called the scale parameter, $b$ is called the shape parameter, and F is called the cumulative distribution function. The shape parameter $b$ can also tell us something about the failure rate. It turns out that when $b < 1$, the failure rate is decreasing, but when $b > 1$, the failure rate increases. Suppose your data come from a Weibull distribution and you find that $b < 1$. That means that as time increases, the failure rate becomes smaller or the time between power quality events becomes less. This might be because you have defects in some of your parts causing high failure rates at the beginning (so-called infant mortality) and by using a power quality mitigation device, the number of power quality events is reduced per unit time compared to no mitigation device. As the defective parts die, the failure rate goes down. Alternatively, if $b > 1$, then the failure rate is increasing. As parts start to approach their maximum possible life, they will begin wearing out, causing an increased failure rate. The $a$ or scale parameter is <u>approximately</u> equal to the mean-time-to-failure and is equal to the mean-time-to-failure when the shape is equal to one. The expectation is that when you incorporate a power quality mitigation device, the shape parameter $b$ is reduced and preferably less than one; however, if $b$=1, the conclusion will be identical to the deterministic calculation.

What effect is there on the NPV if $b$<1? Figure 5-6 shows the shape parameter, $b$, versus the NPV. From Figure 5-6, you can see that a decrease in $b$<2, results in an increase in the NPV. The reason lies in the shape of the Weibull distribution. For example, if the $b$<2 while keeping the scale parameter $a$ the same, the actual mean time between power quality events is decreased resulting in more events per unit time, thus deceasing the mean time between power quality events.

Clearly, from Figure 5-6, the shape parameter of the Weibull distribution can result in a wide range of net present value numbers that directly affect the project balance.

Which is correct? That depends on the best-fit Weibull distribution to the power quality data actually collected. However, as shown in Figure 5-6, the NPV is highly dependent on the Weibull distribution's shape parameter.

Could a normal distribution be assumed for the time between power quality events? If a normal distribution were assumed, the mean value would be equivalent to the NPV using the deterministic method; however, there would be a distribution about the mean value. Depending on the variance, the distribution about the mean could be significant resulting in a wide NPV variation with the most likely value being the mean.

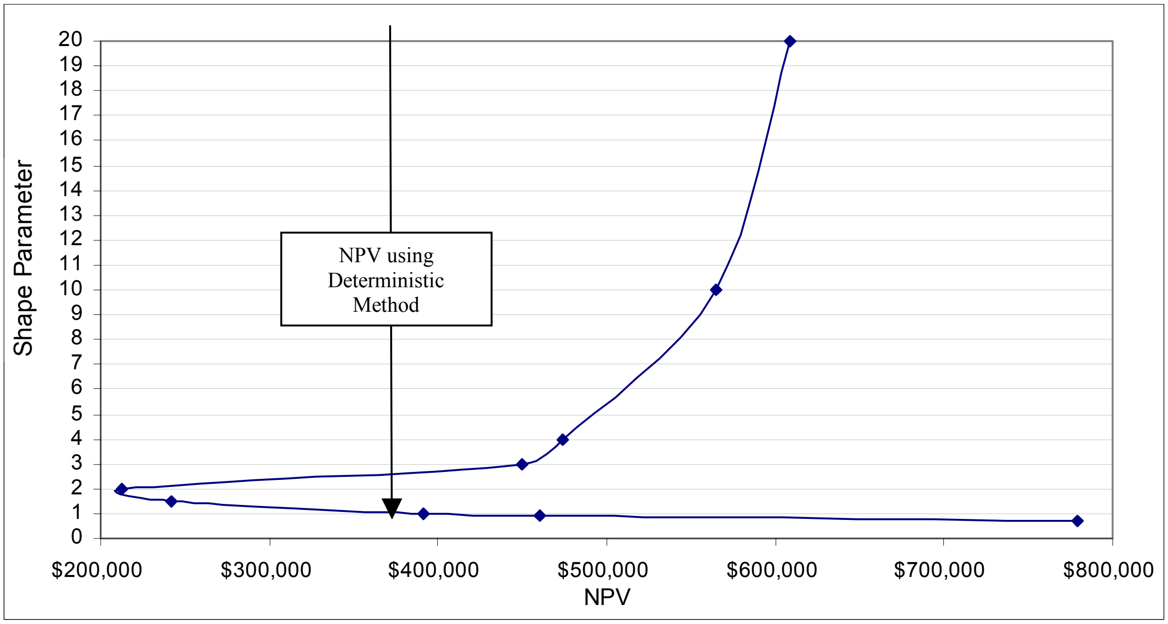


**Figure 5-6**
**Change in the Weibull Distribution Shape Parameter and the NPV**

Which is the most likely Weibull distribution shape parameter? This answer depends on the power quality data. The Weibull distribution shape parameter should be determined based on the best-fit Weibull distribution given the original power quality data. If the power quality data were similar to that shown in Example 1, then clearly the best fit distribution would be the Weibull distribution. The Weibull distribution would then be used as the input in determining the expected benefit shown in Example 2. The Monte Carol simulation can then be used to calculate the expected NPV. Since the time between power quality events is a random variable, this random variable is used in calculating the NPV that will result in the NPV being a random variable with a mean and distribution. Figure 5-7 shows the output distribution of the NPV given a $b$=2. The distribution in Figure 5-7 has a mean of $224,402 and a standard deviation of $211,796. Note that the output distribution shown in Figure 5-7, closely resembles a normal distribution. This is due to one of the most useful theorems in statistics called the Central Limit Theorem.

The Central Limit Theorem says, roughly, that the sum of a large number of independent and identically distributed random variables is approximately normally distributed. Another way of saying this is that the mean is approximately normal for large samples. In the example above, the input variable, time between power quality events, was modeled using a Weibull distribution. Using Monte Carlo simulation, the input values were randomly selected, again using the Weibull distribution, and the output calculated. The Monte Carlo simulation was performed over 10,000 iterations to generate the output shown in Figure 5-7.

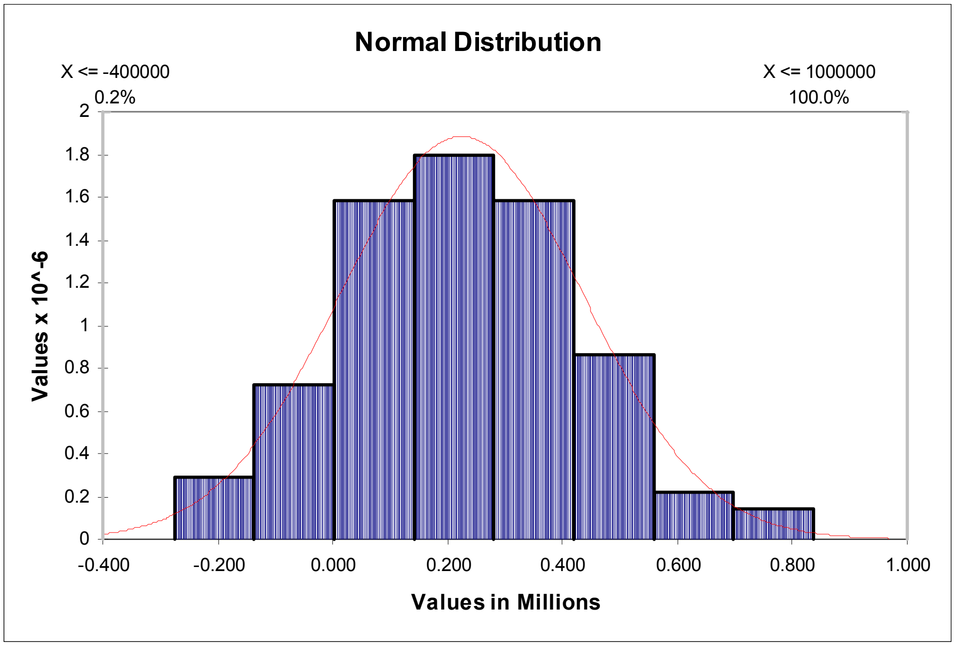


**Figure 5-7**
**Output Distribution for the NPV**

By using the Weibull distribution as the input variable in the economic model in Table 5-2, statistical information contained about the time between power quality events can be incorporated into the economic model and the output evaluated using standard statistical techniques. For example, further analysis of the output distribution in Figure 5-7 indicates the variability about the mean NPV of $224,402. In a deterministic approach, no information concerning the variability is possible other than assuming the variability is zero. By using Monte Carlo simulation, the output can be analyzed and the risk associated with making an economic decision evaluated.

# 6

# CAPITAL BUDGETING USING PROBABILISTIC TECHNIQUES

## Overview of Capital Budgeting

Every organization operates to a plan, whether explicitly or implicitly. Well-managed projects operate to a documented, explicit financial plan called a budget. The best-managed operations also generate detailed reports to compare actual financial performance to their budgets. This chapter focuses on the budgeting process, specifically, the capital budgeting process, and on the role budgets can play in analyzing performance when incorporating the additional information content provided by the probabilistic analysis method discussed in the previous chapters.

Estimates of costs (or benefits derived from the installation of a power quality mitigation system) and expenses--manufacturing costs, selling, marketing, administration expenses, capital expenses, and so forth--provide a base or yardstick against which to compare actual expenditures. The process of setting these standards or benchmarks is called budgeting. Comparing actual financial results with budget expectations (budget analysis) highlights deviations from plan and can help signal appropriate corrective action.

The use of accounting data to make informed operating decisions is referred to as managerial accounting. Since budgeting and budget analysis are key tools for operating managers, this chapter focuses on how operating plans, or budgets, are established, then on how they are used to ascertain whether operations are on plan or require remedial steps to return to plan.

For most firms, the first step in the investment process is the preparation of an annual capital budget, which is a list of planned investment projects and a breakdown of planned investment outlays. In principle, the capital budget should be a list of all positive NPV opportunities open to the firm.

Most firms let project proposals bubble up from project areas, functional areas, or divisions for review by management, then subsequent review by senior management, accounting, and finance. Once all proposals have been reviewed and “make the cut” at the senior management level, the list is consolidated for final review by the senior management and finance. The resulting budget is a list of proposed new projects for the coming time period, usually the next fiscal year. Supporting information usually is provided on standard forms, supplemented by descriptive memoranda for larger projects.

The capital budget focuses on the longer-term need for and generation of investment capital, which is then used to fund capital items such as power quality mitigation equipment. Capital budgets should be developed for several years, typically three and in some cases five, to provide plenty of advance warning of capital shortage or excesses. While it may be difficult or, in some cases, impossible to plan for power quality mitigation equipment years in advance, knowing that such devices likely will be required to support operations is important to consider when developing the capital budget. Again, since most power quality mitigation hardware systems are considered capital items, including such devices in the capital plan is essential when trying to justify the purchase of such devices and eliciting senior management support. In addition, since a power quality mitigation system eliminates costly process disruptions, the justification for a system likely is to be the benefit derived or the expenditures forgone because of implementing the mitigation system. The NPV statistical distribution shown previously in Figure 5-7 should be used as input for the budgeting process.

## The Revenue or Benefit Forecast

The cornerstone of the budget is the forecast of revenue or benefit for the coming period. In a profit-seeking company, the forecast of sales becomes the critical first step in the budgeting process. In budgeting for a power quality mitigation system, the budget is the forecast of the benefit derived from the use of the power quality mitigation system. For example, if the expected benefit is the elimination of ten power quality events that cause disruptions in end-use processes, the benefit is the cost savings associated from eliminating these disruptions. If the expected number of power quality events or the time between power quality events can be represented by a statistical distribution as discussed in Chapter 5, this information should be incorporated into the revenue or benefit forecast as discussed previously.

## History as Prologue

The budgeting process relies heavily on historical financial data. Since an operating budget is simply an estimate of the income statement for a future period, past operating statements are obviously useful guides. Frequently, budget levels for the coming year are established simply by incrementing last year's actual expenditures up or down. In any case, a manager needs a detailed understanding of current expenditure levels as they consider appropriate budget levels for a future period.

When incorporating a power quality mitigation system into the budgetary process, the historical number of power quality events typically experienced at an end-user facility can be used as a good estimate of potential future events.

## Pro-Forma Income Statement and Balance Sheets

An operating budget is a pro forma (estimated in advance) financial statement. Pro forma, or estimated, income statements and balance sheets are also vital to the financial management of a company. Most companies develop pro forma income statements and balance sheets as of various dates in the future, particularly at the end of the year and perhaps at critical dates during the coming year. These pro forma income statements and balance sheets permit management (1) to consider the need for increases or decreases in inventory, accounts receivable, accounts payable, and other working capital items; (2) to judge the adequacy of the company's liquidity at these future dates; and (3) to decide whether to undertake additional short-term borrowing or perhaps repay existing loans. Thus, the ability of the company to finance its projected operations is revealed by pro forma income statements and balance sheets.

The integration of a power quality mitigation system should be incorporated into both the income statement and balance sheet. For example, the benefit derived from the use of a power quality mitigation system should be incorporated into the income statement while the depreciation of the system is included in the balance sheet. The balance sheet also includes an increase in equipment asset and associated liability depending on the specific financing arrangement.

## Sources of Uncertainty in Capital Budgeting

The factors that affect the uncertainty involved in a capital budgeting process are many and varied. It would be almost impossible to list and discuss all of them; however, there are four major sources of uncertainty, which are nearly always present in a capital budgeting problem.

The first factor, which is always present, is the possible inaccuracy of the estimates used in the capital budgeting process. If exact information is available regarding the items of income and expenses, the resulting accuracy should be good. If, on the other hand, little factual information is available and nearly all the values have to be estimated, the accuracy may be high or low, depending on the manner in which the estimated values are obtained. Questions that should be asked concern whether the estimates are based on good information or are merely guesses.

The accuracy of the cash inflow estimates is difficult to determine. If they are based on a considerable amount of past experience or have been determined by adequate market surveys, a fair degree of reliance may be placed on them. On the other hand, if they are merely the result of guesswork, with a considerable element of hope thrown in, they must of course be considered to contain a sizable element of uncertainty.

The second key factor affecting uncertainty is the type of business involved in relation to the future health of the economy. Some lines of business are notoriously less stable than others. For example, most mining enterprises are more risky than large retail food stores. However, we cannot arbitrarily say that an investment in any retail food store always involves less uncertainty than investment in mining property. Whenever capital is to be invested in an enterprise, the nature and history of the business, as well as expectations of future economic conditions (e.g., interest rates), should be considered in deciding what risk is present.

A third factor affecting uncertainty is the type of equipment involved. Some types of structures have rather definite economic lives and residual values. Little is know of the physical or economic lives of others, and they have almost no resale value. A good engine lathe generally can be used for many purposes in nearly any fabrication shop. Quite different would be a special type of lathe that was built to do only one unusual job. Its value would be dependent almost entirely upon the demand for the special task that it can perform. Thus, the type of equipment involved will have a direct bearing upon the accuracy of the estimated income and expenditure patterns. Where money is to be invested in specialized equipment, this factor should be considered carefully.

The fourth, and very important, factor that must always be considered in evaluating uncertainty is the length of the assumed study period. The conditions that have been assumed in regard to income and expense must exist throughout the study period to obtain a satisfactory return on the investment. A long study period naturally decreases the probability of all the factors turning out as estimated. Therefore, a long study period, all else being equal, always increases the uncertainty in an investment.

## Capital Budgeting Using Sensitivity Analysis and the Project Balance Method

The capital budgeting process is an iterative process that begins with a first estimate of projected financial performance. The budget is than refined as additional information from other stakeholders is incorporated into the budgetary process. Finally, a sensitivity analysis is performed to determine how sensitive various input variables are relative to the output variable, NPV for example, as shown in Figure 6-1.

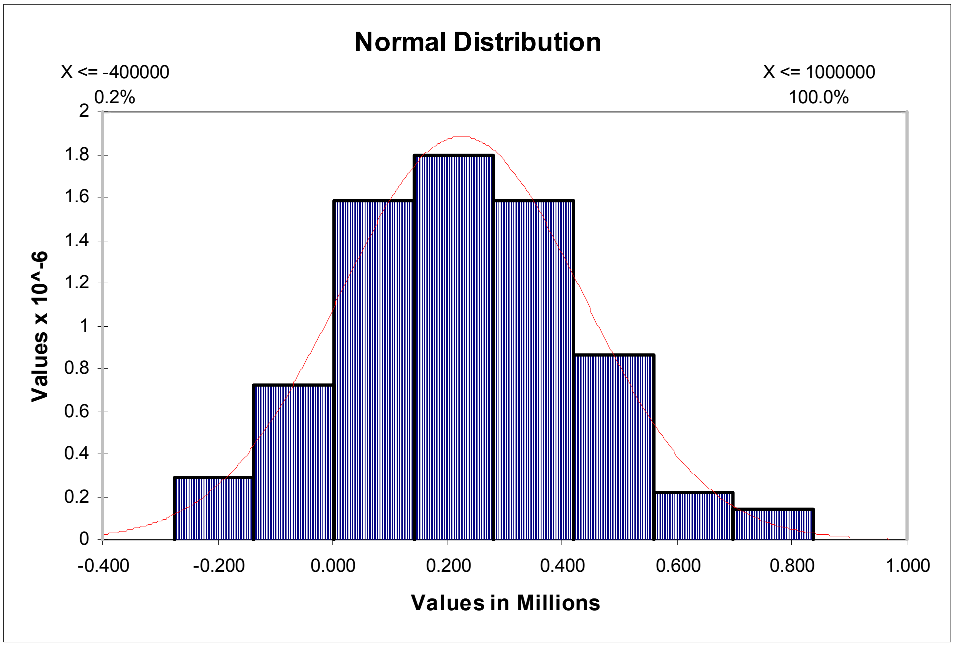


**Figure 6-1**
**Output Distribution for the NPV**

In addition to the statistical distribution of the output variable, NPV, multiple variables can be plotted on a single graph with the horizontal axis being the percent change in the input variable and the vertical axis representing the NPV, as shown in Figure 6-2. Notice that, for some input variables, such as the expected number of power quality events, a small percent change can result in a large change in NPV. To assure successful results, management should monitor such sensitive input variable closely. For other input variables, such as yearly operational expenses, the NPV is much less sensitive to a small change in input; therefore, the attention required by management is much less. The sensitivity profile graph shown in Figure 6-2 shows how sensitive each important input variable is to the NPV or economic variable of interest.

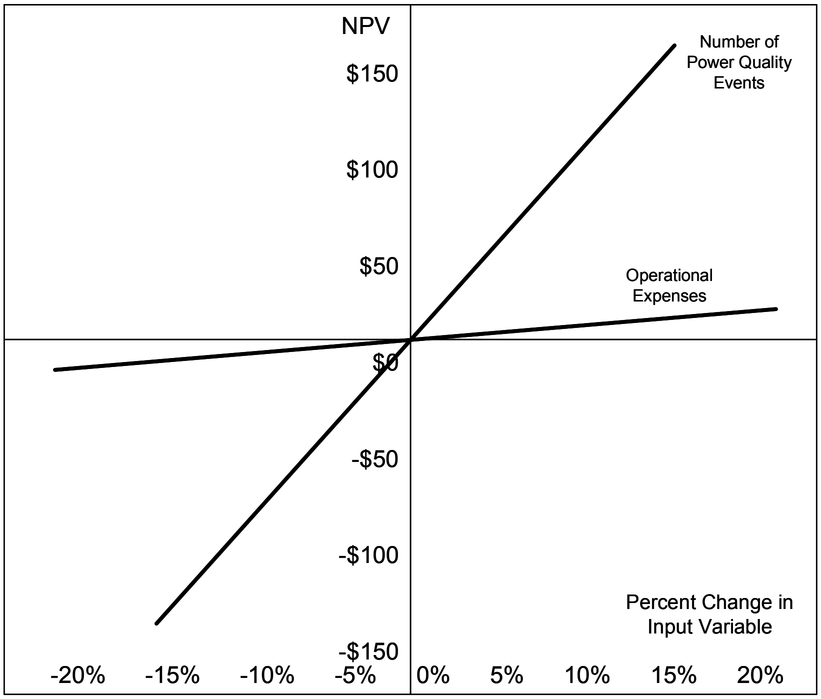


**Figure 6-2**
**Sensitivity Graph**

In many cases, a simple breakeven analysis does not provide adequate information about the potential impact of uncertainty in each estimated factor value. In such instances, and, in fact, in most cases, it is helpful to determine how sensitive the situation is to the several factors of concern so that proper weight and consideration may be assigned to them. Sensitivity, in general, means the relative magnitude of change in the measure of merit (such as number of power quality events) caused by one or more changes in estimated study factor values.

## Information Content and Financial Decision Making

The additional information content provided by incorporating probabilistic techniques into the budgetary process is related primarily to a better understanding of the uncertainty profile. For example, by understanding the statistical nature of the number of power quality events and the expected time between power quality events, the economic benefit derived from installing a power quality mitigation system can be analyzed and presented to management in a format that can be discussed in financial terms.

There are numerous methods for analyzing uncertainty, resulting from the four major sources described earlier. The information about the uncertainty profile provided by each method varies depending on the variable of interest. The five popular methods include:

1. Breakeven analysis
2. Sensitivity analysis
3. Optimistic-pessimistic estimation
4. Risk-adjusted MARRs
5. Reduction of the useful life

Breakeven analysis determines the value of a key common factor, such as utilization of capacity at which the economic desirability of two alternatives is equal. This breakeven point is then compared to an independent estimate of the factor's most likely value to assist with the selection between alternatives. Similarly, sensitivity analysis typically determines the range of values that a key parameter may have without reversing the superiority that the best alternatives has over others being considered. The remaining three procedures for dealing with uncertainty are aimed at selecting the best course of action when one or more consequences of alternatives being evaluated lack estimation precision.

Regrettably, there is no quick and easy answer to the question "How should uncertainty best be considered in an engineering economic analysis?" Generally, simple procedures (e.g., breakeven analysis and sensitivity analysis) allow reasonable discrimination among alternatives to be made on the basis of the uncertainties present, and they are relatively inexpensive to apply. Using probabilistic methods discussed in the previous chapters, additional discrimination among alternatives is possible.

As a summary, Table 6-1 shows a comparison of the difference techniques for both traditional and non-traditional life-cycle and budget analysis.

**Table 6-1**
**Summary Table of Traditional and Non-Traditional Life-Cycle Analysis Techniques**

| Traditional Techniques | Non-Traditional Techniques |
|---|---|
| • Deterministic analysis | • Probabilistic analysis |
| • Non-probabilistic analysis | • Statistical analysis |
| • Certainty analysis | • Monte Carlo Simulation |
| • Breakeven analysis | • Optimization analysis |
| • Risk-adjusted MARRs | • Sensitivity analysis |
| • Reduction of useful life | • Optimistic-pessimistic analysis |

## Bibliography

Gup, Benton E, *Principles of Financial Management*:  John Wiley & Sons, New York, 1983.

Plank, Tom M.; and Blensly, Douglas L., *Accounting Desk Book*: Prentice Hall, Englewood Cliffs, New Jersey, 1989.

Fess, Philip E.; and Warren, Carl S., *Accounting Principles*, 14th Edition:  South-Western Publishing Company, Cincinnati, Ohio, 1984.

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