Asset Population Management and Testing

Methodology Extensions

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PRODUCT DESCRIPTION

Increasing pressure from both customers and regulators to maintain and enhance service reliability, while at the same time controlling costs, has put many utilities' distribution businesses in a classic dilemma of conflicting objectives. For that reason, asset management has become an increasingly important aspect of corporate business strategies. A significant focus of EPRI's asset management research in recent years has been to develop a rational basis for selecting repair or replacement options for specific classes of equipment by balancing the risks of equipment failure against the costs of continued maintenance or capital replacement.

EPRI has published a series of reports that discuss methods for making decisions about aging assets in electric distribution systems. Three reports on guidelines for asset replacement presented the methodology (*Guidelines for Intelligent Asset Replacement, Volume I,* 1002086, December 2003), applied the methodology to inventories of wood poles (*Guidelines for Intelligent Asset Replacement, Volume II,* 1002087, December 2004) and to inventories of underground cables (*Guidelines for Intelligent Asset Replacement, Volume III,* 1002088, December 2005). The purpose of these reports is to provide utilities with sufficient information and methodology in order that specific asset management decisions can be made that will yield least cost asset management strategies.

Background
For many utilities, particularly those focusing on the power delivery business, their distribution system assets such as the transformer inventory, the wood poles inventory, the underground cable inventory, and other system components, represent a substantial portion of their capital assets. Managing these inventories entails significant costs and directly affects the reliability of electric service. Therefore, utilities need cost-effective strategies for maintaining their distribution system assets. In previous reports, as cited above, the methodology for determining such strategies was presented.

Objective
This report consists of three somewhat independent papers addressing aspects of the aging assets management problem developed in prior EPRI work. The purpose of these papers is to extend the decision framework for equipment replacement that forms the heart of EPRI's approach to the aging assets problem.

Approach
EPRI has developed a decision framework that enables utilities to generate business cases for asset management policies. This framework takes a life-cycle costing approach that enables corporate financial managers and regulators to assess the multi-year financial impacts of

maintaining specific classes of power delivery infrastructure assets, such as underground cable, wooden poles, and transformers.

The analytical tools presented in this report share a basic framework for decision-making that specifies the evolution of the condition of the asset population over time, the various decision alternatives that are available, and the basic data needed to support the decision model.

Results

The first paper in this report uses a simple economic model to show how to find the optimal policy for replacing aging equipment using *dynamic programming*. The model illustrates the basic concepts in a straightforward and concrete manner. The principles laid out in this chapter also apply to more general formulations of the optimal equipment replacement decision, used in other EPRI work on aging assets management.

The second paper in this report uses a simple economic model to show how to find the optimal policy for testing and replacing aging equipment using *Bayesian* analysis. Again, the principles laid out in this chapter also apply to more general formulations of the optimal equipment testing and replacement decision, used in other EPRI work on aging assets management.

The third paper in this report addresses a question that frequently arises in using EPRI's aging assets decision framework – what should a company do if the annual cost of optimal replacement policy exceeds the available budget? The models find the optimal stationary policy for equipment replacement, which applies over an indefinite time horizon. Following this policy typically leads to a stable distribution of the asset population among the various ages and condition states. However, initially, the asset population inventory may be quite different than the long-run distribution. This situation may force unreasonably large expenditures in the early years of application of the stationary policy, as the oldest, most deteriorated equipment in the initial inventory is replaced. Thus a transient policy must be applied to the initial asset inventory, in order to move it toward the long-run population distribution without violating the available budget constraints. This third paper discusses how to find such transient policies.

EPRI Perspective

EPRI has been developing methods for distribution planning since 1992. Methodology, software, and equipment failure data have been under development for several years to aid companies in developing economic asset management strategies – strategies that meet customer needs for reliability and power quality at least cost. The objective of this project is to use the information, tools, and experience that have been developed in EPRI's asset management research to deliver general guidelines and strategies for managing specific equipment categories based on the knowledge assembled in previous years' work.

Keywords

Distribution systems, Distribution, Aging assets management, Reliability analysis, Repair and replacement policies

ABSTRACT

Increasing pressure from both customers and regulators to maintain and enhance service reliability, while at the same time controlling costs, has put many utilities' distribution businesses in a classic dilemma of conflicting objectives. For that reason, asset management has become an increasingly important aspect of corporate business strategies. A significant focus of EPRI's asset management research in recent years has been to develop a rational basis for selecting repair or replacement options for specific classes of equipment by balancing the risks of equipment failure against the costs of continued maintenance or capital replacement.

This report consists of three somewhat independent papers addressing aspects of the aging assets management problem developed in prior EPRI work. The purpose of these papers is to extend the decision framework for equipment replacement that forms the heart of EPRI's approach to the aging assets problem.

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1 INTRODUCTION

This report consists of three somewhat independent papers addressing aspects of the aging assets management problem developed in prior EPRI work. The purpose of these papers is to extend the decision framework for equipment replacement that forms the heart of EPRI's approach to the aging assets problem.

The first paper, presented in chapter 2, describes and illustrates the decision framework for equipment replacement. It uses a simple economic model to show how to find the optimal policy for replacing aging equipment using *dynamic programming*, which is discussed in detail in several prior EPRI reports as well as in many text books. The model developed in this chapter does not exhaust the power of dynamic programming methods, but it does illustrate the basic concepts in a straightforward and concrete manner. The principles laid out in this chapter also apply to more general formulations of the optimal equipment replacement decision, used in other EPRI work on aging assets management.

The second paper, presented in chapter 3, extends the decision framework for equipment replacement to include the possibility of diagnostic testing. It uses a simple economic model to show how to find the optimal policy for testing and replacing aging equipment using *Bayesian analysis*, which is discussed several prior EPRI reports as well as in many text books. Again, the model developed in this chapter does not exhaust the power of such analysis methods, but it does illustrate the basic concepts in a straightforward and concrete manner. The principles laid out in this chapter also apply to more general formulations of the optimal equipment testing and replacement decision, used in other EPRI work on aging assets management.

The third paper addresses a question that frequently arises in using the dynamic programming methods that form the basis for EPRI's equipment replacement decision framework – what should a company do if the annual cost of optimal replacement policy exceeds the available budget? This issue arises because the dynamic programming models find the optimal *stationary* policy for equipment replacement, which applies over an indefinite time horizon. Following this policy typically leads to a stable distribution of the asset population among the various ages and condition states. However, initially, the asset population inventory may be quite different than the long-run distribution. This situation may force unreasonably large expenditures in the early years of application of the stationary policy, as the oldest, most deteriorated equipment in the initial inventory is replaced. Thus a *transient* policy must be applied to the initial asset inventory, in order to move it toward the long-run population distribution without violating the available budget constraints. This chapter discusses how to find such transient policies.

2
OPTIMAL EQUIPMENT REPLACEMENT USING DYNAMIC PROGRAMMING

Utilities today face the twin challenges of satisfying increasingly high standards for reliability and service quality while at the same time reducing costs and improving earnings. To meet the challenges, utilities are adopting asset management as their framework for allocating capital and operation/maintenance budgets. Simply stated, asset management consists of decision-making processes which have the goal of deriving the most value from utility assets within the available budget.

Many utilities have aging equipment, such as underground cables and power transformers, that are rapidly deteriorating. In many cases, utilities face substantial replacement costs as cohorts of equipment installed during periods of high load growth in previous decades simultaneously begin to show rapidly increasing failure rates as they reach end of life. The capital required to replace this vital infrastructure represents a substantial financial burden over the coming decade. Thus, equipment replacement decisions represent an importance aspect of asset management.

Managing a population of aging equipment requires considering three distinct phenomena: 1) representing the dynamic processes of failure and replacement of the equipment; 2) projecting changing failure rates as equipment ages; 3) balancing the costs of equipment failure with replacement options to come up with the least-cost equipment replacement policy. This chapter describes and illustrates a decision framework for equipment replacement. It uses a simple economic model to show how to find the optimal (that is, the lowest life-cycle cost) policy for replacing aging equipment. The theory underlying this model is called *dynamic programming*, and it is discussed in detail in several prior EPRI reports as well as in many text books (see the references at the end of this chapter). The model developed here does not exhaust the power of dynamic programming methods, but it does illustrate the basic concepts in a straightforward and concrete manner.

The objective of the economic model is to minimize the lifecycle cost of maintaining the equipment inventory, subject to serviceability requirements. The lifecycle cost comprises the total of the installation, testing (if used), and failure costs throughout a long-term horizon, all taken on a present value basis. The serviceability requirement means that equipment that fails must be replaced.

Modeling the Dynamics of Equipment Deterioration

Over time, the condition of a piece of equipment usually deteriorates as it experiences environmental and operating stresses. The economic model utilized in this note represents the dynamic process of equipment deterioration mathematically, using a set of equations that provide a forecast of future deterioration. The forecast depends on the present condition of the equipment. Information summarizing the current condition is called its *state*. Dynamic equations then represent how the state evolves over time. While many representations of the state may be useful in replacement decisions, in the simple model presented here, it is assumed that the equipment age t is the only state variable.

The economic model represents the decisions regarding equipment replacement as depending on its state. The specification of a decision for each state is called a *policy*, so the model develops a state-dependent policy that minimizes the lifecycle cost of maintaining the equipment population. In the simple *optimal replacement age* model discussed in this note, the decision options are simply to replace now at cost R or to wait one year at cost 0.

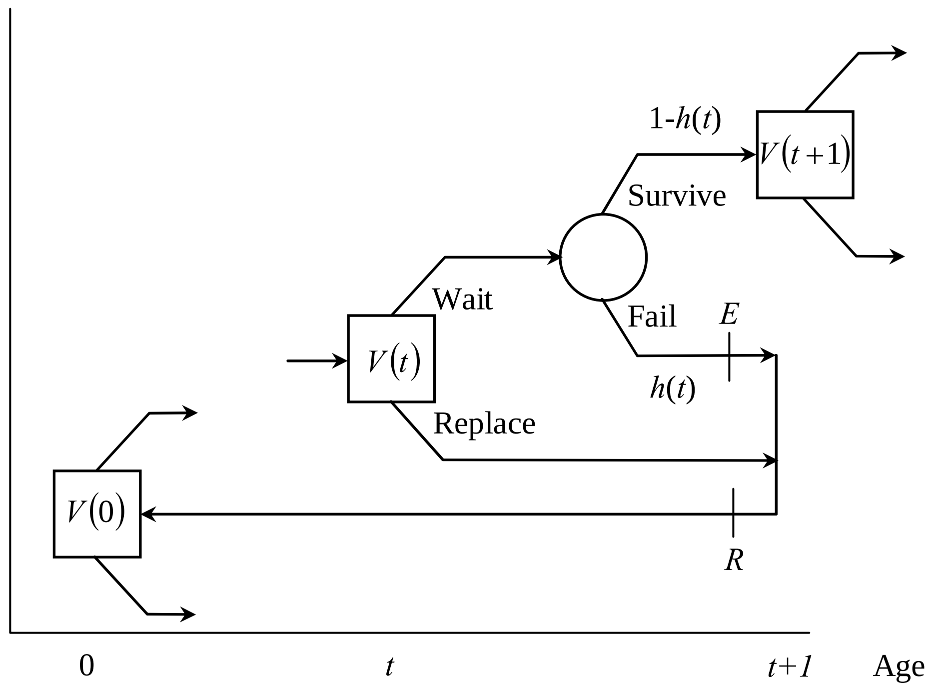

Figure 2-1
Dynamics of Equipment Deterioration

In general, the evolution of the equipment state cannot be predicted with certainty; that is, deterioration is subject to random influences. This fact implies that the dynamic equations must describe the state evolution probabilistically – given the current state, there will be a probability distribution of states in which the equipment might be found a year later. In the optimal replacement age model, the uncertainty is represented by two possibilities: either the equipment survives, in which case it will become a year older, or it fails, in which case it is replaced with new (age 0) equipment. Failure has probability $h(t)$ and incurs an additional failure cost E, and then it must be replaced. Survival has probability $1-h(t)$ and incurs no cost. The failure probability $h(t)$ depends on the equipment age and is called the *hazard function*. (More precisely, $h(t)$ is the probability that equipment that has survived to age t does not survive to age $t+1$). Assume the failure rate increases with age, $h(t+1) \geq h(t)$. The dynamics of equipment deterioration are illustrated in figure 2-1, which shows the uncertainty (indicated by the circular node) and the decision (indicated by the square node) at stage t. The dynamic model of the equipment deterioration over time is constructed by stacking multiple stages of this type into a *decision tree*.

In the optimal replacement age model, the hazard function is represented by a *linearized s-curve*, illustrated in figure 2-2a. This form of the hazard function exhibits the following plausible behavior. When the equipment is relatively new, its likelihood of failure is small and constant. As it ages, it reaches a period, called *burn-out*, during which the failure probability accelerates rapidly. At some later age, the hazard rate levels off again, but at a higher rate than when the equipment was new, reflecting the intuition that equipment which survives to old age is unusually durable. Several empirical studies of power delivery equipment (notably underground cables and wood poles) have confirmed statistically this behavior of failure rates with age. Mathematically, the hazard function has the form

$$h(t) = H_1 + \max\{0, M(t - T_1)\} - \max\{0, M(t - T_2)\}$$

with $T_2 > T_1$, where

$H_1 =$ steady-state failure rate

$T_1 =$ beginning of burn-out

M = failure acceleration rate during burn-out

$T_2 =$ end of burn-out

$H_1 + M(T_2 - T_1) =$ final failure rate

The values of these parameters for the hazard function shown in figure 2-2 are given in table 2-1.

Table 2-1
Hazard Function Parameters

H_1	Steady-state failure rate	0.025
T_1	Beginning of burn-out	25
M	Failure acceleration during burn-out	0.01
T_2	End of burn-out	40

Closely related to the hazard function is the *survivor curve* $S(t)$, which is the probability that the equipment survives (i.e. does not fail prior) to age t. Figure 2-2b illustrates the survivor curve for the hazard function shown in figure 2-2a. Mathematically,

$$S(t) = \prod_{s=0}^{t-1} [1 - h(s)]$$

where $S(0) = 1$.

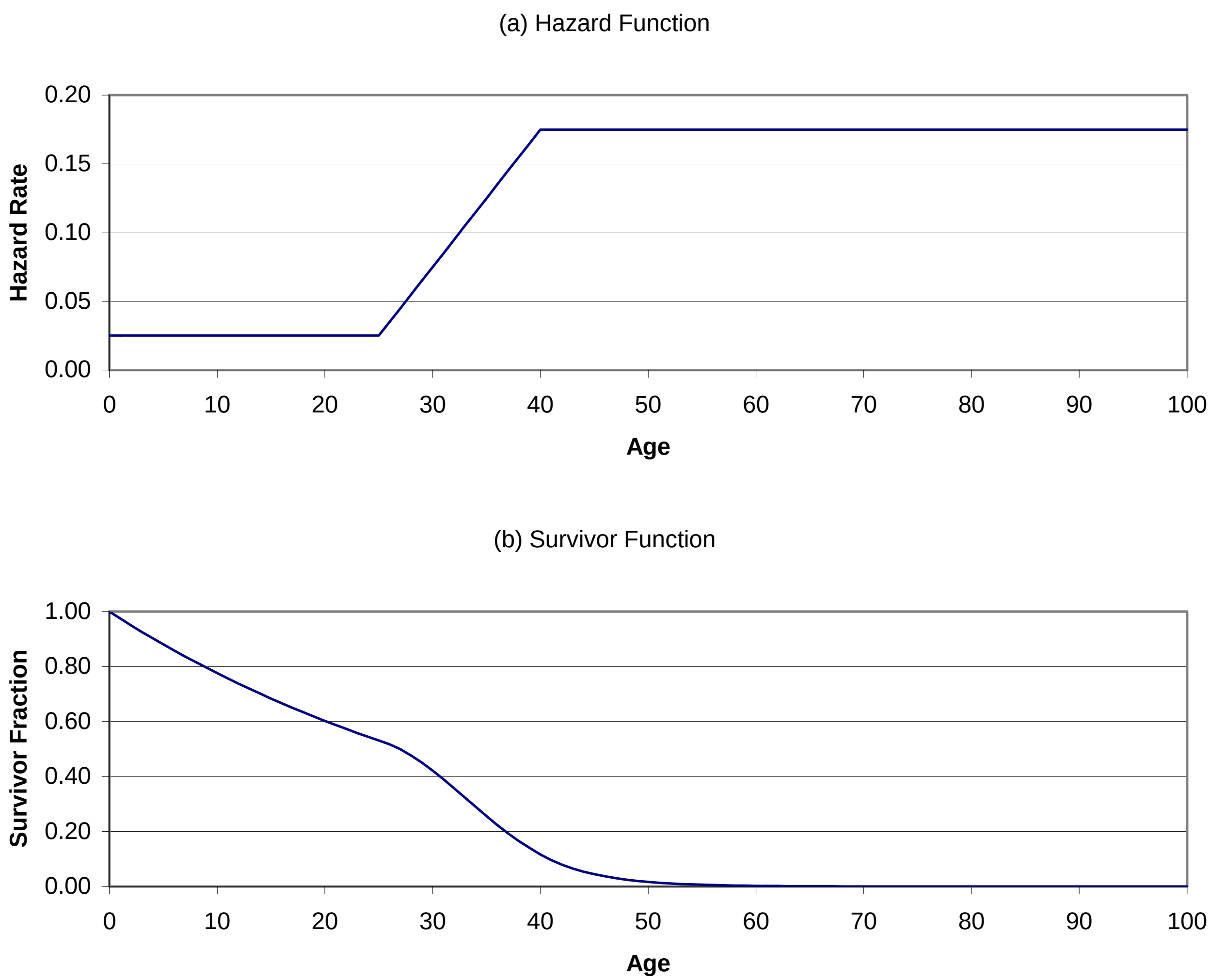

Figure 2-2
Linearized S-Curve Hazard Function and Survivor Curve

Modeling the Replacement Decision

A key concept in modeling replacement is the *cost-to-go*, which is the present value (over an infinite horizon) of following the optimal replacement policy starting when the equipment is age t. Let

$V(t) =$ cost-to-go at age t with an infinite horizon

$\alpha =$ annual discount factor (if the annual discount rate is r then $\alpha = \frac{1}{1+r}$)

At age t, the decision options are to replace immediately or to wait one year. In the former case, the cost-to-go is simply the cost of replacement plus the cost-to-go of the optimal policy starting with new equipment age 0, that is $R + V(0)$. In the latter case, waiting a year creates an uncertainty about whether or not the equipment fails in that period. If it does not fail, the cost is the discounted cost-to-go of equipment one year older, that is $\alpha V(t+1)$. If it does fail, the cost is cost of failure plus the cost of replacement plus the cost-to-go of the optimal policy starting with new equipment age 0, that is $E + R + V(0)$. Thus the cost-to-go of waiting a year is the expected value of these two contingencies weighted by their respective probabilities,
$[1 - h(t)]\alpha V(t+1) + h(t)[E + R + V(0)]$.
Then the optimal decision is to replace at age t if
$R + V(0) < [1 - h(t)]\alpha V(t+1) + h(t)[E + R + V(0)]$
or to wait if the reverse is true. Therefore, the cost-to-go at age t is the minimum of the two decision options, immediate replacement or waiting a year:

$$V(t) = \min\{R + V(0), [1 - h(t)]\alpha V(t+1) + h(t)[E + R + V(0)]\}$$

This relationship is known as *Bellman's equation*, and it establishes a recursive relationship among the costs-to-go at different ages that permits solving for the optimal policy, as will be discussed below. The entire formulation of this problem is called a *dynamic program*.

The cost-to-go depends on the policy used; a policy is a decision rule (or function) d that specifies what action is to be taken in each state; this dependence is indicated explicitly by including the policy in the notation for the cost-to-go as $V(t, d)$.

It is a property of this particular model (that is, age is the only state variable and the failure rate increases with age) that the optimal policy is completely characterized by a single age τ^*, the optimal replacement age. That is, the optimal policy is to wait if $t < \tau^*$ and to replace if $t \geq \tau^*$.

Thus, define a decision policy (not necessarily optimal) to replace at age τ by

$$d(t) = \begin{cases} \text{wait} & \text{if } t < \tau \\ \\ \text{replace} & \text{if } t \geq \tau \end{cases}$$

Since the cost-to-go depends on the replacement age τ, it is computed for this policy by:

$$V(t,\tau)=[1-h(t)]\alpha V(t+1,\tau)+h(t)[E+R+V(0,\tau)] \text{ for } t<\tau$$

$$V(t,\tau)=R+V(0,\tau) \text{ for } t\geq\tau$$

The optimal replacement age τ^* minimizes the cost-to-go. In order to find the optimal replacement age, first these equations are used to derive a formula for $V(t,\tau)$ as a function of $V(0,\tau)$. In particular, for $t<\tau$

$$V(t,\tau)=[V(0,\tau)+R]\left\{\alpha^{(\tau-t)}\frac{S(\tau)}{S(t)}+\sum_{s=t}^{\tau-1}\alpha^{(s-t)}\frac{S(s)-S(s+1)}{S(t)}\right\}+E\left\{\sum_{s=t}^{\tau-1}\alpha^{(s-t)}\frac{S(s)-S(s+1)}{S(t)}\right\}$$

This formula has the following interpretation:

$\frac{S(s)-S(s+1)}{S(t)}$ is the probability that a piece of equipment currently age t fails in service at age $s\geq t$, and $\frac{S(\tau)}{S(t)}$ is the probability that it survives to the scheduled replacement age τ. Whether the equipment fails in service or is replaced as scheduled, it incurs the cost $V(0,\tau)+R$ at the time of replacement, appropriately discounted from the time of replacement to the current age; this explains the first term in the formula. In addition, equipment that fails in service incurs the additional cost E, appropriately discounted, as computed in the second term.

To simplify the notation, let

$$A(t)=\alpha^t S(t) \text{ with } A(0)=1$$

$$B(t)=\sum_{s=0}^{t-1}\alpha^s[S(s)-S(s+1)] \text{ with } B(0)=0$$

Note that recursively $B(t)=B(t-1)+\alpha^{(t-1)}[S(t-1)-S(t)]$

Then for $t<\tau$

$$V(t,\tau)=[V(0,\tau)+R]\left[\frac{A(\tau)}{A(t)}+\frac{B(\tau)-B(t)}{A(t)}\right]+E\left[\frac{B(\tau)-B(t)}{A(t)}\right] \qquad (2\text{-}1)$$

Now, setting $t=0$

$$V(0,\tau)=[V(0,\tau)+R]\{A(\tau)+B(\tau)\}+E\{B(\tau)\}$$

Thus

$$V(0,\tau) = R\frac{A(\tau)+B(\tau)}{1-A(\tau)-B(\tau)} + E\frac{B(\tau)}{1-A(\tau)-B(\tau)} \qquad (2\text{-}2)$$

Solving for the Optimal Policy Using Policy Iteration

In general, the optimal solution to a dynamic program is a specification of a decision for each state, that is, a policy, which satisfies Bellman's equation. Policy iteration is a mathematical programming algorithm that calculates the optimal policy for a dynamic program. It is a general procedure that applies to a wide variety of dynamic programming models. It is an iterative procedure that starts with a trial policy and seeks to improve the policy by changing the decisions in each state. The policy iteration algorithm consists of two general steps:

Step 1: Policy Evaluation: With the current trial policy, compute the cost-to-go

Step 2: Policy Improvement: With the current cost-to-go, test the decision in each state to determine whether changing it would reduce the cost. If so, then revise the trial policy and go to step 1; otherwise stop: the current policy is optimal.

In the optimal replacement age model, the policy is characterized by the replacement age, so the policy improvement step consists of changing the replacement age. More specifically for this model, policy iteration looks like this:

Step 1 - Policy Evaluation:

Assume a trial value for the optimal replacement age, τ'

Compute the trial cost-to-go for each age using formula (2-2) for $V(0,\tau')$ and formula (2-1) for $V(t,\tau')$ with $t < \tau'$ and $V(t,\tau') = R + V(0,\tau')$ with $t \geq \tau'$.

Step 2 - Policy Improvement:

Let $Q(t) = \min\{R + V(0,\tau'), [1-h(t)]\alpha V(t+1,\tau') + h(t)[E + R + V(0,\tau')]\}$.

Only one of the following cases applies:

(1) If $t < \tau'$ and $V(t,\tau') > Q(t)$ then decrease τ'.

(2) If $t > \tau'$ and $V(t,\tau') > Q(t)$ then increase τ'.

(3) If $V(t,\tau') \leq Q(t)$ for all t then stop; the current value of τ' is optimal. Otherwise, continue increasing or decreasing τ' until the respective inequality no longer holds.

Go back to step 1 with the new trial age τ'.

Table 2-3 and figure 2-3 illustrate the policy iteration algorithm for the range of ages of interest, with the linearized s-curve hazard function of figure 2-2. The entries in *italics* in table 3 represent the non-optimality of the current solution (conditions (1) or (2) of policy improvement step). The following are the values of the cost parameters:

Table 2-2
Base Case Cost Parameters

R	Replacement cost	$10,000
E	Failure cost	$5,000
r	Annual discount rate	5%

Note that the algorithm converges to the optimal policy in just three iterations. For the example data, the initial trial value of the replacement age is arbitrarily set at 50 years. On iteration 1, $V(t,\tau') > Q(t)$ for ages between 49 and 37, so the trial replacement age is lowered to 37 for the next iteration. On iteration 2, $V(t,\tau') > Q(t)$ for age 37, so the trial replacement age is raised to 38. On iteration 3, $V(t,\tau') \le Q(t)$ for all ages, so the optimal solution is 38 years. (Note that in figure 2-3, the lines representing the three iterations have been separated for clarity.)

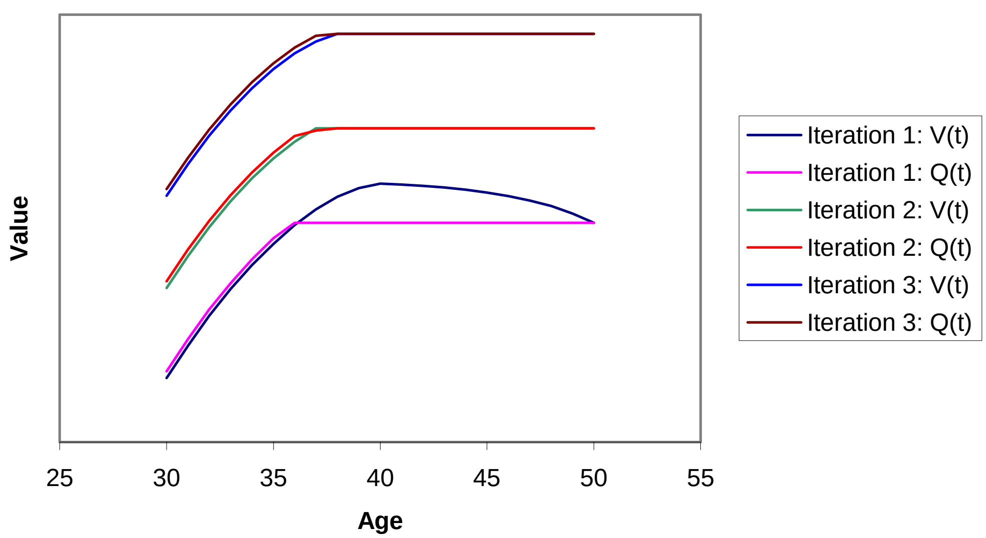

Figure 2-3
Policy Iteration

Table 2-3
Policy Iteration

	Iteration 1		Iteration 2		Iteration 3	
	$\tau' = 50$		$\tau' = 37$		$\tau' = 38$	
Age	$V(t,\tau')$	$Q(t)$	$V(t,\tau')$	$Q(t)$	$V(t,\tau')$	$Q(t)$
30	$18,173	$18,244	$18,121	$18,194	$18,091	$18,164
31	$18,516	$18,585	$18,458	$18,528	$18,425	$18,495
32	$18,828	$18,894	$18,763	$18,829	$18,724	$18,792
33	$19,110	$19,174	$19,035	$19,099	$18,991	$19,056
34	$19,363	$19,425	$19,276	$19,339	$19,225	$19,288
35	$19,588	$19,648	$19,486	$19,547	$19,426	$19,487
36	$19,784	$19,811	$19,662	$19,722	$19,591	$19,652
37	*$19,950*	*$19,811*	*$19,803*	*$19,781*	$19,718	$19,777
38	*$20,081*	*$19,811*	$19,803	$19,803	$19,799	$19,799
39	*$20,175*	*$19,811*	$19,803	$19,803	$19,799	$19,799
40	*$20,221*	*$19,811*	$19,803	$19,803	$19,799	$19,799
41	*$20,210*	*$19,811*	$19,803	$19,803	$19,799	$19,799
42	*$20,196*	*$19,811*	$19,803	$19,803	$19,799	$19,799
43	*$20,178*	*$19,811*	$19,803	$19,803	$19,799	$19,799
44	*$20,156*	*$19,811*	$19,803	$19,803	$19,799	$19,799
45	*$20,127*	*$19,811*	$19,803	$19,803	$19,799	$19,799
46	*$20,090*	*$19,811*	$19,803	$19,803	$19,799	$19,799
47	*$20,043*	*$19,811*	$19,803	$19,803	$19,799	$19,799
48	*$19,983*	*$19,811*	$19,803	$19,803	$19,799	$19,799
49	*$19,907*	*$19,811*	$19,803	$19,803	$21,799	$21,799
50	$19,811	$19,811	$19,803	$19,803	$21,799	$21,799

In general, policy Iteration has the following three extremely desirable features: (1) each trial policy it suggests is guaranteed to be at least as good as the previous policy, (2) the new policy is at least as good in every state, and (3) it is guaranteed to converge to the optimal policy. It also converges extremely rapidly.

Analytical Results

In general, closed-form solutions to dynamic programs are rare (that is, solutions that can be expressed as a formula rather than as the output of an iterative process). However, for the optimal replacement age model, it is possible to find the optimal replacement age by solving a single equation. An approximation to optimal replacement age occurs at $\hat{\tau}$ such that the two decisions (replace now, wait one year) have equal value

$$R + V(0, \hat{\tau}) = [1 - h(\hat{\tau})]\alpha V(\hat{\tau} + 1, \hat{\tau}) + h(\hat{\tau})[E + R + V(0, \hat{\tau})]$$

Rewriting this equation to simplify it gives

$$[1 - \alpha][1 - h(\hat{\tau})][R + V(0, \hat{\tau})] = h(\hat{\tau})E$$

Figure 2-4 illustrates the solution to the example by showing the graphs of the left-hand side (LHS) and right-hand-side (RHS) of this equation. The optimal replacement age occurs where the two graphs cross, 38 years for the example.

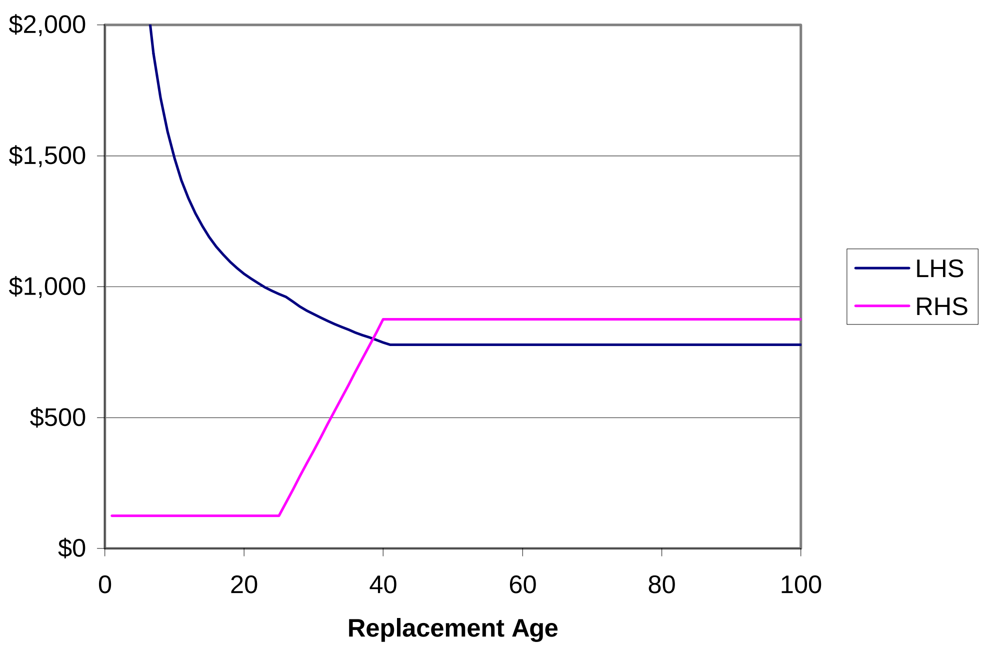

Figure 2-4
Analytical Solution for the Optimal Replacement Age

Note that if the value of E is changed to \$4000, as shown in figure 2-5, the two graphs do not intersect. That is, for the lower failure cost, the optimal replacement age is infinite and the optimal policy is to *run to failure*.

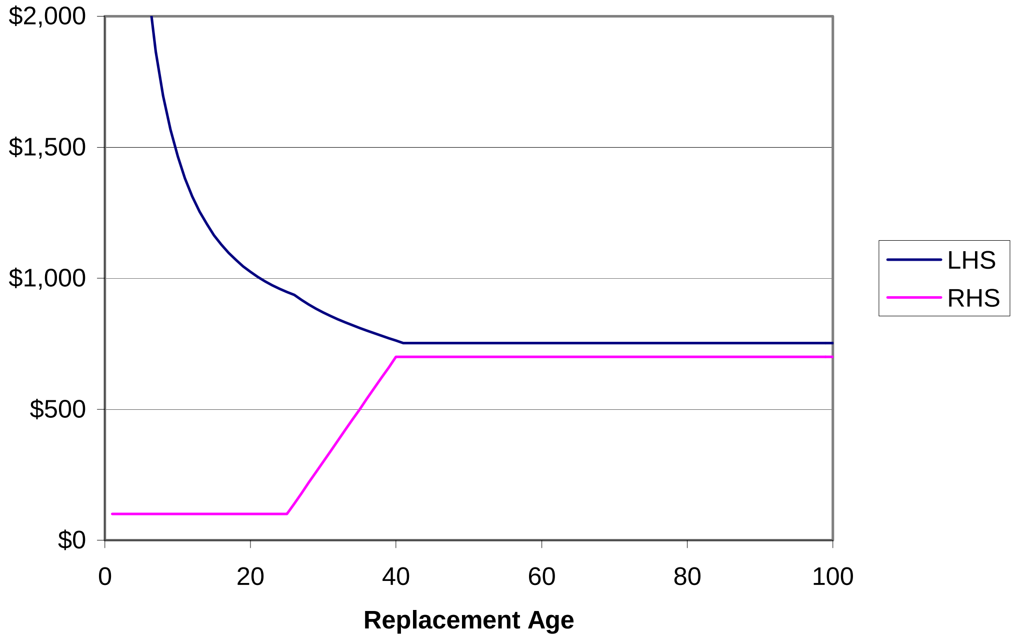

Figure 2-5
Run-to-Failure is Optimal

Steady-State Analysis

After the optimal policy has been applied for a long time, the age distribution of the equipment population stabilizes to a steady-state, regardless of the initial age distribution. This steady state distribution is useful for several reasons. It enables calculating the expenditures due to equipment replacement. It also enables determining the reliability of the equipment population.

Let $\pi(t) =$ fraction of equipment population of age t in the long run (assuming the optimal replacement policy is used)

The probability distribution $\pi(t)$ satisfies the equations

$$\pi(t) = [1 - h(t-1)]\pi(t-1) \text{ for } t = 1, \ldots, \tau^* - 1$$

$$\pi(0) = \sum_{s=1}^{\tau^*-1} h(s)\pi(s) + [1 - h(\tau^* - 1)]\pi(\tau^* - 1)$$

$\pi(t) = 0$ for $t \geq \tau^*$

and the probabilities must add to one

$$\sum_{s=0}^{\tau^*-1} \pi(s) = 1$$

Solving these equations gives

$$\pi(t) = S(t) \left[\sum_{s=0}^{\tau^*-1} S(s) \right]^{-1} \text{ for } t = 0, \ldots, \tau^* - 1$$

Figure 2-6 shows the steady-state age distribution for the example data. Note that approximately 0.7% of the equipment survives till the scheduled replacement age.

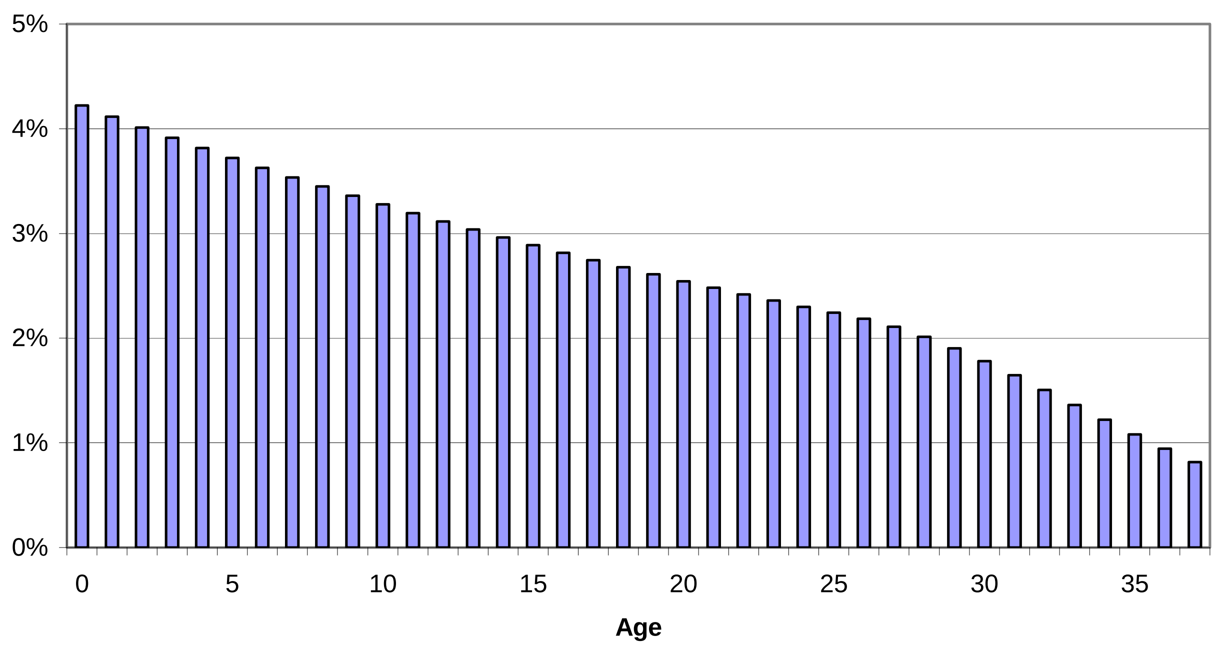

Figure 2-6
Steady-State Population Distribution

The expected annual number of failures per unit of equipment (a measure of reliability) is the sum over all ages of the steady-state population at that age times the failure rate at that age, $\phi = \sum_{s=0}^{\tau^*-1} h(t)\pi(t)$. For the example, $\phi =$ 0.0352; that is, in a population of 10,000, one would expect about 352 failures per year. By contrast, about $\left[1-h\left(\tau^*-1\right)\right]\pi\left(\tau^*-1\right)=$ 70 scheduled replacements would occur.

When $\tau^*=\infty$, the steady state distribution still exists, because

$$\sum_{s=0}^{\infty} S(s)=\sum_{n=0}^{\infty}[n+1][S(n)-S(n+1)]= \text{ the mean time to failure.}$$

In the example, the mean time to failure is about 24.7 years, and $\phi=$ 0.0405, meaning that in a population of 10,000, one would expect about 405 failures per year.

The present value cost, including replacement and failure costs, of the optimal policy is the sum over all ages of the steady-state population at that age times the cost-to-go at that age, $\sum_{t=0}^{\tau^*-1}\pi(t)V\left(t,\tau^*\right)$. For population of 10,000, the present value cost is $128 million or $6.42 million annually $\left(r\sum_{t=0}^{\tau^*-1}\pi(t)V\left(t,\tau^*\right)\right)$.

The cost of a failure E may be interpreted as the value of lost service due to equipment unavailability. As E increases, it becomes optimal to replace at earlier and earlier ages. This trade-off is illustrated in table 2-4 and figure 2-7.

Table 2-4
The Cost vs. Reliability Trade-off

Failure Cost	Replacement Age	Failures per year	Scheduled Replacements per year	Total Replacements	Total Replacement Cost
$4,000	Infinite	405	0	405	$4,054,882
$5,000	38	352	70	422	$4,221,375
$7,000	35	323	111	434	$4,344,846
$9,000	33	302	144	446	$4,463,523
$11,000	31	282	180	462	$4,617,439
$13,000	30	272	199	471	$4,709,111

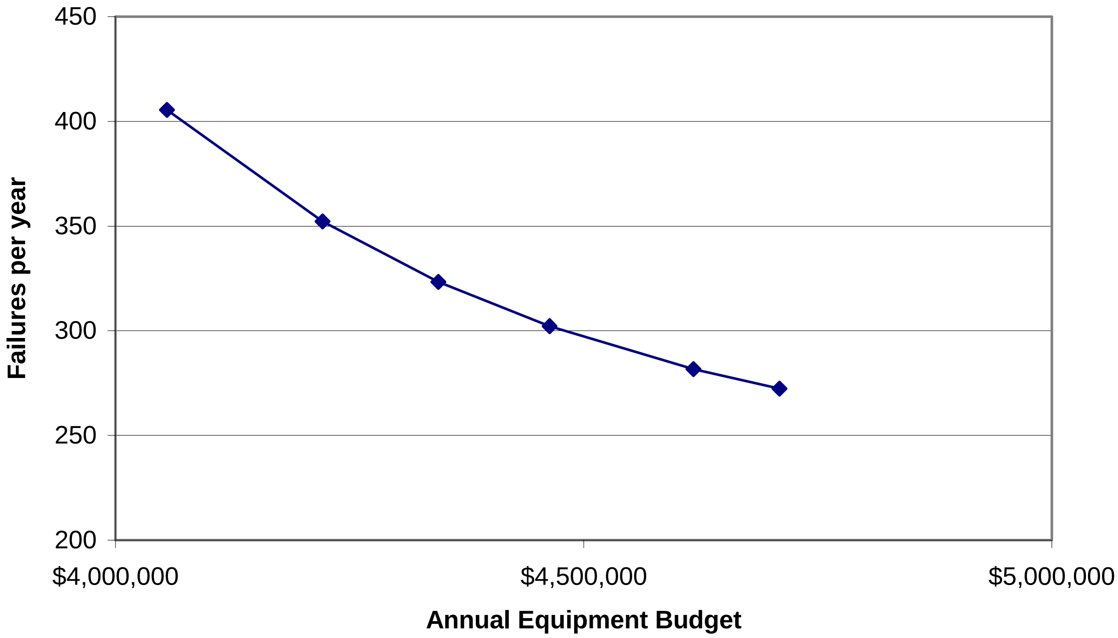

Figure 2-7
The Cost vs. Reliability Trade-off

Conclusion

In general, modeling optimal equipment replacement involves three stages. First, choose an appropriate representation of the equipment condition state and formulate a mathematical description of the evolution of the state with time. In the example described in this note, this stage is illustrated in figure 2-1. Second, gather data to estimate the relevant parameters of this formulation, notably the hazard functions representing the failure behavior of the equipment as it ages. This stage involves using historical data, expert judgment, or a combination of the two. Third, formulate an optimization model using dynamic programming and solve it using the policy iteration algorithm. Solving the dynamic program is essentially a mechanical process that requires solving a system of equations, such as (1) and (2) in this example. In general, the equations will not have a closed form solution, so the solution algorithm will need to invert a matrix in step 1.

These three stages interact with each other. Choosing a representation of the state is a design decision. Usually, the amount of information that the state can represent is limited, for two reasons. First, the amount of historical information available about equipment performance is often limited for reasons of cost or practicality. Second, a complex definition of the state can make the dynamic equations mathematically intractable. The more information that is encoded in the state, the higher the dimension that is needed in the dynamic equations, a phenomenon called the “curse of dimensionality.” Thus, considerations of data available in stage two and mathematical tractability in stage three must influence the state definition in stage one.

This chapter provides a simple example to illustrate the principles that apply to more general formulations of the optimal equipment replacement decision.

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3
THE VALUE OF TESTING IN EQUIPMENT REPLACEMENT DECISIONS

When considering when to replace aging equipment, the likelihood that the equipment will survive another year represents a critical uncertainty. In the simple optimal replacement age model discussed in chapter 2, equipment is replaced when the survival probability becomes low enough that the cost of immediate replacement is less than waiting a year and paying for a failure if the equipment fails during that time. One way to reduce this uncertainty, and thus the cost of the replacement policy, is to use diagnostic tests to determine which equipment is more likely to fail at a certain age, and then to replace only that equipment rather than all the equipment of that age. The value of testing derives from the information it provides about equipment condition that permits better forecasting of future failures. This note describes and illustrates a decision framework for equipment replacement that includes the possibility of testing. It uses a simple economic model to show how to find the optimal (that is, the lowest life-cycle cost) policy for testing and replacing aging equipment. The theory underlying this model is called *Bayesian analysis*, and it is discussed in detail in several prior EPRI reports as well as in many text books (see the references at the end of this chapter). The model developed here does not exhaust the power of such analysis methods, but it does illustrate the basic concepts in a straightforward and concrete manner.

To test or not is a decision, just as is the decision to replace now or wait. In fact, the two decisions are intimately linked, since the value of testing depends on the action taken as a result of the test outcome. A test has no value unless at least one of its possible outcomes changes the replacement decision. That fact implies that it is not always useful to test; at some ages, equipment failure is either so unlikely or so likely that a test would not change the decision, so typically tests are applied only at ages when the failure probability is in some intermediate range. However, testing also introduces its own uncertainty, since most tests are not perfectly accurate. The value of testing depends on its ability to discriminate equipment that is likely to fail from that which is not.

An Example

Consider a population of 10,000 pieces of equipment, all of the same age, with a probability of failing in the next year of 2%. Suppose a test is available that can distinguish equipment that is in good condition from that which is in bad condition. Bad equipment is due to fail in the next year but good equipment will likely survive. However, the test is not perfectly accurate – in fact, the probability that the test result is accurate (that is, the test result says that the equipment is in good condition when it is actually due to survive and it says that the equipment is in bad condition when it is actually due to fail) is 95%. (Note that these two probabilities do not have to be the same.)

After observing the test results, there are four possibilities:

- Equipment due to survive for which the test result is good
- Equipment due to survive for which the test result is bad
- Equipment due to fail for which the test result is bad
- Equipment due to fail for which the test result is good

Table 3-1 illustrates the possibilities.

Table 3-1
Test Quality and Results

	Test good	Test bad
Equipment due to survive	95%	5%
Equipment due to fail	5%	95%

	Test good	Test bad	Total	Probability of condition
Equipment due to survive	9310	490	9800	98%
Equipment due to fail	10	190	200	2%
Total	9320	680	10000	
Probability of test outcome	93%	7%		

For instance, 98% or 9800 pieces of equipment are in condition to survive and 95% or 9310 of them will give a test good result, whereas 5% or 490 will give a bad test result. Overall 93% of the equipment will test good and 7% will test bad.

Based on table 3-1, then, this test would lead you to believe that there are 680 bad pieces of equipment, 190 of which are really due to fail; thus, the failure likelihood given a bad test result is 28%. Furthermore, this test would lead you to believe that there are 9,320 good pieces of equipment, 10 of which are really due to fail; thus, the failure likelihood given a good test result is 0.107%. In other words, the test has changed the probability of failure from 2% prior to the test either to 28% after a bad test result or to 0.107% after a good test result (these probabilities are called the *posterior* failure probabilities given the test outcome to distinguish them from the *prior* failure probability before the test result is known). The discriminatory power of the test is the ratio of these two likelihoods or 260.

The value of the test is determined by what decisions are made based on the test result. Suppose the cost of replacing the equipment is $1000 but the cost of a failure is $5000. Table 3-2 illustrates the economic consequences of the decision to test.

Table 3-2
Economic Analysis of the Testing Decision

	# of Equipment		
Expected cost of not replacing without testing	200	$1,000,000	⇐ Optimal Decision
Expected cost of replacing without testing	10,000	$10,000,000	
Optimal cost without testing		$1,000,000	
Expected cost of not replacing with bad test	190	$950,000	
Expected cost of replacing with bad test	680	$680,000	⇐ Optimal Decision
Expected cost of not replacing with good test	10	$50,000	⇐ Optimal Decision
Expected cost of replacing with good test	9,320	$9,320,000	
Optimal cost with testing		$730,000	
Value of testing		$270,000	
per equipment		$27	

Without the test, the choice is between replacing all 10,000 pieces of equipment (because there is no way to know *a priori* which will fail) at a cost of $10 million or not replacing, in which case there will be 200 failures at a cost of $1 million; clearly the optimal decision is not to replace.

If the test is used, then the decision depends on its outcome. If the test outcome is bad, the choice is between replacing all 680 pieces of equipment with that result at a cost of $680,000 or not replacing, in which case there will be 190 failures at a cost of $950,000; therefore, the optimal decision for equipment that tests bad is to replace. On the other hand, if the test outcome is good, the choice is between replacing all 9,320 pieces of equipment with that result at a cost of $9.3 million or not replacing, in which case there will be 10 failures at a cost of $50,000; hence, the optimal decision for equipment that tests good is not to replace. Thus, the test discriminates the equipment that is more likely to fail, and even though only 28% of the equipment that tests bad will actually fail, that level discrimination is enough to justify replacing it all.

The cost of following the optimal strategy with testing (replace only if the test is bad) is therefore \$730,000, which is less than the cost of the optimal strategy without testing (do not replace) of \$1 million. Therefore, testing is optimal as long as its cost is less than the difference between the two cases, \$270,000 or \$27 per piece.

Testing in the Dynamic Model

The example just discussed is a static or single period model. To extend testing to the dynamic model of equipment replacement discussed in chapter 2 requires several modifications. Most importantly, the underlying failure rate increases with age, so that the posterior failure probabilities resulting from the test also increase with age. This fact implies that there will be a range of ages during which it will be optimal to test. Parallel reasoning to that used to derive the optimal replacement age model in chapter 2 can also be used to formulate an *optimal replacement age with testing* model.

The probabilities prior to testing that equipment age t survives or fails are given by the hazard function

$h(t) =$ probability that equipment fails in the next period.

$1 - h(t) =$ probability that equipment age t survives in the next period.

Test accuracy is represented by the probabilities

$p_B =$ probability that test result is bad given that the equipment is due to fail

$p_G =$ probability that test result is good given that the equipment is due to survive

Table 3-3
Test parameter definitions

	Test good	Test bad
Equipment due to survive	p_G	$1 - p_G$
Equipment due to fail	$1 - p_B$	p_B

Assume that the test accuracy is constant with time and, in particular, that it is independent of the increasing failure rate $h(t)$. Also, it can be assumed that $p_B > 1 - p_B$ and $p_G > 1 - p_G$, that is, the test reveals some information about the true condition of the equipment (unless they all equal $\frac{1}{2}$, in which case the test gives no information). In the example, $p_G = p_B = 95\%$.

Then the total probabilities of the test outcomes are computed using the following table

Table 3-4
Test parameter definitions (continued)

	Test good	Test bad	Total Prior Failure Probability
Equipment due to survive	$[1-h(t)]p_G$	$[1-h(t)][1-p_G]$	$1-h(t)$
Equipment due to fail	$h(t)[1-p_B]$	$h(t)p_B$	$h(t)$
Total probability of test outcome	$h(t)[1-p_B]+[1-h(t)]p_G$	$h(t)p_B+[1-h(t)][1-p_G]$	1

Define

$\lambda(t)=h(t)p_B+[1-h(t)][1-p_G]=$ probability of bad test outcome

$1-\lambda(t)=h(t)[1-p_B]+[1-h(t)]p_G=$ probability of good test outcome

In the example $\lambda=$ 7%.

Then the (posterior) failure probabilities given the test outcome are

$q_B(t)=\dfrac{h(t)p_B}{h(t)p_B+[1-h(t)][1-p_G]}=\dfrac{h(t)p_B}{\lambda(t)}=$ probability that equipment fails given bad test outcome

$q_G(t)=\dfrac{h(t)[1-p_B]}{h(t)[1-p_B]+[1-h(t)]p_G}=\dfrac{h(t)[1-p_B]}{1-\lambda(t)}=$ probability that equipment fails given good test outcome

These equations are specific instances of a general result in probability theory called *Bayes' Theorem,* and so they are often called *Bayesian updating* of the failure rate based on the test result. In the example, $q_B=$ 28% and $q_G=$ 0.107%

Note the following relationships among the test parameters:

- $\lambda(t)q_B(t)=h(t)p_B$
- $\lambda(t)[1-q_B(t)]=[1-h(t)][1-p_G]$
- $[1-\lambda(t)]q_G(t)=h(t)[1-p_B]$

- $[1-\lambda(t)][1-q_G(t)]=[1-h(t)]p_G$

Figure 3-1 shows the behavior of the probability of a bad test outcome, $\lambda(t)$, and the posterior failure probabilities, $q_B(t)$ and $q_G(t)$ with the test parameters used in the example. In general, they all increase with age because the failure rate $h(t)$ increases with age.

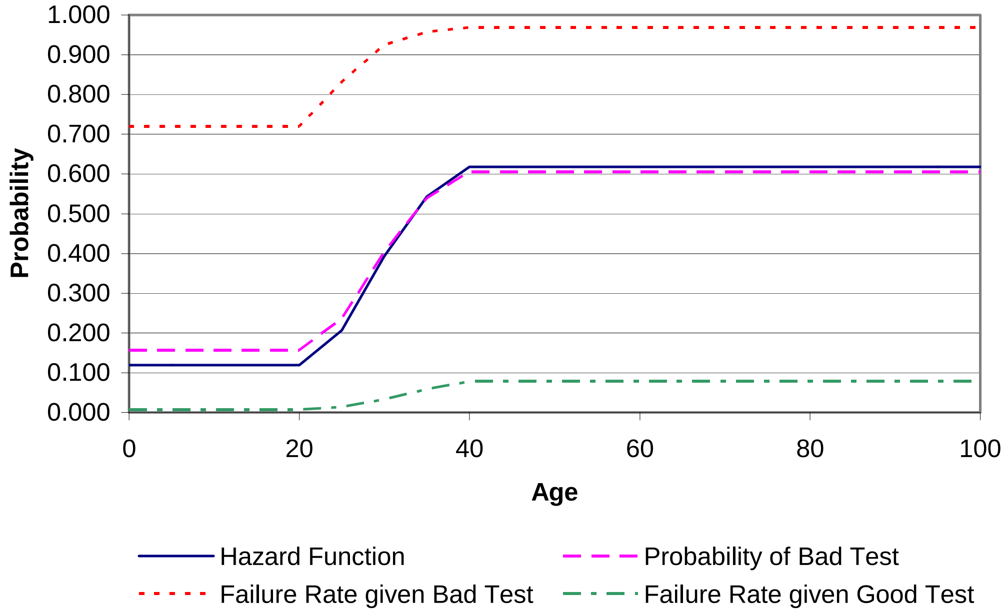

Figure 3-1
Probability parameters of the testing model

Modeling the Testing and Replacement Decision

When testing is a decision option, the dynamic model discussed in chapter 2 has to be modified. One critical issue that has to be resolved in a dynamic model is how much information is revealed by repeated tests, since in a multi-stage model, testing can occur at each stage. A number of alternative assumptions about repeated tests could be considered, but for this simple model it has been assumed that the results of repeated tests on the same equipment are *independent* of each other; that is, the probability of each test outcome does not depend on whether a prior test result was good or bad. In other words, testing has no memory. This assumption makes sense if the interval between successive tests is fairly long. Thus, in this paper it is assumed that the equipment ages in increments of 5 years, in contrast to the model discussed in chapter 2, in which the age increment is one year. The hazard function shown in figure 3-1 is the same as that used in the previous chapter but recomputed with the 5-year age increment.

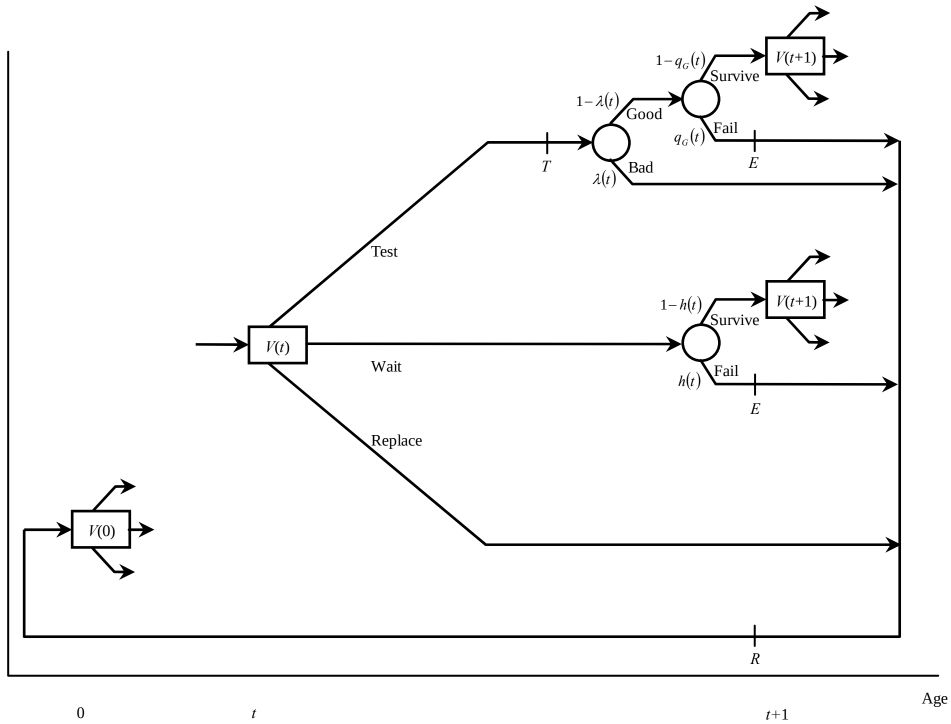

Figure 3-2
Dynamics of Equipment Deterioration and Testing

The optimal replacement age with testing model is formulated as a dynamic program as discussed in the previous chapter; figure 3-2 illustrates the decisions at one stage of the dynamic program. If the decision is not to test, the optimal decision is the same as the basic model discussed in chapter 2 and the cost-to-go is

$$\min\{R+V(0),[1-h(t)]\alpha V(t+1)+h(t)[E+R+V(0)]\}$$

where

$V(0)$ = cost-to-go when equipment is new (age 0)

R = cost of a replacement

E = cost of a failure

$\alpha =$ annual discount factor (if the annual discount rate is r then $\alpha = \frac{1}{1+r}$)

If the decision is to test, the optimal decision depends on the test outcome, and the cost-to-go is given by

Test outcome bad: $V_B(t)=\min\{R+V(0),[1-q_B(t)]\alpha V(t+1)+q_B(t)[E+R+V(0)]\}$

Test outcome good: $V_G(t) = \min\{R + V(0), [1 - q_G(t)]\alpha V(t+1) + q_G(t)[E + R + V(0)]\}$

Let $T =$ cost of test. Then the expected cost to go with testing is

$T + \lambda(t)V_B(t) + [1 - \lambda(t)]V_G(t)$

Then the total expected cost-to-go is

$$V(t) = \min\left\{\begin{array}{c} [1 - h(t)]\alpha V(t+1) + h(t)[E + R + V(0)], \\ T + \lambda(t)V_B(t) + [1 - \lambda(t)]V_G(t), \\ R + V(0) \end{array}\right\}$$

which is Bellman's equation for this dynamic program.

A policy in the optimal replacement age with testing model is characterized by two ages:

$\tau_1 =$ age at which testing begins

$\tau_2 =$ age at which replacement begins regardless of the test result (i.e. testing is no longer useful)

The optimal policy (τ_1^*, τ_2^*) that minimizes the cost-to-go can be found using the policy iteration algorithm discussed in chapter 2.

The cost-to-go is a function of the policy and, particularly, of these two parameters and so it is written as $V(t, \tau_1, \tau_2)$. Then

$$V(t,\tau_1,\tau_2) = \left\{\begin{array}{cc} [1 - h(t)]\alpha V(t+1,\tau_1,\tau_2) + h(t)[E + R + V(0,\tau_1,\tau_2)] & \text{for } t < \tau_1 \\ \hline \begin{array}{c} T + \lambda(t)\{R + V(0,\tau_1,\tau_2)\} \\ + [1 - \lambda(t)]\{[1 - q_G(t)]\alpha V(t+1,\tau_1,\tau_2) + q_G(t)[E + R + V(0,\tau_1,\tau_2)]\} \end{array} & \text{for } \tau_1 \le t < \tau_2 \\ \hline R + V(0,\tau_1,\tau_2) & \text{for } t \ge \tau_2 \end{array}\right\}$$

$$= \left\{\begin{array}{cc} [1 - h(t)]\alpha V(t+1,\tau_1,\tau_2) + h(t)[E + R + V(0,\tau_1,\tau_2)] & \text{for } t < \tau_1 \\ \hline \begin{array}{c} [1 - h(t)]p_G\alpha V(t+1,\tau_1,\tau_2) + \{h(t)p_B + [1 - h(t)](1 - p_G)\}[R + V(0,\tau_1,\tau_2)] \\ + h(t)(1 - p_B)[E + R + V(0,\tau_1,\tau_2)] + T \end{array} & \text{for } \tau_1 \le t < \tau_2 \\ \hline [R + V(0,\tau_1,\tau_2)] & \text{for } t \ge \tau_2 \end{array}\right\}$$

where the second equation results from making the relevant substitutions for $\lambda(t)$, $q_B(t)$ and $q_G(t)$. This formula is essentially a recursive relationship which can be solved for $V(t,\tau_1,\tau_2)$ in terms of $V(0,\tau_1,\tau_2)$. Solution of this recursion equation is needed in order to apply the policy iteration algorithm. After a fair amount of algebra, the solution is

$$
V(t,\tau_1,\tau_2)=
\begin{cases}
\begin{aligned}
&[R+V(0,\tau_1,\tau_2)]\Bigg\{\alpha^{(\tau_2-t)}p_G^{(\tau_2-\tau_1)}\frac{S(\tau_2)}{S(t)}+\sum_{s=\tau_1}^{\tau_2-1}\alpha^{(s-t)}p_G^{(s-\tau_1)}p_B h(s)\frac{S(s)}{S(t)}\\
&\qquad+\sum_{s=\tau_1}^{\tau_2-1}\alpha^{(s-t)}p_G^{(s-\tau_1)}(1-p_G)[1-h(s)]\frac{S(s)}{S(t)}\Bigg\}\\
&+[E+R+V(0,\tau_1,\tau_2)]\Bigg\{\sum_{s=\tau_1}^{\tau_2-1}\alpha^{(s-t)}p_G^{(s-\tau_1)}(1-p_B)h(s)\frac{S(s)}{S(t)}+\sum_{s=t}^{\tau_1-1}\alpha^{(s-t)}h(s)\frac{S(s)}{S(t)}\Bigg\}\\
&\qquad+T\sum_{s=\tau_1}^{\tau_2-1}\alpha^{(s-t)}p_G^{(s-\tau_1)}\frac{S(s)}{S(t)}
\end{aligned} & \text{for } t<\tau_1 \\
\hline
\begin{aligned}
&[R+V(0,\tau_1,\tau_2)]\Bigg\{\alpha^{(\tau_2-t)}p_G^{(\tau_2-t)}\frac{S(\tau_2)}{S(t)}+\sum_{s=t}^{\tau_2-1}\alpha^{(s-t)}p_G^{(s-t)}p_B h(s)\frac{S(s)}{S(t)}\\
&\qquad+\sum_{s=t}^{\tau_2-1}\alpha^{(s-t)}p_G^{(s-t)}(1-p_G)[1-h(s)]\frac{S(s)}{S(t)}\Bigg\}\\
&+[E+R+V(0,\tau_1,\tau_2)]\sum_{s=t}^{\tau_2-1}\alpha^{(s-t)}p_G^{(s-t)}(1-p_B)h(s)\frac{S(s)}{S(t)}+T\sum_{s=t}^{\tau_2-1}\alpha^{(s-t)}p_G^{(s-t)}\frac{S(s)}{S(t)}
\end{aligned} & \text{for } \tau_1\le t<\tau_2 \\
\hline
[R+V(0,\tau_1,\tau_2)] & \text{for } t\ge\tau_2
\end{cases}
$$

This involved formula has an intuitive explanation. Consider, for instance, the formula for ages $t<\tau_1$, before inspections begin. The factor $[R+V(0,\tau_1,\tau_2)]$ represents the cost of a scheduled replacement, that is the cost of the replacement itself plus the cost-to-go of new equipment. The various terms in brackets that multiply it represent the probabilities of the various ways in which equipment would be scheduled for replacement, all discounted appropriately. The first represents the probability that the equipment lasts till the scheduled replacement age τ_2, given that it has survived till age t, times the probability that it tests good at each age between τ_1 and τ_2. The second set of terms represents the probability that the equipment survives and tests good till age $s<\tau_2$, given that it has survived till age t, and then becomes due to fail but correctly tests bad at that time. The final set of terms represents the probability that the equipment survives and tests good till age $s<\tau_2$ but then incorrectly tests bad at that time. The factor $[E+R+V(0,\tau_1,\tau_2)]$

represents the cost of a failure and unscheduled replacement, and the terms in brackets that multiply it represent the probabilities of the various ways in which that type of event could occur. The final term represents the cost of inspections *T* times the expected number of inspections that occur, which is the sum of the probabilities that the equipment survives and tests good at each age $s < \tau_2$. The formula for ages t between τ_1 and τ_2, that is during the inspection period, has an analogous interpretation.

This formula can be simplified somewhat for computation. Define

$$A(t) = \alpha^t S(t)$$

$$B(t) = \sum_{s=0}^{t-1} \alpha^s \left[S(s) - S(s+1)\right] \text{ with } B(0) = 0$$

$$C(t) = \sum_{s=0}^{t-1} \alpha^s p_G^s S(s) \text{ with } C(0) = 0$$

$$D(t) = \sum_{s=0}^{t-1} \alpha^s p_G^s \left[S(s) - S(s+1)\right] \text{ with } D(0) = 0$$

Note that

$$C(t) = \alpha^{t-1} p_G^{t-1} S(t-1) + C(t-1)$$

$$D(t) = \alpha^{t-1} p_G^{t-1} \left[S(t-1) - S(t)\right] + D(t-1)$$

Then after some additional algebra, the cost-to-go is computed as follows:

$$
\begin{aligned}
&V(t,\tau_1,\tau_2)= \\
&[R+V(0,\tau_1,\tau_2)]\left\{p_G^{(\tau_2-\max[t,\tau_1])}\frac{A(\tau_2)}{A(t)}+p_G^{(1-\max[t,\tau_1])}\frac{D(\tau_2)-D(\max[t,\tau_1])}{A(t)}\right. \\
&\left.+\,p_G^{-\max[t,\tau_1]}(1-p_G)\frac{C(\tau_2)-C(\max[t,\tau_1])}{A(t)}+\frac{B(\tau_1)-B(\max[t,\tau_1])}{A(t)}\right\} \\
&+E\left\{p_G^{-\max[t,\tau_1]}(1-p_B)\frac{D(\tau_2)-D(\max[t,\tau_1])}{A(t)}+\frac{B(\tau_1)-B(\max[t,\tau_1])}{A(t)}\right\}+Tp_G^{-\max[t,\tau_1]}\frac{C(\tau_2)-C(\max[t,\tau_1])}{A(t)}
\end{aligned}
\tag{3-1}
$$

for $t<\tau_2$ and

$V(t,\tau_1,\tau_2)=R+V(0,\tau_1,\tau_2)$ for $t\ge\tau_2$

Now, setting $t=0$, further manipulation of this formula gives

$$
\begin{aligned}
&V(0,\tau_1,\tau_2)= \\
&R\left\{p_G^{(\tau_2-\tau_1)}A(\tau_2)+p_G^{(1-\tau_1)}[D(\tau_2)-D(\tau_1)]+p_G^{-\tau_1}(1-p_G)[C(\tau_2)-C(\tau_1)]+B(\tau_1)\right\}Z \\
&+E\left\{p_G^{-\tau_1}(1-p_B)[D(\tau_2)-D(\tau_1)]+B(\tau_1)\right\}Z+T\left\{p_G^{-\tau_1}[C(\tau_2)-C(\tau_1)]\right\}Z
\end{aligned}
\tag{3-2}
$$

where

$$Z=\left\{1-p_G^{(\tau_2-\tau_1)}A(\tau_2)-p_G^{-\tau_1}p_G[D(\tau_2)-D(\tau_1)]-p_G^{-\tau_1}(1-p_G)[C(\tau_2)-C(\tau_1)]-B(\tau_1)\right\}^{-1}$$

These equations are analogous to equations (2-1) and (2-2) of the previous chapter.

Solving for the Optimal Policy Using Policy Iteration

The optimal policy (τ_1^*,τ_2^*) for the optimal replacement age with testing model can be found using the policy iteration algorithm discussed in chapter 2. Using the hazard function of the previous chapter illustrated in figure 3-1, the test parameter values given in table 3-1, and the cost parameters given in table 5, policy iteration gives the optimal policy as τ_1^* = 25 years, τ_2^*> 100 years (effectively infinite).

Convergence of the algorithm is extremely rapid; starting with an initial trial policy of τ_1' = 35, τ_2' =50, it converges in just two iterations. Note that the optimal policy without testing in this example is to replace at age 35; therefore, with testing it is optimal to replace equipment which tests bad starting at age 25, 10 years earlier. Furthermore, with testing, it is never optimal in this example to replace equipment without testing, effectively delaying unconditional replacement indefinitely. The optimal cost-to-go is displayed in table 3-6.

Table 3-5
Base Case Cost Parameters

R	Replacement cost	$10,000
E	Failure cost	$5,000
T	Test cost	$500
r	Annual discount rate	5%

Table 3-6
Optimal Cost-to-Go by Equipment Age

Age t	$V(t,\tau_1^*,\tau_2^*)$	Age t	$V(t,\tau_1^*,\tau_2^*)$
0	$10,995	55	$19,604
5	$11,450	60	$19,604
10	$12,108	65	$19,604
15	$13,061	70	$19,605
20	$14,442	75	$19,606
25	$16,442	80	$19,613
30	$18,157	85	$19,636
35	$19,187	90	$19,716
40	$19,604	95	$20,000
45	$19,604	100	$20,995
50	$19,604		

Steady-State Analysis

After the optimal policy has been applied for a long time, the age distribution of the equipment population stabilizes to a steady-state, regardless of the initial age distribution. This steady state distribution is useful for several reasons. It enables calculating the expenditures due to equipment testing and replacement. It also enables determining the reliability of the equipment population.

Let $\pi(t) =$ fraction of equipment population of age t in the long run (assuming the optimal replacement policy is used)

The probability distribution $\pi(t)$ satisfies the equations

$$\pi(t) = [1 - h(t-1)]\pi(t-1) \text{ for } t = 1, \ldots, \tau_1^* - 1$$

$$\pi(t) = [1 - h(t-1)]p_G \pi(t-1) \text{ for } t = \tau_1^*, \ldots, \tau_2^* - 1$$

$$\pi(0) = \sum_{s=1}^{\tau_1^*-1} h(s)\pi(s) + \sum_{s=\tau_1^*}^{\tau_2^*-1} \{h(t) + [1-h(t)](1-p_G)\}\pi(s) + [1 - h(\tau_2^* - 1)]p_G \pi(\tau_2^* - 1)$$

$$\pi(t) = 0 \text{ for } t \geq \tau_2^*$$

and the probabilities must add to one

$$\sum_{s=0}^{\tau_2^*-1} \pi(s) = 1$$

Solving these equations gives

$$\pi(0) = \left\{ \sum_{t=0}^{\tau_1^*-1} S(t) + \sum_{t=\tau_1^*}^{\tau_2^*-1} p_G^{(t-\tau_1^*+1)} S(t) \right\}^{-1} \text{ if } \tau_2^* < \infty$$

$$\pi(0) = \left\{ \sum_{t=0}^{\tau_1^*-1} S(t) + \sum_{t=\tau_1^*}^{\infty} [S(t) - S(t+1)] p_G \frac{1 - p_G^{(t-\tau_1^*+1)}}{1 - p_G} \right\}^{-1} \text{ if } \tau_2^* = \infty$$

$$\pi(t) = S(t)\pi(0)^{-1} \text{ for } t < \tau_1^*$$

$$\pi(t) = p_G^{(t-\tau_1^*+1)} S(t)\pi(0)^{-1} \text{ for } t \geq \tau_1^*$$

Figure 3-3 shows the steady-state age distributions with and without testing for the example data. Note that with testing, the equipment population is older than without testing; because testing identifies and replaces equipment that is likely to fail, equipment is replaced at older ages.

The expected annual number of failures per unit of equipment (a measure of reliability) is $\phi = \sum_{s=0}^{\tau_1^*-1} h(t)\pi(t) + \sum_{s=\tau_1^*}^{\tau_2^*-1} q_G(t)\pi(t)$. For the example, $\phi =$ 0.0987; that is, in a population of 10,000, one would expect about 987 failures per five-year period or about 197 per year; without testing, one would expect about1520 failures per five-year period or about 304 per year. By contrast, about $\sum_{s=\tau_1^*}^{\tau_2^*-1} \lambda(t)\pi(t) = 926$ scheduled replacements per five-year period or about 185 per year would occur after a bad test outcome; without testing, about 522 per five-year period would survive till the scheduled replacement age, or about 104 scheduled replacements per year.

The present value cost, including replacement, failure, and testing costs, of the optimal policy is $13,556 per unit or $678 annually. Without testing, the present value cost of the optimal policy is $14,156 per unit or $708 annually. Thus, the additional value of testing is $600 per unit (over the $500 cost of the test). In other words, testing is optimal as long as the cost of the test is less than $1,100 per unit.

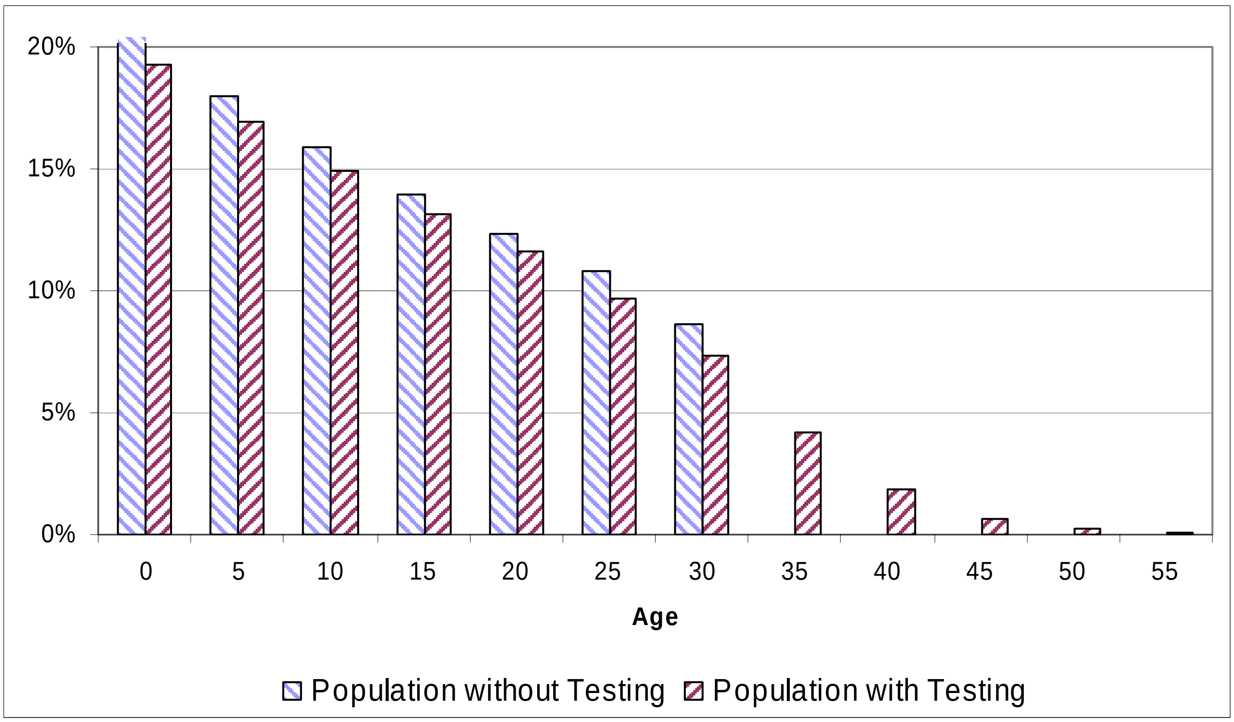

Figure 3-3
Steady-State Population Distribution with and without Testing

Conclusion

Diagnostic tests can play an important role in managing equipment assets. When equipment conditions are not directly observable, such as for underground cables or power transformers, diagnostic tests can provide some information about their condition. However, three issues can limit the usefulness of tests. First, tests are generally not completely accurate; that is, for instance, a cable segment with no insulation degradation may give a test result indicating some level of degradation and vice versa. However, as discussed in this report, even an inaccurate test may provide information useful for asset management decisions. In principle, it should be possible to quantify the level of test accuracy (a subject of on-going research at EPRI), but at present such estimates often rely on expert judgment. Second, the relationship between test outcomes and equipment failure rates is not well established for many kinds of power delivery equipment; that is, how much more likely is a 30-year-old cable segment with moderate insulation degradation to fail in the next year than a cable segment of the same age with no insulation degradation? Again, in principle, it should be possible to quantify this relationship (also a subject of on-going research at EPRI), but at present such estimates rely on expert judgment. Finally, testing is costly (and in some cases may cause failure of the equipment). In fact, these three issues are linked together, raising the question of whether the value of the information, even if somewhat inaccurate, outweighs the cost of obtaining it.

The decision framework discussed in this report models testing using several overall principles. First, whether or not to test is a decision, a part of an overall asset management policy for a particular class of equipment. That is, the value of test information depends on the actions one takes based on the test outcome and on the cost savings attainable over acting without doing the tests. Said another way, a test has value only if its outcome would lead to making a different decision, and even then, unless the value exceeds the cost of the test, one should not use the test. Thus, testing is included among the decision alternatives considered by the framework. Second, the decision framework separates specification of the test outcome from the action taken based upon it. This principle stands in contrast to much of the current industry practice, which is to state the test outcome as a recommendation for action, such as, “replace immediately” or “retest in 3 years.” Separating test outcome from recommended action permits optimizing the asset management policy as a function of test outcome, rather than pre-specifying the decision. Third, test outcomes are specified in terms of the condition state they diagnose; that is, test outcomes are reported as, for instance, “good” or “bad.” This specification is often used because it is fairly easy to understand, it applies across many kinds of tests, and it conforms to another industry practice of rating equipment condition on a qualitative scale (typically with 3- or 4-points). Alternatively, one could specify the test outcome as the specific numerical measurement produced by the test; however, it is not clear that this would improve the overall decision model, and it would make it much more complicated. Fourth, it is frequently more useful to represent *test protocols*, or test banks, rather than individual tests. A test protocol is a bank of related tests applied to an individual piece of equipment either at the same time or in succession over a relatively short period of time (say within 2 or 3 years). This representation conforms to current industry practice that takes advantage of the different kinds of information available from different tests. It also allows representing some of the contingent test policies utilities are using, such as retesting within two years to determine how rapidly degradation is progressing. Finally, through the use of test protocols to represent banks of related tests, the simple decision model discussed in this report assumes that no other dependence exists among tests done on the same piece of equipment at different times; that is, the model does not remember the outcome of a previous test bank. This assumption greatly simplifies the state dynamic equations by not requiring them to represent the test outcome explicitly. It is a reasonable assumption under two conditions: i) test intervals are fairly long relative to the speed of degradation, or ii) the action taken as a result of the test changes the equipment condition so that the previous condition is no longer relevant. However, the representation of test information over time remains an area of possible extension of the decision model, which may be useful for certain kinds of equipment, such as power transformers, that degrade gradually over time.

References

[1] Raiffa, H. *Decision Analysis*, Addison-Wesley, Reading, MA, 1968.

[2] *Guidelines for Intelligent Asset Replacement: Volume 3 – Underground Distribution Cables.* EPRI, Palo Alto, CA: 2005. 1010740.

[3] *Asset Population Model with Testing for Managing Aging Power Delivery Assets*, EPRI, Palo Alto, CA: 2004. 1008562.

4
INITIAL INVENTORIES AND TRANSIENT TRAJECTORIES

Introduction

Utilities today face the twin challenges of satisfying increasingly high standards for reliability and service quality while at the same time reducing costs and improving earnings. To meet the challenges, utilities are adopting *asset management* as their framework for allocating capital and operation/maintenance budgets. Simply stated, asset management consists of decision-making processes that have the goal of deriving the most value from utility assets within the available budget.

Basic framework for decision-making

The basic framework for decision-making that specifies the asset management policies that are applied to power delivery asset inventories has been presented in previous EPRI reports, including 1002086 [*Guidelines for Intelligent Asset Replacement, Vol.1*], 1002087 [*Guidelines for Intelligent Asset Replacement, Vol.2, Wood Poles*], 1002088 [*Guidelines for Intelligent Asset Replacement, Vol.3, Underground Cable*] and 1002257 [*Cable Reliability Management Strategies*]. The following are the elements of the decision framework:

- Objective
- State definitions and dynamics
- Decision alternatives and policies
- Data requirements

Objective

The objective in this decision framework is to minimize the lifecycle cost of maintaining the asset inventory, subject to serviceability requirements. The lifecycle cost comprises the total of the installation, inspection, treatment/maintenance, and replacement costs throughout a multi-year time horizon, all taken on a present value basis. The serviceability requirement means that assets that do not meet minimum performance standards must be replaced or refurbished in some manner.

Dynamic Behavior: Deterioration and Hazard Rates

The dynamic behavior of power delivery system assets is based on the idea of deterioration. Over time, the condition of an individual piece of equipment may deteriorate as it experiences operational loading or overloading, environmental damage, human interaction, animal activity, and other factors. The rate of deterioration depends on the type of asset, the initial installation of the equipment, as well as on the occurrence of these influencing factors, and it also can be retarded by some remedial or preventive maintenance activities. The decision framework represents the dynamic process of power delivery system asset deterioration mathematically, using a set of equations that provide a forecast of future asset conditions. The forecast depends on the present condition of the asset. In general, information summarizing what is known about an asset is called its *state*. This knowledge is sufficient to forecast the future behavior of the asset. The state may include two kinds of information, the *observable state* that can be known with certainty and the *unobservable state* that is not directly observable but can be characterized by a probability distribution and can be inferred based on the outcomes of diagnostic tests.

In present implementations of the methodology, the observable state of an asset may have two components. The first component represents the age of the asset (either the actual age or the so-called *effective age* representing the effects of refurbishment). The second component of the observable state is the number of previous failures of the asset. We assume that these two components of the state are known at any time for all members of the asset inventory.

Now, the decision framework represents the decisions regarding asset testing, maintenance, repair, refurbishment, and replacement as depending on the asset state. The specification of a decision for each asset state is called a *policy*, so the decision framework develops a *state-dependent policy* that *minimizes the lifecycle cost* of maintaining the asset population.

The deterioration of an asset's condition is represented mathematically by a probability distribution called the *hazard function*. The value of the hazard function at a particular age t, called the *hazard rate*, is the probability that a piece of equipment that has survived to that age does not survive to age $t+1$. The reports cited above describe hazard functions and methods whereby hazard functions may be estimated from observed data available at a utility.

The essential tradeoff in asset management is between accepting the risk that an aging asset will fail, where that risk is specified by the increasing hazard rate and the cost of a failure, and repairing or replacing the asset, so that the hazard is reduced at some certain cost.

Decision Alternatives and Inventory Policies: Stationary and Transient Policies

At any point in time, the observable state of the asset is known, the unobservable state of the cable is specified by a probability distribution, and the asset manager must make a decision about the asset. The methodology permits the choice of the following alternatives:

- *do nothing*, which does not change the state of the asset; doing nothing means that the behavior of the asset is governed by the hazard function given by the current state of the cable—which means that the current state is sufficiently satisfactory so that no intervention need be taken;

- *maintain* the asset, which changes the state of the asset; a typical maintenance alternative defined is *rejuvenation*; when an asset is rejuvenated, its unobservable condition is changed favorably; the user of the methodology may define the specific changes; a typical outcome is that the probability that the unobservable condition is in the worst state is changed to zero as a result of rejuvenation;

- *test* the asset, which means that further information about the unobservable state is needed prior to deciding what to do with the asset; the outcome of the test is a claim about the unobservable state; testing is not perfect, so that the test outcome revises the probability distribution on the unobservable state, hence the hazard rate is changed, which in turn changes the subsequent decision;

- *replace* the asset, which means that the present state of the asset was such that the risk of failure was too great to accept; the asset can be replaced with a like asset or a different asset type.

By convention, decisions occur at the beginning of each period up to the planning horizon (which may be infinite).

A *policy* is a complete specification of what decisions to make for the state of the asset in each period up to the planning horizon. .A policy that makes the same decision for a given state regardless of when the decision is taken is called a *stationary policy*. In other words, in a stationary policy, the decision depends only on the asset's state and not on time. A policy that is not stationary, that is, one in which the decisions do depend on time, is called a *transient policy*.

In determining the least cost policy over the indefinite future, the least cost stationary policy is part of the solution to the asset management problem. The other part of the solution to the asset management problem is the transient policy. The need for a transient policy arises because the stationary policy may not be feasible in the short run. For instance, it may not be possible to implement the stationary policy initially because of budget constraints. The transient policy transforms the initial asset inventory into the inventory to which the stationary policy can be applied.

The long-run optimal stationary policy typically leads to a stable distribution of the asset population among the various states (see for instance, figure 2-6 in chapter 2). However, initially, the asset population inventory may be quite different than the long-run distribution. For example, a typical stationary policy might be to replace all assets over forty years of age. This policy never changes. After a single application of this policy, there would be no assets over forty years old in the inventory. But suppose that the initial inventory contains a large number of assets over forty years old. The replacement costs associated with a single application of the stationary policy may be too great for the utility to accept. Therefore, a policy that more slowly steers the inventory toward the stationary condition, with no assets over forty years old, must be adopted. If there were no constraints on the inventory policy, then the stationary policy can be applied immediately, and the initial asset inventory would be transformed into the stationary inventory as quickly as possible. If there are constraints, then the stationary policy must be modified. This modification comprises the transient policy.

The transient policy depends on three aspects of the problem: the stationary policy, the initial inventory, and the constraints on policy costs or other attributes of the policy. The purpose of this report is to specify the transient policy.

In the next chapter, an algorithm for finding the transient policy is given. The algorithm is illustrated by an example. Then follows a brief concluding chapter, a list of references, and an appendix that presents some of the aspects of the underlying mathematical theory that guides the development of both the transient policy and the optimal steady-state policy.

The Transient Trajectory

This chapter specifies the transient trajectory for an asset inventory. The first step in determining the transient trajectory is to identify the stationary trajectory. This can best be illustrated by example.

Example: Stationary Trajectory for a Cable Inventory

The inputs to the analysis that determines the optimal stationary policy are the following.

Inventory

Consider an inventory of underground cable that consists of 20,000 cable segments, each of length 500 feet, that are distributed by age and by failure history as shown in Table 4-1.

Table 4-1
Underground Cable Inventory

INITIAL CABLE INVENTORY (Segments/State)

Failures									
2	0	0	1000	1000	1000	1000	1000	1000	1000
1	0	1000	1000	1000	1000	1000	1000	0	0
0	1000	1000	1000	1000	1000	1000	1000	0	0
Age	0	5	10	15	20	25	30	35	40

Strategic Alternatives

The optimal stationary policy will control this inventory over the foreseeable future by choosing, as a function of the observable state, which is the pair (age, past failures), one of the strategic alternatives

- No Action
- Replace
- Rejuvenate

The effect of rejuvenation is to reduce the *effective* age of a cable segment. The effective age of a cable segment that has never been rejuvenated is the actual age of the cable segment. Rejuvenation transforms the effective age such that the effective age after rejuvenation is 25% of the effective age prior to rejuvenation. The effect of rejuvenation is to delay onset of burnout (see the hazard function in Figure 4-1, where the age at onset of burnout is denoted T).

Costs

The relevant costs are the following.

- Replacement cost is $23,000 per segment (500 ft)
- Rejuvenation cost is $15,000 per segment
- Failure cost is $18,500 per occurrence, which includes both utility cost and estimated customer cost.
- The discount factor is 0.05 per year.

Hazard Rate

Failure of cable segments is governed by a hazard function. In this example, the hazard function is piecewise linear, as shown in Figure 4-1. The piecewise linear hazard function is characterized by three parameters:

- the steady-state failure rate h_{ss} = 6.90 x 10^{-5} failures/ft/yr = 0.3642 failures/mile/yr
- the onset of burnout T = 14 years
- the slope of burnout m = 0.361 failures/mile/yr, which is equivalent to setting the doubling time during burnout = 2.77 years.

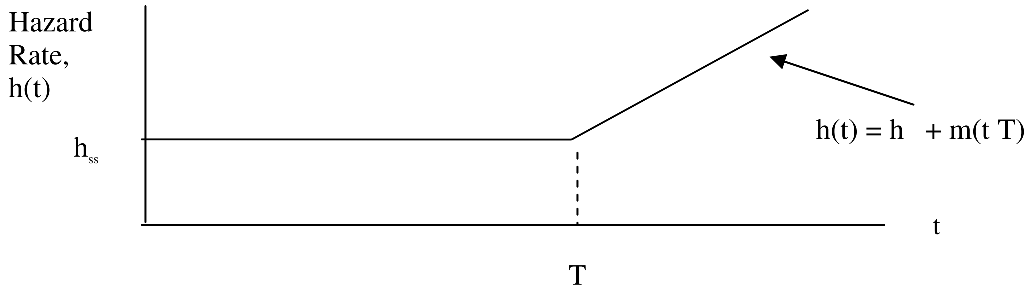

Figure 4-1
Piecewise Linear Hazard Function

This completes the specification of the inputs.

Example (cont'd): Optimal Stationary Policy

The optimal stationary policy for this example is given in Table 4-2. Note that the optimal policy replaces cable segments that have not failed when the age is 25 years. Rejuvenation is optimal for cable that has failed more than once and is 15 years old. The optimal policy is stationary because it does not change over time.

Table 4-2
Optimal Stationary Policy

OPTIMAL CABLE POLICY

	Failures		
Age	**0**	**1**	**2**
0	No Action		
5	No Action	No Action	
10	No Action	No Action	No Action
15	No Action	No Action	Rejuvenate
20	No Action	Replace	Replace
25	Replace	Replace	Replace
30	Replace	Replace	Replace
35	Replace	Replace	Replace
40	Replace	Replace	Replace

The optimal stationary policy requires that all cable segments over twenty-five years of age or older be replaced, as well as cable segments that are twenty years old and have failed at least once. The steady-state inventory that corresponds to this policy is given in Table 4-3.

Table 4-3
Optimal Steady-state Inventory

STEADY STATE INVENTORY UNDER OPTIMAL POLICY

	Failures		
Age	**0**	**1**	**2**
0	3,578		
5	3,011	567	
10	2,775	954	90
15	2,335	1,243	241
20	1,630	1,471	477
25	834	797	0
30	0	0	0
35	0	0	0
40	0	0	0

The steady-state investment (replacement plus rejuvenation) cost of this policy is the sum of the replacement costs plus the rejuvenation costs. In steady-state, there are 834 + 1471 + 797 + 477 = 3578 (due to rounding) segments that are replaced each period and 241 segments that are rejuvenated each period. That is a total cost of 3579($23,000) + 241($15,000) = $85.9 million. This cost does not include the cost of failures in steady-state. This cost applies if the inventory were in the optimal steady-state as given in Table 4-3.

In the example, the inventory does not begin in the optimal steady-state. Instead, it is given by the distribution presented in Table 4-1. If the optimal policy were applied to this inventory, the number of required replacements is 10,000 and the number of required rejuvenations is 1,000. This is because these are the numbers of segments that are in the (age, previous failure) states that are replaced or rejuvenated under the optimal policy given in Table 4-2. The cost of replacing and retiring these units is $230 million + $15 million = $245 million, a cost that is nearly three times greater than the steady state cost.

One may expect that the periodic cost is subject to a budget constraint. One may also expect that this budget constraint is insufficient to permit application of the optimal steady-state policy for all initial inventories. Therefore, the transient trajectory must be determined. The transient trajectory transforms the inventory, given by (in this example) Table 4-1 into the optimal steady-state inventory, given by Table 4-3, while following the optimal steady-state policy, given by Table 4-2, as closely as possible without violating the budget constraint.

The following algorithm specifies the transient trajectory. The algorithm will be applied to the example.

Algorithm for Determining the Transient Trajectory

An algorithm for determining the transient trajectory is given in the following steps. The basic idea of this procedure is that the optimal stationary policy specifies which assets in the population should be replaced. Among these assets, a replacement priority is assigned according to their projected failure rates, with those having the highest failure rate assigned the highest priority. Then the replacement budget is used to replace these assets in this priority order. If the budget is sufficient to replace all assets scheduled under the optimal stationary policy, then the inventory moves immediately to its long-run distribution, and the stationary policy continues to be applied indefinitely. If, on the other hand, the budget limits replacements, those with the greatest risk of failure are replaced, and replacement of the remainder is deferred. The asset inventory thus moves closer to the long-run distribution. The prioritization procedure is repeated for subsequent budgets until the all the assets scheduled for replacement under the stationary policy have in fact been replaced, and thence the stationary policy continues to be applied indefinitely.

Notation

- Let $\underline{x}$ denote the state of the cable. Recall that $\underline{x}$ is at present the pair (t, f), where t is cable age and f is number of previous failures.
- Let $h(\underline{x})$ denote the hazard rate for cable in state $\underline{x}$.
- Let B_k denote the budget available for cable replacement at time k in the planning period. It is this collection of budget values that constrains the transient trajectory. B_o denotes the initial budget available.
- Let c_{repl} denote the cost of replacement for each unit of asset; e.g., cost per cable segment.
- Let $(k, \underline{x})$ denote the fraction of the cable inventory that is in state $\underline{x}$ at time k. The initial inventory distribution is $(0, \underline{x})$. The total initial inventory is I_o. The total number of segments in each state $\underline{x}$ initially is $I_o\ (0, \underline{x})$. Note that in the present model, the total inventory (I_o) is constant.

Procedure

Step 1. Identify the *Replacement (and Rejuvenation) States*. Replacement states are the collection of states $\{\underline{x}\}$ that would be replaced (or never achieved) if the optimal steady-state policy were adopted. Similarly, the rejuvenation states can be identified. (For simplicity, the term Replacement States will be understood to mean a state that will either be replaced or rejuvenated.)

Step 2. Order the Replacement States by decreasing hazard rate. This yields the sequence of states $\{\underline{x}_j = (t_j, f_j) : j = 1, 2, 3, \ldots\}$ such that $h(\underline{x}_j) \geq h(\underline{x}_{j+1})$, (with ties between states with equal hazard rates broken by giving priority to the older equipment and to equipment with more prior failures if the ages are equal). This is the *Replacement Priority*.

Step 3. Modify the Replacement Priority if the optimal policy includes *Testing*. Let *"$R(\underline{x})$"* denote the test outcome that indicates replacement under the optimal policy when the state is $\underline{x}$ (as discussed in EPRI report 1002088, the quotation marks indicate that the test outcome is a claim about the true condition, but because of inaccuracies in the test, this claim is not 100% reliable). As discussed in chapter 3, the information about the unobservable part of the state revealed by the test allows one to update the hazard rate. Denote the updated hazard rate conditional upon this test outcome as $h(\underline{x}|\text{“}R(\underline{x})\text{”})$. Revise the ordering in Step 2 such that all states $\underline{x}$ in which the test would be applied under the optimal policy have $h(\underline{x})$ replaced by $h(\underline{x}|\text{“}R(\underline{x})\text{”})$. This provides the *Test-modified Replacement Priority*, the sequence of states $\{\underline{x}_j = (t_j, f_j) : j = 1, 2, 3, \ldots\}$.

Step 4. Initial Replacement. Following the *Replacement Priority* (*Test-modified* as appropriate), replace until the initial budget B_o is exhausted. If assets in state $\underline{x}$ are to be tested, subject to both the Replacement Priority and the budget constraint, then replace the expected number that will have test outcome *“R(x)”*, which is $I_o\ (0, \underline{x})\, p(\text{“}R(\underline{x})\text{”})$, where $p(\text{“}R(\underline{x})\text{”})$ is the probability of that test outcome. The remaining expected number of assets, $I_o\ (0, \underline{x})\,[1-p(\text{“}R(\underline{x})\text{”})]$, will not be replaced. They will have hazard rate $h(\underline{x}|\text{“}R(\underline{x})\text{”})$ for the first year in the planning period.

The number of assets replaced is equal to $\sum_{j=1} I_o \pi(0, \underline{x}_j) \leq B_o / c_{repl}$, where the sum is equal to the maximal number of replacements, in Replacement Priority order, that does not exceed the constraint.

Step 5. Dynamics. If the initial budget is sufficiently large such that all assets have been replaced according to the Replacement Priority, then invoke the optimal policy and no further transient analysis is required. (This assumes that the steady-state budget is available is no greater than B_o.)

If the initial budget is not sufficiently large, then the optimal policy cannot be invoked. In that case, perform the following.

5a. Apply the failure dynamics given by the appropriate hazard rates, either $h(\underline{x})$ or $h(\underline{x}|\text{“}R(\underline{x})\text{”})$.

5b. Find the new population distribution $I_o\ (1, \underline{x})$.

5c. Perform Step 4 with k=1 for budget B_1.

5d. Return to top of Step 5. Continue for k=k+1.

Example (cont'd): Optimal Transient Policy and Trajectory

The transient trajectory given by the algorithm above is optimal in the sense that it transfers the inventory to the optimal steady-state as quickly as possible, and at each stage of the algorithm, the riskiest assets are replaced (or rejuvenated).

In the present example, suppose the periodic replacement (including rejuvenation) budget is \$100 million, a value somewhat greater than the steady-state cost. Thus, B_k = \$100 million for each period k in the analysis.

The periodic hazard rates $h(\underline{x})$ are presented in Table 4-4.

Table 4-4
Periodic Hazard Rates

Age	Failures 0	1	2
0	0.1584	0.1584	0.1584
5	0.1584	0.1584	0.1584
10	0.1584	0.1584	0.1584
15	0.3019	0.3836	0.5485
20	0.4886	0.6332	0.8401
25	0.6255	0.7817	0.9434
30	0.7257	0.8701	0.9799
35	0.7991	0.9227	0.9929
40	0.8528	0.9540	0.9975

Step 1 of the algorithm requires that the replacement and rejuvenation states given in Table 4-2 be identified. These states are $\{\underline{x} = (t, f)$: (25,0), (30,0), (35,0), (40,0), (20,1), (25,1), (30,1), (35,1), (40,1), (20,2), (25,2), (30,2), (35,2), (40,2), (15,2)*}. The asterisk denotes that (15,2) is a rejuvenation state. There are fifteen replacement states.

Step 2 of the algorithm orders the replacement states by decreasing hazard. The ordering is

1. (40,2)	6. (35,1)	11. (25,1)
2. (35,2)	7. (30,1)	12. (30,0)
3. (30,2)	8. (40,0)	13. (20,1)
4. (40,1)	9. (20,2)	14. (25,0)
5. (25,2)	10. (35,0)	15. (15,2)

which follows from inspection of Table 4-4.

Step 3 is not operative in this example. Testing is not part of the optimal policy.

Step 4 is the initial replacement. The initial inventory $I_o\ (0, \underline{x})$ is given in Table 4-1. Note that there are 20,000 segments in the initial inventory. Note also that the optimal policy applied to the initial inventory would result in 10,000 replacements and 1,000 refurbishments. The cost of these decisions exceeds the budget constraint.

The ratio B_o/c_{repl}, the number of initial replacements possible, is \$100 million/\$23,000 = 4,347. Based on the replacement ordering, the initial replacements are 1000 segments in state (40,2), 1000 segments in state (35,2), 1000 segments in state (30,2), 0 segments in state (40,1), 1000 segments in state (25,2), 0 segments in state (35,1), 347 segments in state (30,1). The total number of replacements is 4,347, and the constraint is binding.

Step 5. This step requires that the failure dynamics be applied to the initial inventory that is not replaced. Applying Step 5a, the remaining segments experience failures given by the periodic hazard rates of Table 4-4. Hence the inventory that begins the next period is given in Table 4-5. (For example, there are 1000 segments in state (10,1) in the initial inventory. The hazard rate in this state is 0.1584. Thus, the number of segments that fail and therefore transition to state (15,2) is 158.4. The number of segments that do not fail and therefore transition to state (15,1) is 851.6. But also, there are 1000 segments in state (10,0) in the initial inventory, with hazard rate 0.1584. Therefore, the number of those segments that fail and therefore transition to state (15,1) is 158.4. Hence, the total number in state (15,1) at the beginning of the next period is 851.6 + 158.4 = 1000. Similarly, all the segments that were in state (10,2) at the beginning of the period must transition to state (15,2) because failures are not accumulated in the state variable beyond two. Therefore, the number of segments in state (15,2) at the beginning of the next period is 1000 +158.4 = 1158 (rounded).) Table 4-5 is the result of Step 5b, the first period inventory, or the population distribution $I_o\ (1, \underline{x})$.

Table 4-5
Inventory Dynamics: First Period Inventory

	Failures		
Age	**0**	**1**	**2**
0	4347		
5	842	158	
10	842	1000	158
15	842	1000	1158
20	698	918	1384
25	511	855	1633
30	375	844	782
35	274	811	568
40			

The steps repeat from this point. The budget for period 1 is B_1= \$100 million, the number of replacements is the same as in the initial iteration, the replacement ordering is the same. Hence, upon inspecting Table 4-5, the replacements are 568 segments from state (35,2), 782 segments from state (30,2), 1633 segments from state (25,2), 811 segments from state (35,1), and 553 (out of 844) segments from state (30,1). These comprise 4347 replacements. The remaining segments behave according to the hazard rates. The result is the second period inventory, $I_o\ (2, \underline{x})$, as shown in Table 4-6.

Table 4-6
Inventory Dynamics: Second Period Inventory

	Failures		
Age	**0**	**1**	**2**
0	4347		
5	3658	689	
10	709	266	25
15	708	975	317
20	588	870	1542
25	357	678	1965
30	192	507	669
35	103	310	253
40	55	219	0

Repeating the steps, the replacements are, in priority order, 253 + 669 + 219 + 1965 + 310 + 507 + 55 = 3978. The next state in priority order is (20,2), but there are only 4347- 3978 = 369 replacements that are feasible. Therefore, the inventory in state (20,2) becomes 1542 – 369 = 1173. Applying the hazard function, the third period inventory is shown in Table 4-7.

Table 4-7
Inventory Dynamics: Third Period Inventory

	Failures		
Age	**0**	**1**	**2**
0	4347		
5	3658	689	
10	3078	1159	109
15	597	336	67
20	494	815	691
25	301	606	1724
30	134	371	530
35	53	139	0
40	21	82	0

The third period inventory replacements are 530 + 82 + 1724 + 139 + 371 + 21 + 691 + 53 + 606 + 130 = 4347. The state (30,0) contains 134 – 130 = 4 segments that could not be replaced because of the budget constraint. The fourth period inventory is given in Table 4-8.

Table 4-8
Inventory Dynamics: Fourth Period Inventory

	Failures		
Age	**0**	**1**	**2**
0	4347		
5	3658	689	
10	3078	1159	109
15	2590	1463	293
20	417	387	196
25	253	540	516
30	113	188	0
35	1	3	0
40	0	0	0

It is now feasible to replace segments by applying the complete optimal policy. The total number of segments replaced is 516 + 3 + 188 + 196 + 1 + 540 + 113 + 387 + 253 = 2197 and the total number of rejuvenations is 293. The fifth period inventory is shown in Table 4-9. The optimal policy is feasible at this stage as well.

Table 4-9
Inventory Dynamics: Fifth Period Inventory

	Failures		
Age	**0**	**1**	**2**
0	2197		
5	3658	689	
10	3078	1159	109
15	2590	1463	293
20	1808	1684	561
25	213	204	0
30	0	0	0
35	0	0	0
40	0	0	0

Thus, the policy has converged to the optimal policy. The inventory will converge to the optimal steady-state inventory, but that will require many more iterations.

This concludes the example and the description of the algorithm.

Conclusions

This report has presented a method that will transfer any initial asset inventory to the optimal steady-state inventory while not violating a budget constraint imposed on the total policy management (replace, refurbish, and test) cost in any period. The method is specified by the algorithm given in chapter 2. The main feature of the algorithm is that the riskiest assets are replaced in order, until the budget is exhausted. Therefore, the transient policy is the least-cost policy subject to the budget constraint. If the budget constraint is not less than the periodic cost of the optimal steady-state policy, the transient policy will converge to the optimal steady-state policy as quickly as possible. The greater the budget constraint, generally speaking, the more quickly the policy will converge.

Technical Appendix: Dynamic Theory

The development of the transient trajectory as a method of transferring any initial inventory of assets to the optimal steady-state inventory is based on an underlying mathematical model of the dynamic behavior of aging assets. The aging assets problem is formulated as an optimal control problem on an infinite horizon. The simplest form of this problem is the autonomous problem, formulated as

$$\text{minimize} \int_0^\infty e^{-\rho t}\ell(x(t),u(t))dt$$

$$\text{subject to } \dot{x}(t) = f(x(t),u(t))$$

$$x(0) = x_o$$

$$(x(t),u(t)) \in \mathrm{X}\times\Omega\forall t$$

The autonomous problem has an objective kernel that depends on time only through the discount factor $e^{-\rho t}$. The variable *x* is the state variable and the variable *u* is the control variable. Perhaps the simplest way to view this problem is that the solution is a controller *u(t)* defined over the time interval [0, •) that drives the state from its initial condition x_o to an arbitrary terminal state at t = •, subject to the constraints that the state and control values are within some set for each instant in the planning period, $(x(t),u(t)) \in \mathrm{X}\times\Omega\forall t$, and that the controller directs the state according to the first order differential equation, while minimizing the objective functional.

The main result of the theory of this problem is that there is an optimal steady-state, a so-called *turnpike*, the pair (x^*, u^*), such that $f(x^*, u^*) = \underline{0}$, that is an attractor for all optimal trajectories. That is, all optimal trajectories converge to the turnpike over the infinite horizon, and all optimal trajectories over finite time horizons, with terminal conditions given, spend an arbitrarily large part of the planning period in a neighborhood of the turnpike (Radner [1961], Samuelson [1965]).

An important result indicates how to find the turnpike without having to solve the dynamic problem.

It is clear that for the undiscounted problem ($\rho = 0$), the turnpike is the steady-state that minimizes the kernel of the objective, the solution to the (typically nonlinear) programming problem

$$\text{minimize } \ell(x,u)$$

$$\text{subject to } f(x,u) = \underline{0}$$

$$(x,u) \in X \times \Omega$$

The characterization of the optimal steady-state for the discounted problem is given by the so-called implicit programming problem (Feinstein and Luenberger [1981])

$$\text{minimize } \ell(x,u)$$

$$\text{subject to } f(x,u) - \rho(x - x^*) = \underline{0}$$

$$(x,u) \in X \times \Omega$$

The unique feature of the implicit programming problem is that the state component of the turnpike, x^*, is present in the constraint. That is, the constraint is defined implicitly by the solution to the problem itself. Note that the entire null space of f is feasible for the undiscounted problem, while only a subset of that null space is feasible for the implicit programming problem. (This is easy to see if one replaces x^* in the constraint of the implicit programming problem with a parameter c. The solution for each c is then $(x^*(c), u^*(c))$ and the feasible points of the implicit programming problem are the fixed points of the mapping $c \rightarrow x^*(c)$.) Hence the optimal objective value of the implicit programming problem is inferior to (greater than) the undiscounted optimal objective value.

This theory can be applied to the Markov decision problem with discounting. We make the correspondence $\ell(x,u) = r(u)^T \pi$, where $r(u)$ is the vector of state-occupancy return rates that apply if policy u is chosen and is the (steady-state) probability vector; and $f(x,u) = Q^T(u)$, where the probabilistic state dynamics are given by $\dot{\pi} = Q^T(u)\pi$. The implicit programming problem is then

$$\text{minimize } r(u)^T \pi$$

$$\text{subject to } Q^T(u)\pi - \rho(\pi - \pi^*) = \underline{0}$$

$$\underline{1}^T \pi = 1$$

The columns of the matrix Q^T are the various conditional probability rates that govern the next transition. Each control u is a complete selection of columns, so that u determines Q^T.

Therefore, the implicit programming problem is a linear program with a special structure. The solutions determine the optimal steady-states for different values of the discount rate.

For discrete time, with periodic discount factor δ, we make the correspondence $Q^T(u) \rightarrow (P^T(u)-I)$, because discrete-time dynamics are $(k+1)=P^T(u)\ (k)$, and $\rho \rightarrow (1/\delta)-1$. The discrete-time implicit programming problem is also a linear program.

The solution to this problem determines the optimal steady-state. This is the problem that is solved in the EPRI asset management methodology (which is described in the cited EPRI reports). That methodology identifies the optimal steady-state policy.

This steady-state policy is the turnpike or the attractor for all optimal trajectories. Such information is critical because it specifies where any transient trajectory converges. Therefore, it is possible to identify a cost-effective, if not optimal, transient solution that transfers any initial inventory to the optimal steady-state inventory. Indeed, the algorithm presented in this report accomplishes precisely that transfer.

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