A Primer on Electric Power Flow for Economists and Utility Planners

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REPORT SUMMARY

It is increasingly important for economists, regulatory policymakers, and non-engineering planners to be familiar with the basic engineering and economics of transmission planning and operations. This primer provides a starting point for understanding how the engineering and technical considerations of electric power flow control impinge on the economics of providing utility service.

Background
Transmission operation and control issues lie at the heart of the debate over the future structure of the electric utilities industry. The debate focuses on the movement toward open access, unbundled arrangements with common carriers, or other regulation of the transmission function while retaining the traditional operational efficiencies and network reliability enjoyed in the past. In this debate, it is critical that all participating parties have an appreciation of how transmission system operations and constraints can influence the economics of generation capacity planning, bidding for nonutility generation, the attractiveness of demand-side management, potential decentralized supply technologies, and the terms and prices for wheeling power.

Objectives
To provide a basis for understanding how the engineering and technological considerations of electric power flow control affect the economics of providing utility service; to provide a "one-stop" shopping tutorial spanning the elementary physics of electricity through widely used transmission modeling techniques; to place transmission planning and policy questions in technical perspective.

Approach
The primer's designers presented four topics in sequential manner. First, they identified some key terms and concepts of electrical engineering by describing the basic physics of electric circuits. Second, they presented a typical electric utility system configuration and addressed pertinent control issues. Third, they described steady-state methods for resolving how power will flow from supply sources to demand centers. Fourth, they combined engineering and economics by formulating and solving the optimal flow problem. Throughout the text, they included numerical examples along with graphical and algebraic solutions.

Results

This report is unique because it provides in one document the technical concepts and information readers need to understand how electric power flow control affects the economics of providing utility service.

The report also provides some insights into the often counter-intuitive effects that transmission considerations can have on short-run marginal costs. The research demonstrates that:

- Marginal costs are specific to distinct locations on the network and prevailing operating conditions.
- Location-specific marginal costs can differ significantly from each other and from the variable costs of the highest cost generator dispatched on the system (due to Kirchhoff's Laws and binding flow or capacity constraints on utility resources).

EPRI Perspective

This primer–cosponsored by Incentives Research Inc. and EPRI–arose as a result of the contractor's experience with industry restructuring issues, first in the natural gas pipeline industry and now in the electric power industry. Such experience sensitized them to the possibility that the technical considerations of controlling electric power flows might have a significant effect on the economics of providing utility service. This primer provides a basic understanding of the technological issues associated with electric power flow control and an appreciation for the general nature of transmission modeling procedures. In specific, the primer places transmission planning and policy questions in technical perspective for decision makers who do not have a background in electrical engineering, without taking a position on strategic or regulatory policies for open access. The reader may be interested in related EPRI research that addresses transmission services costing: *Transmission Services Costing Framework: Volume 1: Interim Report on Technical and Economic Fundamentals*, *Revision 1*, October 1994, TR-104266-R1.

Interest Categories

Power system planning and engineering.
Power system operations
Utility planning methods
Utility planning studies

Keywords

Electric circuits
Optimal power flow
Economic analysis
Marginal costs
Transmission costs

ABSTRACT

The main purpose of this report is to put transmission planning and policy questions in technical perspective for decision makers who do not have a background in electrical engineering. This report provides all of the technical concepts and information necessary for the reader to develop an understanding and appreciation of the effects that controlling electric power flow can have on the economics of providing utility service. The report also provides some insights into the often counter-intuitive effects that transmission considerations can have on short-run marginal costs. The research demonstrates that:

- Marginal costs are specific to distinct locations on the network and prevailing operating conditions.
- These location-specific marginal costs can differ significantly from each other and from the variable costs of the highest cost generator dispatched on the system (due to Kirchhoff's Laws and to binding flow or capacity constraints on utility resources).

The report covers four topics. The key terms and concepts of electrical engineering are identified by describing the basic physics of electric circuits. A typical electric utility system configuration is presented and the control issues are identified. Third, some "steady state" methods, for solving how power will flow from scheduled supply sources to demand centers, are described. Finally the engineering constraints and economic objectives for system operations are combined by formulating and solving the optimal power flow problem. Throughout the text several related numerical examples are presented in complete detail, with graphical and algebraic solutions provided.

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Within IRI, Frank C. Graves was the principal investigator and is the author of this report. Simon-Pietro Felice and Charles R. Clark made important contributions, including performing the system simulations and calculations for the numerical examples throughout the report. Steve Thomas and Glen Graves provided valuable editorial suggestions, and Martha Folger and Yumiko Shinoda assisted extensively with the design and production of the report.

INTRODUCTION

It is increasingly important for economists, regulatory policy makers, and planners throughout electric utility organizations to be familiar with the basic engineering and economics of transmission planning and operations. Transmission system operations and constraints can have a significant influence on generation capacity planning, bidding for non-utility generators (NUGs), the attractiveness of demand-side management (DSM) and potential decentralized supply technologies, and the terms and prices for wheeling power. Transmission pricing and control issues are also at the heart of a debate over the future structure of the industry, concerning how to move toward an open-access, unbundled arrangement with common carrier or other regulation of the transmission function while retaining the operational efficiencies and network reliability that have been enjoyed in the past under traditional utility service.

This primer does not attempt to break any new conceptual ground on these planning and policy questions. Indeed, it does not even attempt to answer them, since they involve many institutional issues (such as stranded investment recovery and market strategy for new service design) that must be addressed in specific contexts. Instead, it provides a starting point for understanding how the engineering and technological considerations of controlling electric power flows impinge on the economies of providing utility services. It is written for economists, policy makers, and utility managers who are involved in any of the above planning problems but who have only a modest or rusty background in electrical engineering or economic modeling of networks. Many textbooks on electrical engineering and the economic theory of pricing provide more detailed and sophisticated presentations of the concepts herein, but these sources are often not well-suited for the planner who does not want or need a rigorous derivation of every result. Moreover, such materials often assume a very technical background, and they may provide little economic motivation or interpretation for the engineering problems they address. Accordingly, this primer provides "one stop shopping" from the elementary physics of electricity up through the transmission modeling techniques that are in widespread use by power systems engineers, with economic implications along the way.

Overview of the Text

The primer begins with a chapter that puts transmission planning in the context of other utility planning problems - what kinds of economic decisions are increasingly affected by transmission considerations, and how and when transmission constraints enter the planning procedures used by most utilities. The second chapter then lays a foundation for all subsequent chapters and analysis of utility power management, by providing definitions of key terms and concepts of electrical engineering and by describing the basic physics of electric circuits. This background is sufficient for understanding how the flow of current (or power) can be predicted on networks with fixed voltage sources and electric loads. It also explains the important distinction between "real" and "reactive" power. Many readers will be aware of only the former, but the latter is equally crucial to the safe and effective management of power systems.

The next chapter describes a typical utility system configuration and some of the power control issues that arise. While the basic physics of electricity described in the prior chapter still apply, there are some stark differences in degree of complexity between elementary circuits and utility systems. For instance, utilities usually generate and transmit three waves (rather than one) of alternating current power simultaneously at each generator, in the interest of transmission economies. They must take into account the electrical properties of the transmission lines themselves, which are significant because of the very long length of the conductors. The generators must be able to detect and respond almost instantly to changes in demand. Since several generators will share the responsibility for responding to load changes, power plant adjustments are not independent of each other. They must be adjusted in a coordinated fashion in order to assure stable operations. Finally, utilities incrcase their reliability by interconnecting with each other, but this adds further coordination complexities of keeping actual intercompany power exchanges in line with intended, contractual agreements.

Input voltages are usually fixed for simple circuits, while they are a controllable variable for electric utility systems. This makes utility power flow planning over the many lines of a transmission grid a nonlinear problem (involving many simultaneous equations). Some "steady state" methods for solving how power will flow from supply sources to demand centers are presented in the fourth chapter. These methods apply only when the pattern of power demand is known with certainty, though they can be used to analyze many different hypothetical circumstances. For instance, incremental transmission line usage (such as the "loop flows" that can arise from a wheeling transaction) can be forecasted with these techniques.

The final chapter is where most of the economic policy and service planning implications come together. Readers with a background in electrical systems may be able to focus on just this chapter. It combines the operating economics and engineering, by formulating and solving the "Optimal Power Flow" problem. This is a means of identifying the mix of generators to dispatch in order to achieve least-cost operations while honoring network constraints. Some of the technical concerns that determine the operating and reliability constraints are explained in enough depth for the reader to appreciate why a transmission system could superficially appear underutilized, yet have no slack capacity available for certain new transactions. In addition to producing a recommended mix of generation, the optimal power flow method computes short run marginal costs for each resource used in providing power, which can be helpful in the design of transmission pricing and expansion policies. To provide some insight into the behavior of these often counter-intuitive marginal costs, a detailed numerical example and several sensitivity cases are presented. These demonstrate the sometimes counter-intuitive marginal effects on costs, voltages, and power flows that can arise from changes in real and reactive power demands and from wheeling obligations. (To our knowledge, there is no comparable explanation of how transmission marginal costs can behave in the electrical engineering or economics literature).

The chapter closes with a few caveats on what kinds of questions are not addressed by the methods of analysis described in this primer. In particular, many issues related to long range planning and system reliability (such as transmission expansion) require additional or different forms of analysis. Nonetheless, these kinds of questions should be answered with appropriate recognition of the power flow considerations that determine which manner of operating is economic in the short run, that is, in recognition of the topics explained in this primer.

Presumed Background of the Reader

Much of the material explained in this text is inherently mathematical. Mathematical notation is not only more efficient but also a more effective means of explaining electrical engineering than lengthy verbal descriptions and imprecise analogies. Thus basic comfort with mathematical exposition is assumed, though the required math skills need not be particularly advanced nor in a state of polished readiness. Basic trigonometry is used to describe the cyclical time pattern of variation in alternating current and rudimentary calculus (mostly derivatives) is used to derive how changes in one dimension of power delivery, e.g. current level, affect the levels of related factors, such as voltage. Where the algebraic manipulation of trigonometric functions becomes awkward, a notation called "phasors" (similar to vectors, but using complex numbers) is explained that is much more convenient. Finally, some methods for solving many power delivery equations simultaneously where the allowable solutions are subject to constraints are described but not derived or proved. Such techniques are needed for appraising how a large number of network elements such as generators and transmission trunks can be jointly used to meet total load at least cost. These optimization methods will be familiar to most economists, but perhaps not to many business planners or regulators.

It is not necessary to understand all of the mathematics in order to benefit from this book. Each chapter begins with a non-technical discussion of the key points. The derivations that are presented emphasize intuition over rigor. They usually present most of the intermediate steps, rather than simply assert that it is possible to obtain the desired form of an equation from another, already explained form. This has the desirable effect of showing precisely how a result is obtained, but the undesirable effect of making the text appear more mathematical. The reader can skim or dive into these derivations, according to temperament. Numerical examples and graphics provide a non-algebraic basis for understanding many of the key points. Each chapter closes with a few observations on utility planning or regulatory policy issues that are related to the engineering issues just described.

The text does not present any "cookbook" solution procedures for evaluating transmission economic issues. Full development of planning methodologies would require significantly more description and technical preparation. It also does not develop a theory of transmission pricing, which must depend on much more than just engineering concerns. Instead, it provides a basic understanding of the technological issues and an appreciation for the general nature of transmission modeling procedures, so that the reader can interact productively with transmission planning specialists. Numerous references on both electrical engineering, economics, and mathematics are provided in the footnotes and bibliography for those who would like more background or a more sophisticated treatment of these topics.

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TRANSMISSION CONSIDERATIONS IN UTILITY OPERATIONS AND PLANNING

This chapter reviews how transmission planning problems are becoming more important to utility managers and how planning methods are typically organized for recognizing transmission constraints and performance objectives. It illustrates the growing number of operational and service-design problems in which transmission capacity is a key factor. These problems include transmission siting constraints on new generation or power contracts, opportunities to use third-party suppliers or to conserve power, operational implications of wheeling of power across utilities, issues in voltage control and reactive power management, and the very broad question of what form of open-access procedures and pricing policies would be desirable for the industry. These matters are surveyed in the first half of this chapter, while the second half describes how utility planning methods vary with the time frame of interest. In the very short term (seconds to minutes), transmission constraints are paramount. As the horizon stretches out to a year or more, transmission considerations become more secondary as attention shifts from flow planning to investment planning, but they still cannot be ignored.

Applications of Transmission Economics

Capital costs and siting constraints on new transmission lines have become a significant determinant of what kinds of generation capacity are feasible. By the mid-1980's, the cost of new high-voltage lines was $125,000 - $750,000 per mile, adding substantially to the cost of a new, large generation facility and making transmission expansion itself a significant investment.[1] Perhaps more significantly, it can be extremely difficult to obtain regulatory and environmental approval for adding a line even when financing is not a constraint. This may limit generation expansion to plant sizes and technologies that require only capacity upgrades on existing transmission lines, or it may incrcase the attractiveness of on-site generation, storage, or conservation.

Many utilities are planning to rely heavily on non-utility generators (NUGs) for a large portion of their power supply additions over the next few years. The attractiveness of NUG facilities depends in part on their proposed locations, because they impose a power-handling requirement on the transmission network. Accommodating additional power flows can be costly if transmission must be over lines that already operate near their capacity limits. The cost of adjusting dispatch and transmission controls (or even system reinforcement) to avoid overloading transmission lines and reducing network reliability can be several mills/kWh. In recognition of this issue, at least one utility is utilizing NUG bidding procedures that include a premium or penalty for location on the transmission network[2]. A related problem has been addressed in the United Kingdom, where electric power suppliers am chosen via a day-ahead procedure that can be inconsistent with transmission network operating constraints. The resulting costs of out-of-merit dispatch, standby, and reactive power are aggregated into a term called *uplift* that is added to the half-hourly Power Purchase Price (PPP). Uplift is typically about 10% of the PPP, for an annual cost of £400mm-£600mm in a system roughly the size of the Pennsylvania-Jersey-Maryland power pool.[3] (This amount does not include capacity payments for the transmission grid itself, just its operating costs.)

To date, most NUGs are not fully controllable by the buying utility, especially when they are cogeneration units. For instance, they are generally not dispatchable or available for spinning reserve, and they may not participate in system frequency or voltage control. When the amount of self-scheduled NUG capacity is small, it can be treated by system planners and operators much like a reduction in demand with some associated uncertainty. Other, centrally-controlled generating units can be manipulated to assure that no transmission lines are pushed toward their MW or voltage limits of secure, stable operations as net demand and NUG outputs vary. But as dependency on NUGs increases, the burden on the controllable units becomes more acute. One solution might be to insist that more NUGs be controllable, but their small size and large numbers pose a significant transmission planning and control problem on utility operators, since the scale of the network modeling problem grows rapidly with the number of plants to be analyzed.

The economic attractiveness of certain demand-side management (DSM) activities and of emerging technologies such as decentralized electric storage devices also may depend in part on transmission and distribution considerations. Such technologies are selected based on how much they cost per MW displaced relative to a utility-supplied alternative. The latter can include the incrcased use or expansion of the transmission distribution system. These long run marginal costs per kW or kWh can vary significantly between different locations, especially when the load factor of the demand being served is small. The opportunity to avoid such costs in regions where they are large determines much of the economic attractiveness of the decentralized technology.[4]

In the past, most utilities sought to maintain supply self-sufficiency, relying on imports from other utilities only for short periods of time under conditions of system duress such as a major plant outage or an unexpectedly high peak demand. Transmission interties between utilities were used at a fraction of their maximum flow capacity, with the residual being held in reserve to cover unplanned system disturbances on either system. For a variety of reasons, electric supply and demand in several regions of the country becane less well-balanced by the 1980s, making it attractive for utilities with substantial generation capacity reserves to sell power to other utilities on a long-term basis. As a consequence, the volume of "wheeling" transactions - shipping of power between a seller and buyer over third-party utility transmission lines - has grown significantly over the last decade.

Growth in the volume of wheeling has several potential impacts on system economics, aside from the revenues for the wheeling utilities and the production cost-shifting for the power buyers and sellers. First, the incrcased use of tie-lines for energy exchange can reduce reliability, because the interties become less available for emergency flow capacity. Second, wheeling can create additional flow management burdens. For instance, the magnitude of line losses (power dissipated as heat over the transmission system) can incrcase more than proportionally to the incrcase in wheeling loads. Voltages can also be affected by the wheeling loads.

Typically, the price structure of wheeling contracts involves an annual fixed fee per maximum kW allowed under the wheeling contract. The fee size is often derived from the annualized accounting costs of a specific line nominally designated to carry the third party power, a practice referred to as *contract path* billing.[5] Parties to wheeling contracts have always recognized that electric currents do not follow solely the shortest or least-heavily utilized path, despite what may be specified in a wheeling contract. Instead, power flows over all available paths between supply and load, a phenomenon known as "loop flow." The Federal Energy Regulatory Commission (FERC) has expressed willingness to allow utilities to set wheeling rates that reflect the redirect and marginal costs associated with loop flow, such as incrcased line losses, the need to dispatch less economic plants, and reductions in system reliability.[6] The implementation of these regulatory policy changes can be controversial. Their resolution requires an appreciation for how transmission systems are operated.

The production, transmission, and use of AC electric power all involve the temporary (within-cycle) storage of some electrical and magnetic energy in the transmission circuits themselves or in the end-use device. This temporarily diverted power is called reactive power, and it is especially common in industrial motors. It must be supplied to maintain voltages at desired levels. Wheeling loads can incrcase the amount of reactive power required by the system. In contrast, "real power" is the power available for performing useful work, such as producing heat or light. There is a tradeoff between how much real vs. reactive power a system can supply. The mix of both types of power must be managed by how capacitive and inductive devices in the transmission circuits are switched on and off and by how generators are used. Several utilities have proposed to unbundle the cost of reactive power management for the purpose of setting rates that signal the network impacts of reactive power loads.

Perhaps the most visible and critical application of transmission economics and engineering is in the regulatory debate over the best means of providing "open access" to transmission networks. This debate is very broad, spanning such questions as what aspects of utility operations or networks comprise a natural monopoly, what benefits might be forthcoming from incrcased competition among generation companies, whether transmission capacity is sufficient for supporting short-term energy exchanges in addition to assuring network reliability, where to draw the line between generation and transmission (since ancillary services, like spinning reserve, frequency control, and reactive power supply are provided, at least in part, by generators but are crucial to transmission system performance), and whether it is administratively possible to establish prices for transmission services (and/or markets for the trading of transmission capacity rights) will be perceived as fair and that will signal the full costs of unbundled power transactions. Such questions cannot be approached without a significant understanding of transmission network operations and economics. For instance, transmission "capacity," e.g. to accommodate a wheeling transaction, is not a simple sum of the unused capacities of each of the components of the systems. Instead, it is a changing function of how the system is being used (where the congestion is), the locations of the inputs and outputs for the wheeling transaction, and how any capacity slack on components of the system would be needed if a serious disturbance were to occur anywhere on the system. Numerous power flow studies are required to determine what incremental transmission services are feasible and what they might cost.

Finally, transmission expansion planning requires detailed engineering analysis to determine what is economic. Because of loop flows, increasing the capacity of any line will reduce the power flow on all parallel paths. Transmission capacity can be incrcased by means other than just adding new lines. It may be sufficient to add capacitors that improve voltage control on a heavily loaded line, or even to add devices that partially direct flows down alternative paths. In general, changes to part of the transmission system will affect other parts. Such indirect but significant impacts must be recognized by utility planners and also by policy makers in order to satisfy the requirement that utilities expand whenever the system cannot honor a request for new service without impairing performance to other customers.

Significance of Transmission to Planning

The planning and managing of utility operations is an analytically sophisticated task. Nearly every aspect uses some form of engineering or financial/accounting modeling to help identify good operating and investment practices. As a rule of thumb, the shorter the horizon of the planning problem, the more it will involve transmission considerations. However, it is becoming more important for longer-term models to reflect transmission concerns as well. Figures 1.1 and 1.2 provide a summary of the hierarchy of issues and planning methods involving transmission.

<u>Real time</u> problems are those that arise in controlling the second-by-second and minute-by-minute operation of generation and transmission facilities. Power production levels must be adjusted continuously in response to shifting electric loads

and unpredictable disturbances to the transmission or generation facilities. This is done partly without human intervention by system feedback mechanisms called "automatic generation control" (AGC) that rapidly detect small load variations and redistribute the burden of power production among available generators in a fashion that keeps the system operating within tight performance standards. In particular, all generators must operate at a frequency very close to 60 Hz, and intercompany tie-line flows between utilities must be consistent with power contracts. A major planning problem is determining what rules for adjusting generation and transmission should be used by these AGC systems.

Ideally, AGC would accommodate changing loads by altering the use of on-line power plants in their economic dispatch order, but this is not always feasible. It may be impossible to incrcase the output of the cheapest available plant as rapidly as load increases, necessitating that other, more expensive plants be used at least temporarily. Or, relying entirely on the economically most attractive plant might overload a portion of the transmission network, threatening system stability (the ability of the generating plants to stay in synchrony with each other, crucial to avoiding black-outs.) The extent of line losses also depends on the pattern of network flows and must be recognized in altering the system dispatch. Thus a complex tradeoff must be made between the operating costs and transmission system performance objectives.

To calibrate the AGC mechanisms, several kinds of models are used. "Optimum Power Flow" (OPF) models simulate economic dispatch of the system as constrained by the physical laws for how power will flow on the network and by engineering capacity limits on individual transmission lines. Such analysis is performed by a few utilities as often as every half hour to identify economic rules of operations under the conditions projected for the immediate future. (On the other hand, many utilities do not use OPF at all, due to its computational intensity when analyzing many generation plants, transmission lines, and demands.)

In order to accommodate uncertainty about those future conditions, e.g. to decide how much of which lines should be set aside for absorbing potential disturbances to the system, "contingency analyses" are performed. These involve detailed simulation of system flows for each plausible major disruption to the system, such as unplanned power plant outages or a lightning strike that could disable a portion of the network. If some disruptive event would be especially problematic, e.g. inducing a level of power loading on some of the generators or lines that they cannot sustain, dispatch patterns may be altered to reduce the risk exposure. A transmission system is said to be "secure" if it could continue to provide power that meets demand even if a contingency were to occur. The appropriate adjustments are sometimes identified through "distribution factor" studies which involve a linearized evaluation of how real power flows will be redistributed following an outage of a generator, transformer, or line. These analyses can be embedded in economic studies to find a set of least-cost adjustments to current operations that would make the system likely to remain viable under most foreseeable contingencies. Contingency analyses must be performed at frequent intervals, since system conditions change quite rapidly even under normal operations.

Short-term planning problems have a horizon of one day to one week. The primary concerns are "unit commitment" and how to coordinate the scheduling of utility-controlled thermal plants with hydroelectric and non-conventional forms of generation. The unit commitment problem is the question of which plants to make available in order to achieve the least costly, most reliable operations over a range of foreseeable conditions. This problem is difficult because many thermal plants have significant startup and shutdown costs, in terms of both dollars and lag times to reach full power. To avoid these transition costs, their temperatures and steam pressures can be sustained throughout the day or week at fairly high levels as "spinning reserve" units. However this uses fuel and so becomes uneconomic unless a plant is fairly likely to be used frequently or is critical to network security (ability to tolerate an unplanned disturbance). That tradeoff between the costs of standing by versus the potential benefits of being used must be analyzed with dynamic programming (DP) methods such as optimization. Network constraints are not usually included explicitly in such analyses, but transmission considerations arise in that spinning reserve units must be physically distributed throughout the network in such a fashion that any major line outages would leave essentially self-sufficient subsystems.

The primary seasonal planning issue is maintenance scheduling of large generating units. This is also an optimization problem, but it generally is not one that involves numerous transmission considerations. The shifting of power supply from baseload units on maintenance to less economical "shoulder" units not normally run at a high load factor can involve significant changes in power flow patterns. This might affect the amount or location of transmission capacity available for wheeling or other inter-utility exchanges.

Annual problems, spanning a one-to-three year time frame, include operational budgeting, certain types of fuel procurement and inventory planning, maintenance scheduling, and the design and pricing of utility services. A large portion of utility planning staff is involved in these activities. Among the most familiar planning tools for these purposes is the production costing model, which simulates the projected economic dispatch of a utility system subject to uncertainty about unplanned forced outages of generation facilities. Transmission considerations are usually incorporated as two rules of thumb: must-run plants and average line losses. A few plants in critical locations are simulated as must-run units regardless of their relative operating costs, in order to offset transmission bottlenecks that prevent those regions from being served by more remote plants to avoid violating network security constraints. Average line loss factors capture the need for generation to exceed end-use demand by 1-3%, since some power will always be dissipated as heat in delivery. A more detailed analysis of line losses would reveal that the loss rate is not constant throughout the network or over time, but varies with line loading and can be as much as 7-8% in some circumstances.

Electric rates, excluding fuel and purchased power recovery, are usually set for a one to three-year horizon. They are designed by combining accounting cost (revenue requirements) information with data on the size and time patterns of customer demand. If patterns of customer use (especially at the wholesale level) become more short-term and flexible in the future, more efficient prices based on marginal costs may become important. For transmission services, short run marginal costs that feed into rate calculations can be obtained from applications of the OPF models that are also used for short-term operational planning. Recent FERC rulings and regulations allow electric utilities to be compensated for wheeling opportunity costs under conditions where normal system operations must be adjusted in order to accommodate the third-party transmission.[7] Averaging of OPF marginal costs across the range of economic conditions that could occur over the life of the wheeling contract could be a part of this opportunity cost calculation.

Some proponents of open access to electric transmission have suggested that very short term, essentially spot, prices for electricity could be derived with similar transmission-constrained dispatch models, adjusted to meet a revenue requirement constraint.[8] A pricing regime for signaling both short and long run opportunity costs is needed so that both current system use and future utility (as well as customer) investment planning can be performed well. A potentially thorny issue is how the various and varying engineering constraints related to system security can be reflected in prices, especially when they are not yet binding.[9]

Planning systems themselves can be revised or replaced over a few-year horizon. Whenever a major change of operating or regulatory environment is anticipated, new data collection and modeling capabilities must be developed a few years in advance. Today, at least two potential changes in the utility environment have his portent - the continuing reliance on NUGs, which may require significantly more powerful real time network modeling and control technology, and the prospects for more open-access unbundling of transmission services (possibly down to the retail level), which may require network marginal cost and security analyses in order to set protocols for access and/or efficient prices. Again, the ideal system would be efficient with regard to both short run operations and long run incentives.

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[7] *See the Penelec decision, FERC Docket No. ER91-313-001*, 1992.

[8] . See Hogan, *1991 or Schweppe et al*, 1988.

[9] A well-known result of static optimization theory is that a resource has an opportunity cost of zero unless and until it use fully utilized. By this narrow interpretation of value, a standby or contingency resource has no value for much of the time, even though it is providing a useful insurance-like function. A richer framework would assign a value as proximity to constraints increased. Part of the difficulty in assigning economic values to security analysis are too complex to incorporate economic considerations, since they must evaluate many different scenarios for how the system could be disturbed. See Ilic 1992, for a proposed process of iterating between engineering and economic evaluations to find a compromise between the two types of objectives.

Multi-year capacity expansion plans are made every few years with as much as a forty-year outlook, a duration needed to capture the full lifecycle costs of new system facilities. Typically, the present values of revenue requirements (PVRR) for competing expansion alternatives are compared to identify the least-cost capacity plan. These analyses must consider the need for (or opportunity to avoid) transmission expansion by appropriate generation siting, DSM, or distributed storage technologies. Network adequacy and reliability itself must be evaluated periodically, by identifying where the network is most vulnerable to disruption of service and evaluating how possible transmission capacity expansions would alter the patterns of steady and contingency flows.

This brief survey of the way electric power flow and transmission issues enter into utility economics and planning shows that while their influence depends on the time frame and type of decision under consideration, they can never be ignored.

Figure 1.1

Time Frame	Planning and Control Problems	Transmission Considerations	Planning Topics
Real Time (seconds to minutes)	Preserving system security while meeting continuously varying demands economically	Line electrical characteristics, thermal and voltage limits	Automatic Generation Control (AGC)
	Maintaining system frequency and voltages very near fixed levels	Network-constrained dispatch	Optimal Power Flow (OPF)
	Anticipating disturbances to transmission or generation capacity that could disrupt system; curtailing load in emergencies	Flow capacity reserves for contingencies	Load-flow sensitivities, Distribution factors
Short Term (few hours to 1 week)	Unit commitment tradoff between generation startup/shutdown and operating costs	Transmission bottlenecks that affect location and sizing of spinning reserves	Dynamic Programming (DP)

Annual (1-3 Years)	Production budgeting, e.g. for fuel procurement Seasonal hydro and maintenance scheduling Rate design, e.g. for reactive loads, wheeling	Must-run plants, average line losses Short run marginal costs of wheeling or of providing reactive power Fixed cost recovery	Production costing OPF across many scenarios Cost of service, pricing theory
Multi-Year (3-40 Years)	Power supply planning, e.g. NUG bidding Network expansion for growth, reliability Long-term wholesale power of wheeling contracts Open-access regulatory policies	Transmission construction costs and siting constraints Location specific penalties for new generation, based on transmission consequences Protocols and prices for transmission	Mixed Integer Programming, PVRR scenario analyses OPF, security studies Financial analyse`s, long run avoided costs (LRACs)

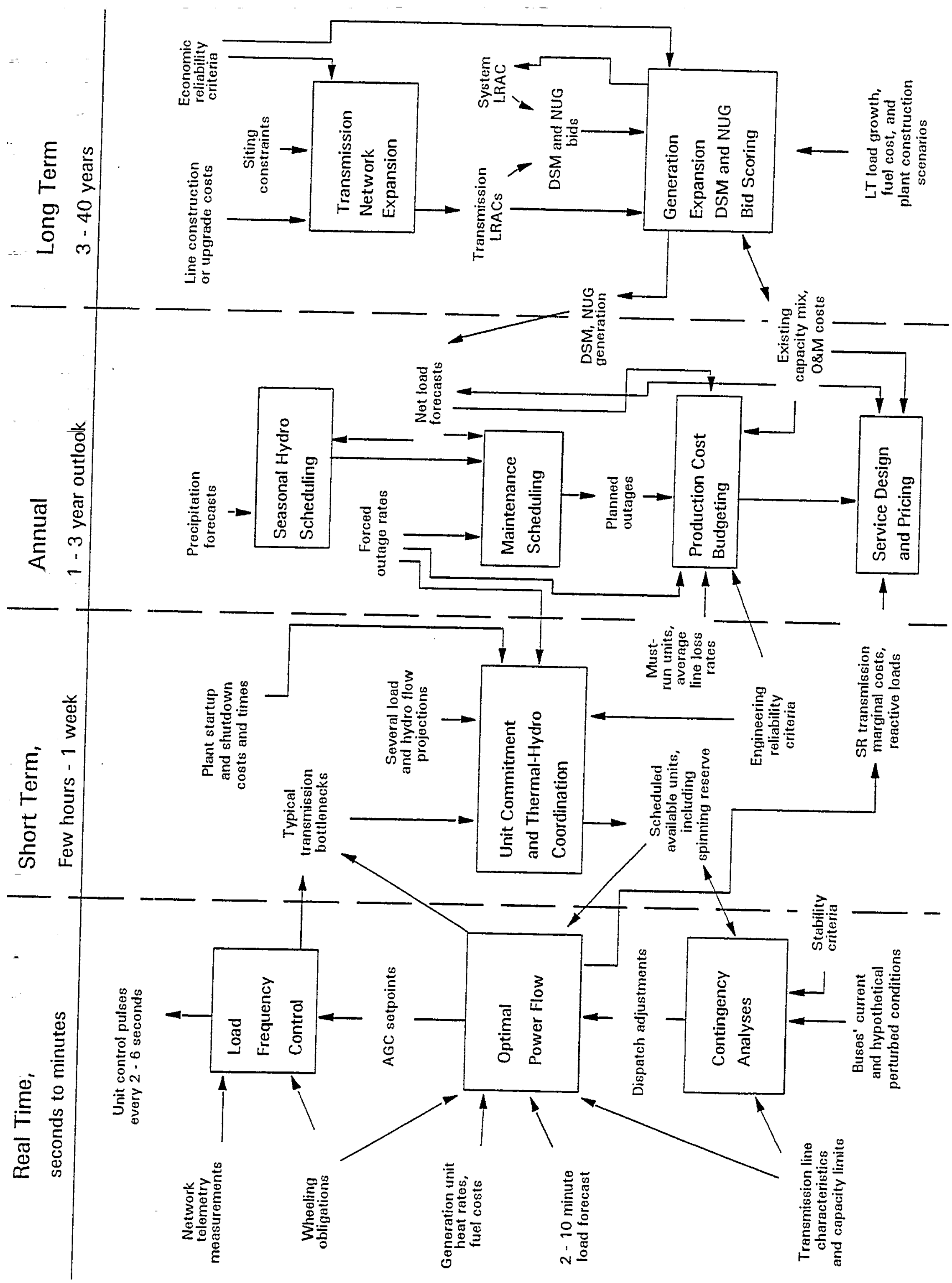
Long Term
3 - 40 years
Annual
1 - 3 year outlook
Short Term,
Few hours - 1 week
Real Time,
seconds to minutes
Economic reliability criteria
Line construction or upgrade costs
Siting constraints
Transmission Network Expansion
Transmission LRACs
System LRAC
DSM and NUG bids
Generation Expansion DSM and NUG Bid Scoring
LT load growth, fuel cost, and plant construction scenarios
DSM, NUG generation
Existing capacity mix, O&M costs
Net load forecasts
Precipitation forecasts
Seasonal Hydro Scheduling
Forced outage rates
Maintenance Scheduling
Planned outages
Production Cost Budgeting
Service Design and Pricing
Plant startup and shutdown costs and times
Several load and hydro flow projections
Typical transmission bottlenecks
Unit Commitment and Thermal-Hydro Coordination
Must-run units, average line loss rates
Engineering reliability criteria
Scheduled available units, including spinning reserve
SR transmission marginal costs, reactive loads
Unit control pulses every 2 - 6 seconds
Load Frequency Control
AGC setpoints
Optimal Power Flow
Dispatch adjustments
Contingency Analyses
Stability criteria
Buses' current and hypothetical perturbed conditions
Network telemetry measurements
Wheeling obligations
Generation unit heat rates, fuel costs
2 - 10 minute load forecast
Transmission line characteristics and capacity limits

Figure 1.2

2

BASICS OF ELECTRIC CIRCUITS

A prerequisite to understanding utility power systems is a background in the physics of simple electric circuits such as those found in the home: a stove, a lightbulb, a vacuum cleaner motor. This chapter provides an overview of how the amount of power and pattern of current flows in such a circuit depends on the type and position of electrical components in the circuit. Key points include:

- Current is the flow of charged particles (electrons) induced by an electromotive force (voltage). This voltage can be either constant or variable; a common form in the U.S. is a voltage that varies sinusoidally over time (alternating voltage) at sixty cycles per second. The flow of current in a circuit (network of interconnected paths made of conducting material) occurs in a fashion that satisfies certain conservation rules called Kirchhoff's Laws. These state that the net current into and out of any point on a circuit will (must) add to zero, and that the sum of the individual voltage changes in a closed loop will equal the applied voltage in that loop.
- The flow of current is influenced by the pattern of paths in the network and by the mix of power-using elements distributed throughout it. Resistors, for example, convert electrical power into heat. If a resistor can be "bypassed" by providing a parallel path of less resistance, current will flow through the new path, but also through the resistor. All else being equal, total current is incrcased by decreasing resistance or by adding more alternative paths.

  Capacitors and inductors are devices that temporarily convert electrical power into stored electromagnetic energy. In a circuit supplied with alternating voltage, these conversions can alter the timing of the flow of current, causing significant changes in power availability in their region of the circuit. The combined effects of resistors, capacitors, and inductors on power availability is called impedance.

  When circuit elements are connected in series (all lying in the same path), the total impedance is the sum of the individual component impedances. When elements are arranged in parallel (each on an alternative path), the inverse of their impedances may be added to find the inverse of the total impedance.
- In a circuit with alternating voltage and only resistive elements, voltage and current will always be in phase, i.e. they will achieve their maxima and minima in synchrony.

However, the temporary conversion of electrical power to stored energy in capacitors or inductors can cause the current and voltage in the region to be pushed out of phase. Current and voltage will both vary sinusoidally at the same frequency, but their peaks and valleys will not coincide. This noncoincidence necessitates keeping track of not only the magnitudes of voltages and currents but also of the relative separation in the timing of their waveforms, called a phase angle. A convention called phasor notation is a computationally convenient means of representing voltages and currents that are out of phase.

- Instantaneous power, the rate at which work can be done using electrical energy in the circuit, is the product of instantaneous voltage and current. In an alternating circuit, instantaneous power varies sinusoidally over time though average useful power is constant over any number of full cycles. Average power is reduced when the phase angle between voltage and current is nonzero.

  The effect of phase angles on average power is easiest to understand and analyze when power is decomposed into two parts: real power (measured in watts) that is available to resistive elements, and reactive power (measured in volt-amps reactive, or VARs) that is diverted by inductive and capacitive elements. The reactive power component can be either incrcased or decrcased by judicious use of inductors and capacitors. Generally, the input power required to serve an end-use device will be lowest when reactive power in a circuit is kept close to zero.

Power flows on a utility system are governed by the same laws as a simple circuit, as described in this chapter, though there are several additional problems related to coordinating the production from several remote generators to match a changing total demand in real time. (These complications are the topic of subsequent chapters.) In particular, the concepts of voltage, impedance, Kirchhoff's Laws, and real and reactive power are the primary elements of the equations used in models for transmission planning.

Energy and Power

Energy is the capacity to do work, e.g. to move a locomotive, to raise an elevator, or to produce heat or light. In the case of a locomotive, engine-produced energy is used to overcome inertia and the frictional forces between the moving parts of the train and the wheels. The energy used to overcome the inertia is changed into *kinetic energy*, the energy of an object in motion. Friction causes some of the energy supplied to the locomotive to be converted into heat, so that energy must be added to the system to keep the locomotive moving. *Power* is the time rate of doing work- in this case, the time rate of adding energy to overcome the frictional forces that tend to slow the train's motion.

In a rising elevator, energy must be added continually as work is being done to raise the elevator against Earth's gravitational pull. This energy is stored in the elevator in a fixed amount (proportional to its height above the starting point) that does not depend on how long it took the elevator to get there. However, the power required to raise it does depend on the rate of ascent. The energy stored in an elevator on an upper floor is called its *potential energy.* This can be converted to kinetic energy by letting the elevator drop, or it can be tapped and put to other uses by various intermediate devices, such as pulling up another elevator on a pulley. There are many examples of stored or potential energy of interest to utilities, such as compressed air reservoirs or the elevated masses of water behind hydroelectric dams.

Electrical power production requires an energy source. Chemical energy is the source in bakeries and fuel cells. For the electric utility, the source may be a hydroelectric dam, a windmill, fossil or nuclear fuel, solar or geothermal energy. The generator of electrical power converts these diverse forms of energy into a controlled, transmittable form of electromagnetic energy that many devices are designed to exploit. Electrical power is very convenient because it can be generated at the time of need and then moved almost instantaneously across a utility service area. However, electric power transmission is not without its own costs and compromises since moving it requires joining the generators (sources) and loads (uses) by long lines, which themselves have properties that govern how efficiently power can be transmitted.

In this section, power will be denoted by the symbol *P* and will be measured it watts.[1]

Current and Conductors

Electric *current*, denoted by the symbol *I*, is the flow of charged subatomic particles, typically *electrons* in the same direction on a path. For electrons to flow, they must be passed from atom to atom in a *conductor*. In principle, electrons can flow through any matter. However a good conductor, such as copper wire, is one through which electrons flow very easily. A poor conductor, called an *insulator*, is one through which electrons do not flow readily, such as porcelain, rubber or air.

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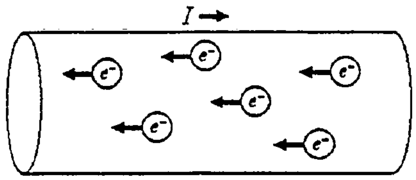
[1] The watt (W) is named for the Scottish engineer James Watt (1736-1819). All units will be denoted in the SI (Systeme lnternational) metric system, which is based on the meter, the kilogram and the second. This system is therefore also known as the MKS system. One watt equals one joule (the SI unit of work and energy) per second. The joule (J) is named for English physicist James P. Joule (1818-1899). In engineering units, $1\ W = 1\ kg\text{-}m^2/s^3$



An electric *circuit* has a source of electrons, at least one path (conductor) through which the electrons (current) can flow, and a destination for the electrons. The direction of the electron current is away from the electron source and toward the electron destination.[2] The question arises: what can cause a current to flow?

A simplified explanation lies in the fact that like charges repel while unlike charges attract, and in the recognition that various objects can be made to sustain large charge separations through chemical and mechanical means. These electricity producers (e.g. batteries or generators) will have one point (terminal) where there is a large net negative charge and another where there is a large net positive charge. When a pathway (conductor) is provided between these two terminals, a current of electrons will flow under the influence of the repulsive force at one end and the attractive force at the other, as long as the charges at the terminals can be maintained or replaced.

Before the charges start to flow, the source and destination terminals have a *potential difference* between them- in effect, an "electrical pressure" or *electromotive force* of repulsion that pushes electrons toward the positive termina. This potential difference can be measured–either between the two terminals themselves, or individually with respect to some common reference point (or ground).[3] . When the difference between the *potentials* of two objects is zero, no current flows. At that moment, the electromotive force pushing from one end of the conductor exactly counters the emf from the other end. The symbol for emf is *E*. In most cases, it is unimportant to make the distinction between emf and potential difference when speaking of circuit operation. The catch-all term *voltage* is used for either in these circumstances, and the unit for its measurement is the volt (*V*).[4]

[2] Based on early terminology and beliefs concerning the nature of electricity, current was assumed to flow from locations designated as "positive" to others designated as "negative." This direction is called the conventional current direction. Conventional current direction is the usual orientatioa of arrowheads on a circuit diagram and is a well-entrenched standard despite present knowledge that the actual electron flow is in the opposite direction

[3] Engineers often speak of the potential of a point in a circuit. The point of zero potential is called ground, which ordinarily has the same potential as the ground under our feel The potential of other points is measured relative to ground.

[4] The volt (V) is named for Italian physicist Count Alessandro Volta (1745-1827).

The unit for current is the *ampere* (A).[5] When one ampere of current flows through a conductor, the charge that passes the cross-sectional area on that conductor in one second is one *coulomb* (C), an amount of charge equal to $6.24x10^{18}$ electrons.[6] *Direct current* is a current that flows in one direction only. This kind of current can be supplied by a voltage cell such as a battery, an example of a *direct voltage source.*

Using these definitions, it is possible to relate the voltage (potential difference, *E*) between two points on a conductor to the current (electron flow,*I*) between these points. One *volt* is defined to be the potential difference between the ends of a conductor when one watt of power (*P*) is dissipated (converted to another form of energy) during the flow of one ampere of current through the conductor: 1 Volt = (1 Watt)/(1 Ampere) and $E = P/I$.

We illustrate this relation with a simple circuit analysis: How much current flows through a battery-powered lamp dissipating 0.25 watts? The answer depends on the potential difference across the terminals of the battery. If a 9 V battery is used, the current will be

I = (0.25 w)/(9 V) = 0.028 A = 28 mA (milliamperes).

A battery with a lower voltage would require that more curreut be supplied to the circuit. To supply the same power with a 3 V battery the current flow would be three times as large:

I = (0.25 W)/(3 V) = 0.083 A = 83 mA.

(This could be more than a dry cell battery can provide, in which case the lamp would not receive its design power and would not be fully lit.)[7]

Resistance and Ohm's Law

Since power is dissipated in a circuit, some force that is opposed to the flow of current must be at work. The example of a locomotive is analogous in some respects. In a frictionless, gravitation-free world, once the initial inertia had been overcome and the

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[5] The ampere, named for French physicist Andri Marie Ampere (1775.1836), is a fundametal unit in the SI (that is, MKS) system. It is defined as the amount of current which, when flowing in the same direction in each of two infinitely long, parallel conductors one meter apart, causes a repulsive force of $2x\ 0^{-7}$ kg,m/s^2 per meter of length to act on each conductor. (This force is due to electromagnetic induction, explained later.)
[6] Named after French physicist Charles Augustin de Coulomb (1736-1806).

[7] For a basic introduction to the electricity and magnetism used throughout this chapter, see Halliday & Resnick, 1978 (or any more recent edition). For the methods of circuit analysis, see Bell, 1988. -

locomotive had been given momentum, the locomotive would continue indefinitely its motion at a constant speed without further work being required. But in the real world, power must be applied continually to compensate for frictional forces opposing the direction of motion in order to maintain the locomotive's speed.

A similar problem arises within a conductor. The electrical analogue to the coefficient of friction is *resistance* (symbol *R*). The resistance of a conductor is a measure of its opposition to current flow; it is a physical property that varies with the conductor's shape, temperature, and other factors. The unit of resistance is the *ohm* (usually signified by the upper case Greek letter omega, Ω).[8] Certain resistance in a conductor can be understood by regarding resistance as being the result of electron-atom "collisions" within the conductor.[9] When current flow is incrcased, more collisions occur and more power is dissipated. The resistance of a copper wire is halved when its cross-sectional area is doubled because fewer electron-atom collisions will occur in a thicker wire. A wire with twice the length will also have twice as many collisions and therefore twice the resistance.

A good conductor allows electrons to flow easily, but they do not necessarily flow fast. The average electron drift velocity is equal to the current per cross-sectional area of a wire divided by the electron density. Algebraically: $v = i/(Ad)$. Applying this formula we discover, perhaps surprisingly, that the speed of electrons in a current-carrying conductor is very low. ln copper there are roughly $1.38\text{x}10^{24}$ free electrons per cubic inch; a #14 AWG copper wire (which is about 1/16 inch in diameter) has a cross-sectional area of $3.23\text{x}10^{-3}$ square inches. For a current of ten Amperes the election drift velocity is only v = 0.000356 meters per second, or 1.28 meters per hour![10] Of course, an immense number of electrons is moving at this lethargic pace. Moreover, the electric impulses (voltage propagation) associated with these electron movements travel at nearly the speed of light, $c = 2.978\text{x}10^{8}$ meters per second. Each electron's movement induces a sequence of movements in other electrons downstream in the wire, with the result being current (or power) that propagates at an enormous speed.

Good conductors have small values for *R*. A measure of the lack of opposition to current in a conductor, *conductance*, is the reciprocal of the resistance of the conductor: G = 1/*R*. The SI unit of conductance (*G*) is the *siemens* (S) or the mho.[11]

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[8] Named for German physicist George Simon Ohm (1787-1854).

[9] Technically, the electrons and conductor atoms do not collide like billiard balls, but the movement of the electrons is deflected in a fashion similar to collisions by the electrical fields surrounding the electrons of the conductor atoms.

[10] 1.28 m/h = [10C/s]•[1/$3.238\text{x}10^{-3}$ in ^{2}]•[in/1.38xl0=nelectrons]•[0.254 m/inch]•[1 electron/1.602x 10^{19} C]
[11] The siemens is named for British engineer Sir William Siemens (1823-1883).

A *resistor* is a circuit element that has a known resistance. Electronic circuits often include numerous resistors of different sizes. Their combined effect can be determined using the following rules of combination for resistors:.

The effective resistance of *n* resistors in *series* (placed sequentially in a single path) is the sum of their individual resistances:

$$R_{total} = R_1 + R_2 + \ldots + R_n$$

The effective conductance of *n* resistors *parallel* (placed side by side, creating alternative paths) is the sum of their individual conductances:

$$G_{total} = G_1 + G_2 + \ldots + G_n$$

or equivalently,

$$I/R_{total} = I/R_1 + I/R_2 + \ldots + I/R_n$$

Note that this R_{total} will be smaller than any of the individual parallel resistances.

Individual conductors have a fairly constant resistance under normal operating conditions.[12]

Ohm's Law states that the current through a conductor is directly proportional to the applied voltage, where the proportionality constant is the conductance of that conductor: $I = EG = E/R$. One ohm is defined as the resistance that permits a current of one ampere to flow when a potential difference of one volt is applied across the resistance: IΩ = (1 V)/(1 A). One way to think of resistance, current, and voltage is to compare a resistor to a hole in a leaky balloon. The air in a balloon will escape faster (incrcased current) if the hole is widened (reduced resistance). Another way to incrcase the rate of escape is to incrcase the pressure differential (incrcased voltage) between the gas inside the balloon and the gas outside the balloon.

As stated previously, voltage is the ratio of power to current ($E = P/I$). Multiplying both sides of this equation by I yields $P = EI$, so the power dissipated by a circuit is the product of the voltage and the current. Substituting $I = E/R$ from Ohm's Law into the

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[12] For the purposes of this chapter, conductors will be treated as idealized, having no resistance, while circuit elements will be treated as having a constant R that does not vary with conditions of use. In practice, temperature can affect the resistance of both conductors and electronic components.

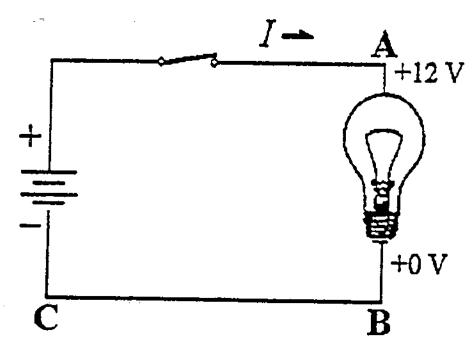
power equation shows that $P = E \cdot (E/R) = E^2/R$. Alternatively, substitution of Ohm's Law in the form $E = IR$ yields $P = IR(I = I^2R$. Thus for a circuit with resistance R,, power dissipation increases as the square of voltage or current, e.g. doubling the applied voltage doubles the current but quadruples the power dissipation.

The above rules on combining resistances imply that if a voltage is applied to resisters in series, the current through each of the resisters will be the same, $I = E/R_{total}$. If the resisters are in parallel, each resistor of a different size will have its own corresponding current, but the total current will be $I_{total} = E.G_{total.}$

Kirchhoff's Laws

Almost any interesting circuit consists of several paths serving several kinds of devices. Kirchhoffs Laws define physical constraints governing how power or current will flow in any network with several paths:

- *Kirchhoff's Current Law* (KCL)) states that the sum of the currents entering a point in an electric circuit must equal the sum of the currents leaving that point. Equivalently, the net current into and out of any point must be zero.
- *Kirchhoff's Voltage Law* (KVL) states that in any closed electric circuit, the sum of the voltage drops across the circuit elements must equal the sum of the applied emf's around every closed loop.



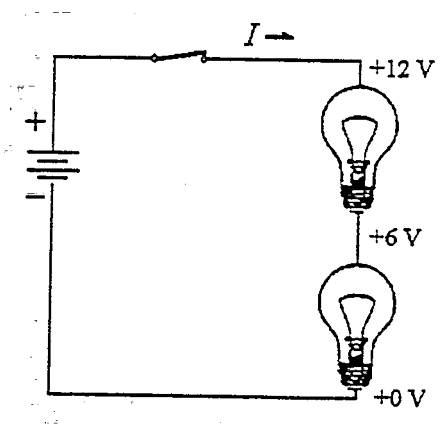
With the use of these laws, the flows on a simple circuit can be predicted. The circuit at right includes a direct voltage source (12 V battery), a resistive load (a lamp rated 3 W at 12 V), a switch, and a few conductors (wires) linking these components in series. The resistance of the wires and internal resistance of the battery can be ignored since they are very small compared to the resistance of the lamp.

When the switch is open, there is no path through which the current may flow. Closing the switch creates a path. KVL states that the net voltage around the closed loop must be zero. Since the battery is providing 12 volts, the voltage drop across the resistor must also be 12 volts. That is, the potential difference between points A and B is 12 V, but between B and C it is zero. The amount of current that flows can be determined with the voltage-defining equation $E = P/I$. The lamp draws 3 W at 12 V, so the current flow

$I = P/E = (3W)/(12V) = 0.25A$.

The lamp is a special kind of resistor, the resistance of which can be deduced from Ohm's Law:

$R = E/I = (12V)/(0.25A) = 48\ \Omega$.



Adding another 48 Ω lamp in series to the existing circuit elements will alter the circuit in several respects. The direct voltage source still provides 12 V, but the total resistance is doubled, to 96 Ω, since series resistance is additive. The entire supply current flows through both of the resistors (lamps) but is now smaller:

$I = E/R_{total} = (12\ V)/(96\ \Omega) = 0.125\ A$.

The voltage (drop) across the original lamp is now half as large:

$E = IR = (0.125\ A)(48\ \Omega) = 6\ V$.

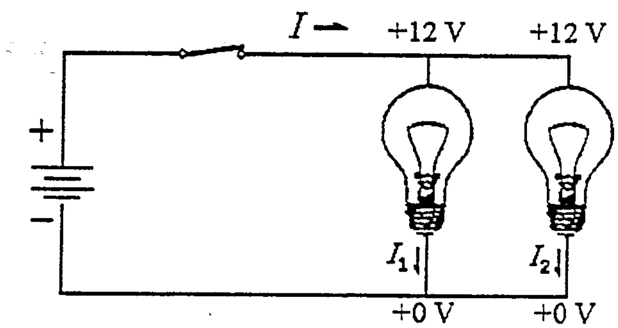
We could have used Kirchhoff's Laws to establish these results: By KVL, the voltage drop across the two lamps must add up to 12 volts. Since the two have the same resistance value, the voltage drop across each must be the same, i.e. 6 volts. By KCL, the current flowing in the loop must be the same as the current flowing across each resistor:

$I = E/R = (6\ V)/(48\ \Omega) = 0.125\ A$

Total power dissipated with two lamps in series is:

$P = EI$ = (12 V)(0.125 A) = 1.5 W,

versus 3 W with only one lamp. Thus less light is enjoyed than would have been produced by one lamp alone.



Placing the second 48 Ω lamp in parallel instead of in series produces a different result. Total conductance is additive for parallel resistors, so

G_{total} = $1/(1/R_1 + 1/R_3)$

= 1/(1/45 + 1/48) = 1/24 siemens

The effective resistance of the two lamps in combination is therefore

$R_{total} = 1/G_{total}$ = 24 Ω.,

one-half of the resistance of either lamp by itself. By KVL, the voltage across each lamp is the same and equal to the source emf, 12 V, so the current through the original lamp is

$I_1 = E(G_1$ = (12 V)/(48 Ω) = 0.25 A.

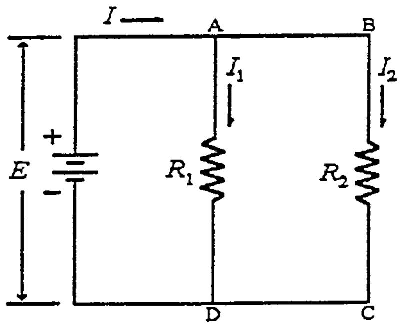
Since G_2 is the same as G_1, I_2 also equal 0.25 A. By KCL the total current supplied by the battery is:

$I = I_1 + I_2$ = 0.25 A + 0.25 A = 0.5 A.

Total power dissipation in the form of light is now $P = EI$ = (12 V)(0.5 A) = 6 W, twice as much as with one lamp alone (so now our battery will not last as long).

Thus we see that the number and location of circuit elements has a critical effect on currents, voltages, and power dissipation in the circuit. In this example, adding the second lamp will either halve or double the total power dissipation in the circuit, depending on whether the new lamp is installed in series or parallel. Accordingly, for any planned circuit, the designer must anticipate that both of Kirchoff's Laws will hold when choosing the location of circuit elements.

KCL is true because electrons are never destroyed or created at any point in the circuit. KVL is a consequence of the fact that current always flows through a circuit in a fashion that minimizes power consumption (I^2R losses). To demonstrate this connection to minimal power losses, consider the following circuit:



Power dissipation $P = I_1^2R_1+I_2^2R_2$

$$= I_1^2R_1+(I-I_1)^2R^2$$

$$= I_1^2R_1+I^2R_2-2II_1R_2+I_1^2R_2$$

P is now a function of I_1 having a minimum where

$$dp/dI_1 = 2I_1R_1 - 2I_1R_2+2IR_2 = 0$$

$$I_1R_1 = (I-I_1)R_2 = I_2R_2$$

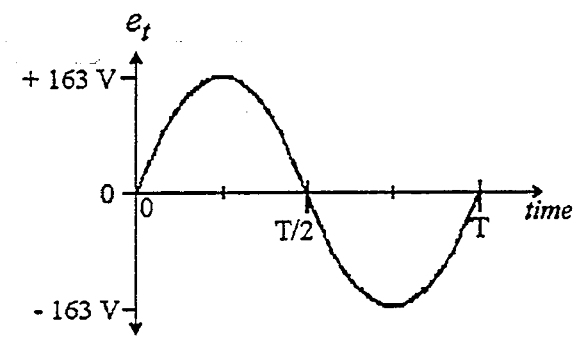
which is KVL for the closed loop ABCD.

Alternating Voltage

To this point, the discussion of voltage sources has been limited to direct voltage sources such as batteries. Appliances that use power from 115 V wall outlets, in contrast, are designed to use power derived from alternating voltage sources.

A wall outlet has two slots for the two prongs on a plug. These slots are like the terminals on a battery, with a major difference: these terminals *alternate* being the "plus" and "minus" terminal sixty times a second, i.e. with a *frequency* f = 60 Hz. In fact, the potential difference between the slots varies smoothly as a sine wave with repetition period $T = 1/f = 1/60$ sec. The *amplitude* (absolute value of the maximum or minimum) of this sine wave is approximately 163 V.[13] The amplitude of an alternating voltage is given the symbol E_m.

The potential difference between the left slot and the right slot of the outlet exhibits the following pattern over one period of 1/60th second:



Time	Voltage
0	*0*
T/4	*+163*
T/2	*0*
3T/4	*-163*
T/4	*0*

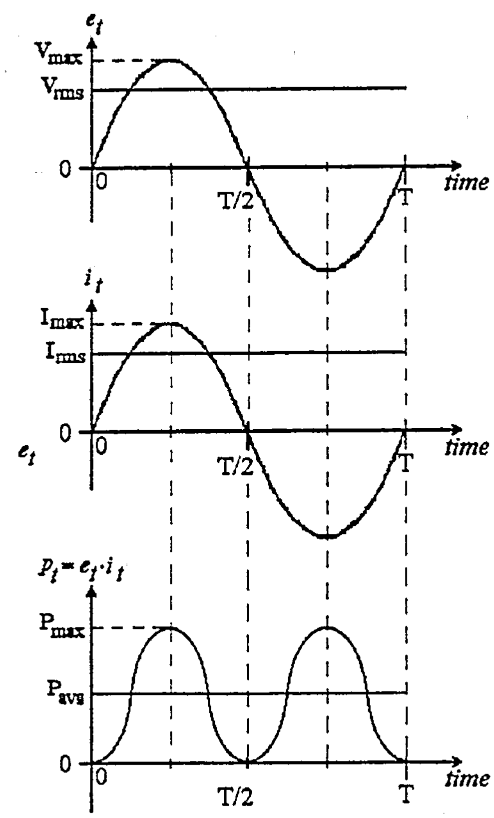
As seen in the figure, the instantaneous voltage difference e_t varies smoothly over time according to the formula $e_t = E_m \, sin(2(ft)$. It is often more convenient to express $2\pi f$ as angular velocity, conventionally denoted ω (lower case omega).

Current can flow in either direction through a resistor. Consider a 1630 Ω resistor attached to a plug. When the prongs are inserted into a household wall outlet, alternating voltages varying between -163 and 163 V are applied across the resistor. At time t, the instantaneous potential difference, between the left prong and the right prong changes in sign (direction), so the current flow determined by Ohm's Law must also change in sign (direction) with the same frequency.[14] Therefore, a resistive circuit with an alternating voltage source is also an alternating current (AC) circuit. Since $I = E/R$ from Ohm's Law, the *instantaneous current* in a resistor is given in this example by

[13] In Europe the frequency of alternation current is fifty cycles per second,and the maximum voltage is 340 volts.

[14] As was the case for DC power, the term "flow" for electrons is somewhat misleading Physically, each electron merely oscillates back and forth in the same fixed location. What "flows" is the influence each such wiggling electron has on its neighbors. Again, this wave of influence (emf) travels at nearly the speed of light around the circuit.

$i_{t\text{-}}= (Em/R)\ (\ sin(2(ft) = I_m\ (\ sin((t)= 163/1630(\ sin((t) =0.1(\ sin(377t)$



Instantaneous power is the product of instantaneous voltage and current: $P_t = e_t i_t$. In a resistive circuit, $p_t = (E_m^{\ 2}/R)(\ sin^2((t) = P_m(\ sin^2((t)$, which is always non-negative and has half the period of the voltage and current waveforms. That is, instantaneous power alternates between 0 and $E_m^{\ 2}/R$ at 120 times a second rather than 60 times. A measure of power that is more useful than the instantaneous power or the maximum power is the *average power* dissipation of an AC circuit. In this case, the average power is

$$P = P_m\ /2,$$

which is evident from the symmetry of the graph of p_t about the horizontal line at one-half its maximum. For an AC resistive circuit to dissipate the same amount of power over time as an equivalent DC circuit, the AC peak power dissipation P_m must be twice the constant DC power dissipation, *P*.

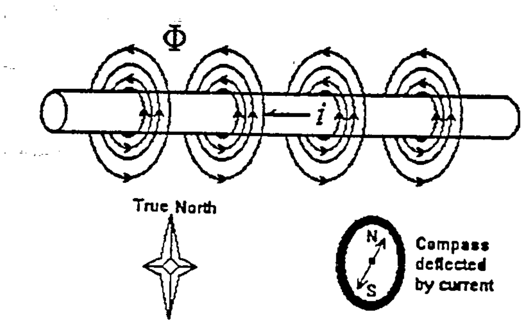
Since AC average power $P = P_m/2= E_m^{\ 2}/2\ R$ (in a resistor) and also $P = E^2/R$, it follows that $E^2 = E_m{}^2/2 = \left(E_m / \sqrt{2}\right)^2$, so the effective or *root mean-squared* (rms) voltage of an AC circuit is $E = E_m / \sqrt{2}$ Similarly, the AC rms current is $I = I_m / \sqrt{2}$ using the rms values of voltage and current does not change Ohm's Law or Kirchhoff's Law, so rms values are generally used in electronics unless stated otherwise. For instance, the rms voltage of a residential wall outlet is $163/\sqrt{2} = 115\,V$.

Ferromagnetism

Everyone is familiar with bar magnets. Because they tend to align themselves with the magnetic field of the Earth, one end of such bars is called a magnetic *North pole* and the other a *South pole*. Like magnetic poles are repelled by each other while opposite poles are attracted[15] . This behavior is explained by magnetic loops of force called *magnetic flux* (symbol () that run parallel through the bar, out the North end, around the outside, and back in through the South end. These flux lines indicate the path that a hypothetical, idealized isolated magnetic North pole would tend to move along if placed in proximity to the bar magnet.[16] When the flux lines of the adjacent magnets are in opposition, e.g. when like poles are juxtaposed, the observed force is repulsive. The attraction between opposite poles occurs when the flux lines of the magnets point in the same direction.

Electromagnetism

Magnetism also occurs whenever a current flows in a conductor. A magnetic flux is created around the conductor, the direction of which is determined by the direction of the current. The *right-hand rule* states that if the right hand were to grasp the conductor such that the conventional flows in the direction of the outstretched thumb, then the magnetic flux will curl around the conductor in the direction of the other fingers of the right hand. One can observe the associated magnetic field by placing a pocket compass near a current-carrying wire.



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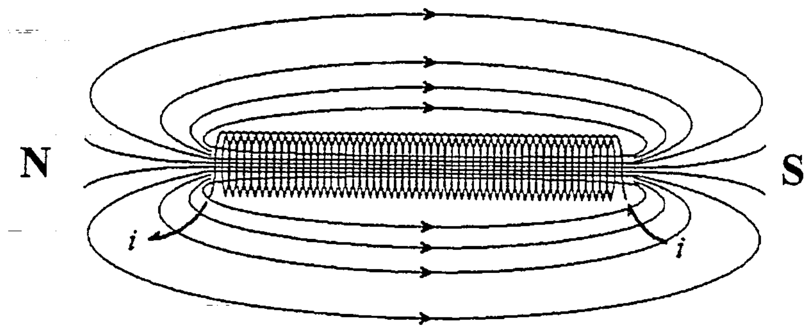
[15] The magnetic "North" pole of a compass points toward the northern pole of the Earth. Thus, the northern pole of the Earth is, by this naming convention, a magnetic "South" pole.

[16] At present, no isolated single magnetic pole or “monopole” has been observed, though the possibility of their existence is currently being studied in the fields of cosmology and very high energy physics. Their discovery could have important implications for theories of the origin of the universe and of the nature of subatomic forces.



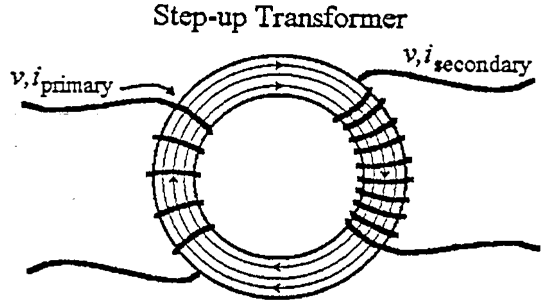
By winding a current-carrying conductor as a coil, a *solenoid* is created. The magnetic flux that is inside the coil is primarily in one direction while the flux on the outside of the coil is in the other direction. As a result, such coils exhibit a North and South pole whenever current flows. If the current switches direction, the poles reverse. if the current alternates, the poles of the solenoid reverse with the same frequency.

Electromagnetic Induction

This relation between electricity and magnetism works in reverse as well: a magnetic flux can create a current in a nearby conductor. When a coiled conductor is moved toward a pole of a bar magnet, the loops of the coil will "cut" the magnetic flux of the magnet. An emf is created in the coil whenever it is cutting flux, such as when it is in motion with respect to the magnet (no matter which one is moving). An emf is induced in any coil that is moved through any magnetic field.

Transformers

The strength of the magnetic flux in a solenoid increases proportionally with the number of turns of the conductor comprising its coil. In a *transformer*, two coils are wound around a core of material, often iron, that causes the magnetic flux to remain primarily within the limited region of the core. When an alternating voltage is applied to the first coil (*primary winding*), alternating current will flow in that coil and a corresponding flux will be created throughout the core. This constantly charting flux then induces an alternating emf in the second coil (*secondary winding*).



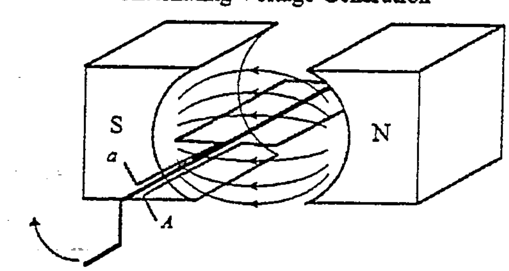
The ratio of the voltage across the primary winding (the input voltage) to the voltage across the secondary coil (the output voltage) is the same as the ratio of the number of turns in the primary coil to the number of turns in the secondary coil: $E_P/E_s=N_P/N_s$, where *N* is the number of turns per coil. Thus, a transformer can be used *to step-up* (incrcase) or *step-down* (decrcase) alternating voltage by using either more or fewer secondary than primary turns.

Since energy is always conserved, the current must behave in the opposite fashion: $I_p/I_s = N_s/N_P$. Thus, a step-up transformer induces a larger voltage but a smaller current in the secondary wiring. This effect is very important to economical long-distance transmission of power, as will be explained in subsequent chapters.

AC Generation

Electrical outlets in the wall provide *single-phase* (one voltage waveform) alternating voltage. The source of this alternating voltage is a power generation station that exploits the phenomenon of electromagnetic induction: a loop of wire is rotated at a constant speed (60 cycles per second) in the field of a magnet. Because of the loop's rotation, its cross-sectional area is periodically aligned and periodically perpendicular to the magnetic field, causing it to cut flux at a variable rate. This induces an emf at the ends of the loop (*A* and *a* in the diagram) that has a sinusoidal waveform of constant frequency.

Alternating Voltage Generation



The source of energy for rotating the loop can be boiler steam, hydro-flow, or any other *prime mover* driving a turbine. The result is electric power that can be transmitted and used throughout the circuit connected to the loop.

Inductors

Suppose a coil that is not carrying current is moved towards a stationary solenoid that is carrying current (hence has a magnetic field). This will cause flux lines to be cut by the moving coil. As a result; a current will be induced in the moving coil, and that current will circulate in the direction opposite to the current in the stationary solenoid. This induced current in the moving coil will also have its own associated magnetic flux, whose direction will be opposed to the direction of the flux emanating from the end of the stationary solenoid. If the induced (coil) flux were not in opposition to the source (solenoid) flux, their effects would be naturally reinforcing violating energy conservation.

There is another way of inducing a current in the first coil without moving it. If the current in the stationary solenoid is varied, its magnetic flux will charge proportionally. This changing flux will induce a current in the first coil (if it is nearby) whose magnetic flux will oppose the initiating flux. A similar induction behavior occurs even within a single coil within which an alternating current flows: the continual changes in magnitude and direction of the flux induced by the current flow in each winding will induce a *counter-emf* inside the coil that opposes the applied emf. (This is easier to envision if you regard a multi-turn coil as being several contiguous smaller coils of a few turns each; each subcoil affects its neighbors.) Such a coil is called an *inductor* and the emf effect is called *self-inductance*, *L*, measured in Henrys (H).[17]

Counter-emf is 180 degrees out of phase to the supply voltage, i.e. counter-emf is at a maximum when applied emf is at a minimum, and vice versa. Consequently an inductor is a device that opposes, or dampens, changes in the current in a circuit. The magnitude of counter-emf depends on the physical properties of the inductor (its self-inductance, denoted *L*) and on the rate of change *di/dt* of the original current waveform:

$e_L = L((di/dt)$

If $i = I_m(sin((I)$ then

$di/dt = (I_m(cos((t)$, *so*

$e_L = L(\ (\ (\ I_m\ (cos((t) = ((L)I_m(sin((t+(/2).\ ($

This shows that a counter-emf in an inductor causes the current waveform to *lag* the applied voltage waveform by 90°, i.e. the voltage reaches its peak $\pi/2$ radians before the current reaches its peak. Physically this occurs because inductors store energy as the applied current increases and relcase energy as the current decrcases, thereby delaying the current waveform inside the inductor. Counter-emf in the loops of a generator creates a force that tends to slow down the rate of rotation. This must be overcome by the energy of the prime-mover (steam, hydro, etc.) that drives the generator.

Inductors placed in series have a total inductance that is the aura of their individual inductances: $L_{total} = L_1 + L_2 + ... + L_n$ While inductors in parallel have total inductance

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[17] Named for American physicist Joseph Henry (1797-1878)

given by $1/L_{total} = 1/L_1 + 1/L_2 + ... + 1/L_{n.}$ Note that these rules have the same form as the rules for combinations of resistors.

The voltage/current ratio in a purely inductive AC circuit is called *inductive reactance*, $X_{L.}$ It is a function of self-inductance and the frequency of the voltage waveform: $E/I = X_L.=2(fL=(L$. Because current lags the supply voltage in an inductor by π/2 radians, the instantaneous current in a purely inductive circuit for which

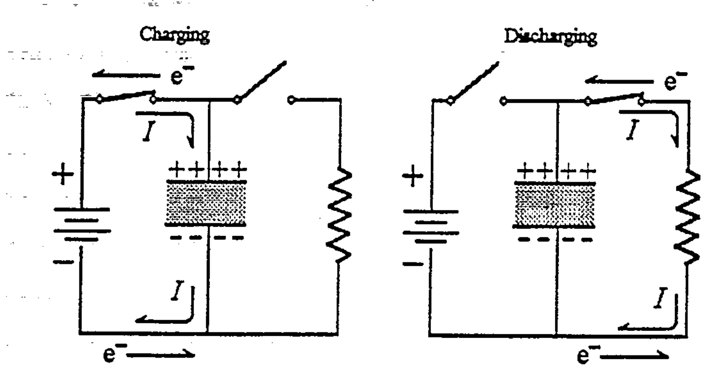
$v_t = E_m sin((t)$ *is:*

$i_t = (E_m/X_L)\ (sin((t-(/2)$.

Inductive reactance X_L has the same fundamental dimensions as resistance, so it is expressed in units of ohms. Note that X_L determines the amplitude of i_t much as a resistance would, but unlike a resistance, it causes current to lag the supply voltage.

Capacitors

Capacitors are pairs of parallel metal plates that sandwich an insulating material called a *dielectric.* The measure of the ability to store electric charge on these plates is called *capacitance*, C, expressed in farads (F), which is proportional to the ratio of plate area to the thickness of the dielectric.[18] In a voltage source (DC or AC) is applied across the capacitor plates, charge accumulates until the plates are "full" of charge, i.e., until the potential difference across the capacitor terminals equals the applied voltage. This will occur when the quantity of accumulated charge $Q = C \cdot E$, where 1 coulomb = 1 F · 1 V. When the applied voltage is removed and an alternative path is created (above), the charge that accumulated on the plates is no longer maintained and current flows in the circuit so as to dissipate the charge on the capacitor plates.



[18] Named for American physicist Joseph Henry (1791-1867)

Capacitors store electric potential energy as the applied voltage increases and discharge energy when the supply voltage decrcases. Expressed mathematically,

$i_c = dQ/dt = d(Ce)/dt = C (de/dt.$

If $e = v_t = E_m(\ sin((t)$, *then*

$i_c = C. E_m - (- \ cos((t) = wCE_m \cdot sin((t + (/2).$

Therefore the effect of capacitance in an AC circuit is to cause the applied voltage to lag the current waveform (or the current to *load* the voltage), opposite to the impact of an inductor. This is because the effective current across the capacitor is greatest when it is starting to build up charge, i.e. when the applied voltage is zero and climbing rapidly. As alternating voltage approaches its peak, the already deposited charge on the capacitor retards and eventually stops the deposition of additional charges. Current in the circuit goes to zero when the potential across the capacitor equals the peak of the applied voltage. This is analogous to the mechanical behavior of a spring, which becomes more difficult to extend the further it is stretched. (Note that no electrons move through the dielectric across the capacitor, but currents on either side have the same sign and size as they would if the electrons actually moved through the dielectric with the current loading the voltage.)

Capacitors are unlike resistors and inductors in the way they are combined algebraically:

For series capacitors, $1/C_{total} = 1/C_1 + 1/C_2 + ... + 1/C_n.$

For parallel capacitors, $C_{total} = C_1 + C_2 + ... + C_n.$

The voltage/current ratio in a purely capacitive AC circuit is called *capacitive reactance*, X_c. It is a function of capacitance and of the frequency of the voltage waveform:

$E/I = X_c = 1/(2(fc) = 1/((C)$

Because the current loads the supply voltage in the capacitor by $\pi/2$ radians, the instantaneous current in the purely capacitive circuit driven by $v_t = E_m \ sin((t)$ is given by

$i_t = (E_m / X_c)(\ sin((t+(/2).$

The units of X_C are ohms, and X_C determines the magnitude of the observed current much like a X_L or R, except now with a loading current.

Power in RLC Circuits

We now have discussed three types of components that affect the relationship between voltage and current in AC circuits. Recall that in an AC circuit with only resistive elements, current and applied emf are always in phase. When capacitors or inductors are included in a circuit, this may no longer be the case. To remember the timing effects of the other two elements, it is convenient to use the mnemonic "ELI the ICE man":

- Inductors - the voltage (*E*) in an inductive circuit (*L*) loads the current (*I*)
- Capacitors - the current (*I*) in a capacitive circuit (*C*) loads the voltage (*E*).

Because inductors and capacitors have opposite effects on the relative *phase angle* between current and voltage, the addition of series inductors to a capacitive circuit (or of series capacitors to a inductive circuit) will tend to diminish the loading (lagging) of the current waveform. In general, the phase angle in a circuit containing resistive, inductive and capacitive elements will not be zero or ±90°, but will vary from place to place in the network. The total effect of resistors, inductors, and capacitors is reflected in the ratio of voltage to current in AC circuits. This ratio is called *impedance* (*Z*). It captures the combined effects of *R*, X_L, *and* X_C, and it is measured in ohms.

The reason for concern over the relative timing of current and voltage is that power levels, given by their product, will be reduced if the two are not strictly in phase. We saw previously that when current and voltage are in phase, AC power is always positive with an average of $P_m/2$. The graphs on Figure 2.1 show that if voltage is out of phase from current by an angle *a*, instantaneous power levels will be periodically negative (when the inductive and capacitive components are feeding power back into the system), scaling the average power down by an amount equal to cos(α), called the *power factor.* In this graph, the phase angle a is equal to 45°. (The numerical values in this figure are arbitrary. Voltage and current are simply assumed to have the stated magnitudes and to be 45° out of phase with each other. Had they been in phase, the average power would have been 5.0 instead of 3.54.)

This power factor can be derived trigonometrically.[19] If $v_t = V_m \cdot \cos(\omega t+\alpha)$ and $i_t = I_m$ *cos((t+() then* P_t *is given by:*

$P_t = vt(\ it = V_{m.}I_m(\ cos((t+()cos((t+()$

$= 1/2V_m(\ I_m\{cos((-() + cos[2((t+()-((-()]\}$

$= 1/2V_{m(}\ I_m\{cos((+() +[cos((-()cos(2(t+2() + sin((-()sin(2(t+2()]\}$

---

[19] Two identities are used in these derivations: cos(a)cos(b) = I/2[cos(a-b)+cos(a+b)], and cos(a+b) = cos(a)cos(b) - sin(a)sin(b)

$= 1/2V_m (I_m cos((-()[I+cos(2(t+2()] + 1/2V_m I_m sin((-()sin(2(t+2()$

Over an entire cycle, any terms that involve *2(t* will average to zero. Thus the average power is just the first term, $1/2V_m(I_m$ (cos((-()=$1/2P_m cos($ where (=(-(is the phase angle between voltage and current. This term is given the name *real power* and will be denoted *P.* The term $1/2V_{m.} I_{m'}$ sin(α-δ) is *called reactive power*, denoted Q. Substituting these new labels, we have:

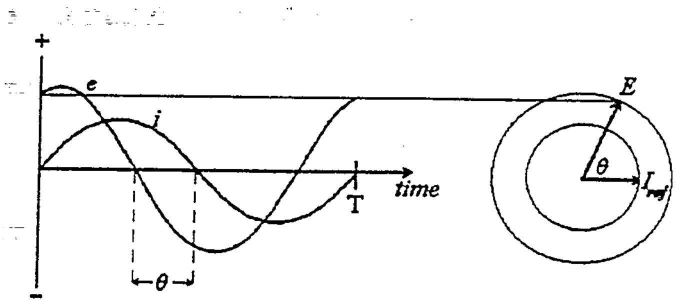
$P_t = P[1+cos(2(t+2()] + Qsin(2(t+2().$

Thus instantaneous power has two components. The first part, *P*[1+*cos*(*2*(*t*+2()*]*, is never less than zero, so its average is positive. This represents power that is consumed by resistive elements in the circuits. The second part, *Qsin*(*2*(*t*+2(), has a positive magnitude *Q* but an average value of zero. This is power diverted to inductive and capacitive elements in the circuit, and it is always 90 degrees out of phase with the first part (sin rather than cos). This timing difference between the two components can be seen in the lower portion of Figure 2.1, where p_t has been decomposed into the resistive and reactive parts that add up to the same curve $p_t = v_t \cdot t_t$.

Real power is "real" in that it can be transformed into useful work. It is expressed in units of Watts. Reactive power is never available for useful work, because its average effect is zero - only alternatingly storing and returning energy to the circuit as charge accumulations or magnetic fields in the capacitive or inductive components, respectively. Nonetheless, reactive power is quite important (and quite real), because it can alter the performance of electrical systems. For instance, it can cause unwanted local voltage changes on transmission networks, and it requires higher levels of current for a given amount of real power to be supplied than would be necessary in its absence. *Q* is expressed in units of Volts-Amps-Reactive, or VARs (which have the same engineering dimensions as Watts), in order to distinguish it from real power. Note again that real power P is an <u>average</u>, while *Q* is a <u>magnitude</u>; neither is time-variant like $v_t i_t$ or p_L

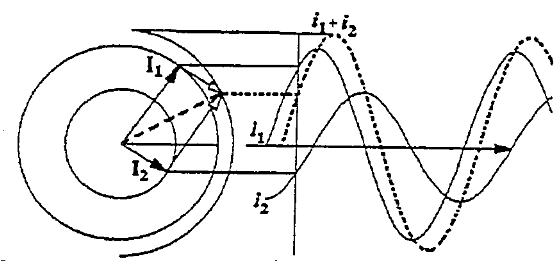
Phasors

In principle, it is always possible to analyze power levels with manipulations of the trigonometric equations, as was done above. However, this approach can become extremely awkward for even fairly simple networks. A more convenient method of keeping track of the size and effects of phase angles on power must be used. The solution involves defining a "reference" current or voltage (somewhat arbitrarily, according to what is convenient) and then measuring all other phase angles relative to that reference waveform. It then is possible to decompose any other current or voltage waveform into two component waves, one in-phase with the reference and one 90° out-of-phase with it. These components can then be combined with each other with simpler algebra than is required for the trigonometric expressions. These decomposed waveforms are called *phasors*.[20]



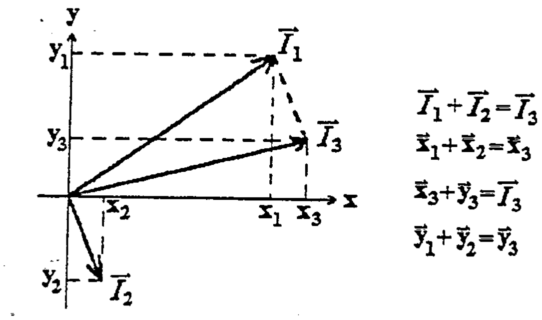
The phasor associated with a waveform such as $e_t = E_m$ *cos*((*t*+() is a two-dimensional quantity like vector, having a magnitude, here E_m, and a direction, given by the phase angle (relative to some reference current (or voltage) at time *t* = (. A phasor can be depicted by a diagram consisting of the locus of points that its vector (E_m() would sweep through over time as it rotates at the AC rate (. (Note that (does not change as the phasor rotates, since the reference phasor is also rotating at the same frequency.)

Phasor Addition



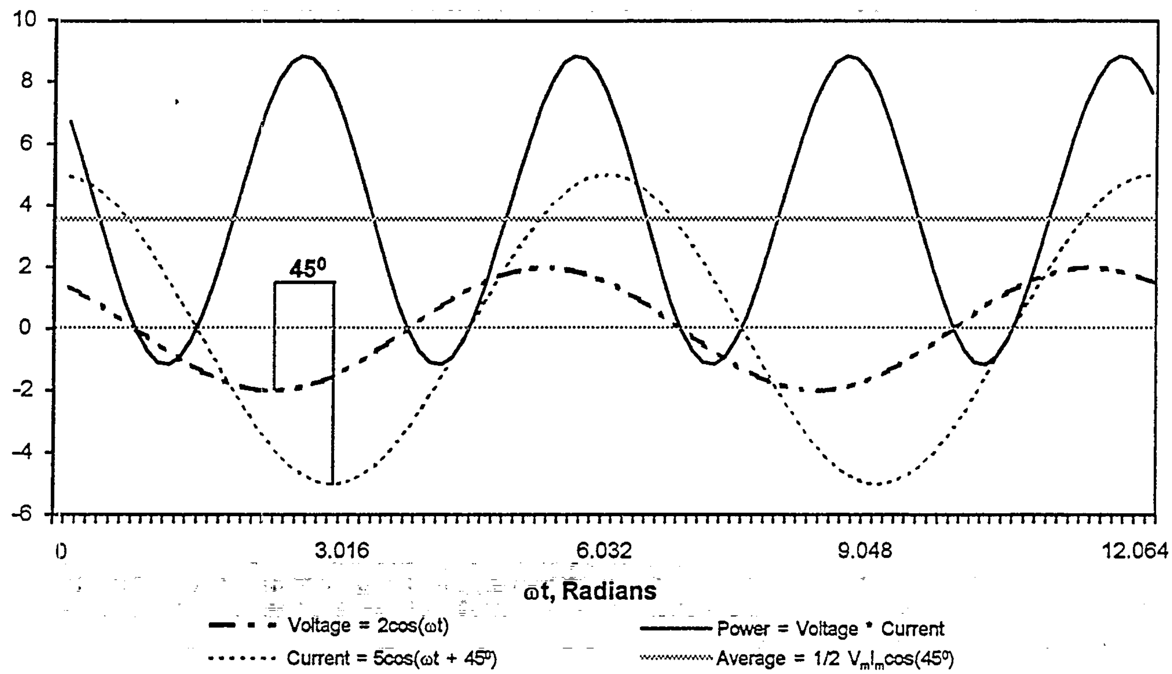
There are two mathematically equivalent notations for a phasor. The first is called "rectangular" notation. It is most useful when phasors are to be added together, e.g. when currents merge from different paths at a junction of conductors. Phasors with the same frequency of rotation can be added in the same manner as vectors are added: by connecting them "head to toe" while preserving their magnitudes and relative directions.

[20] See Schatf and Behness, 1991, pp. 1-86, for a nice development of the mathematics and analytic convenience of complex members and phasors for representing electrical quantities in Ac systems.

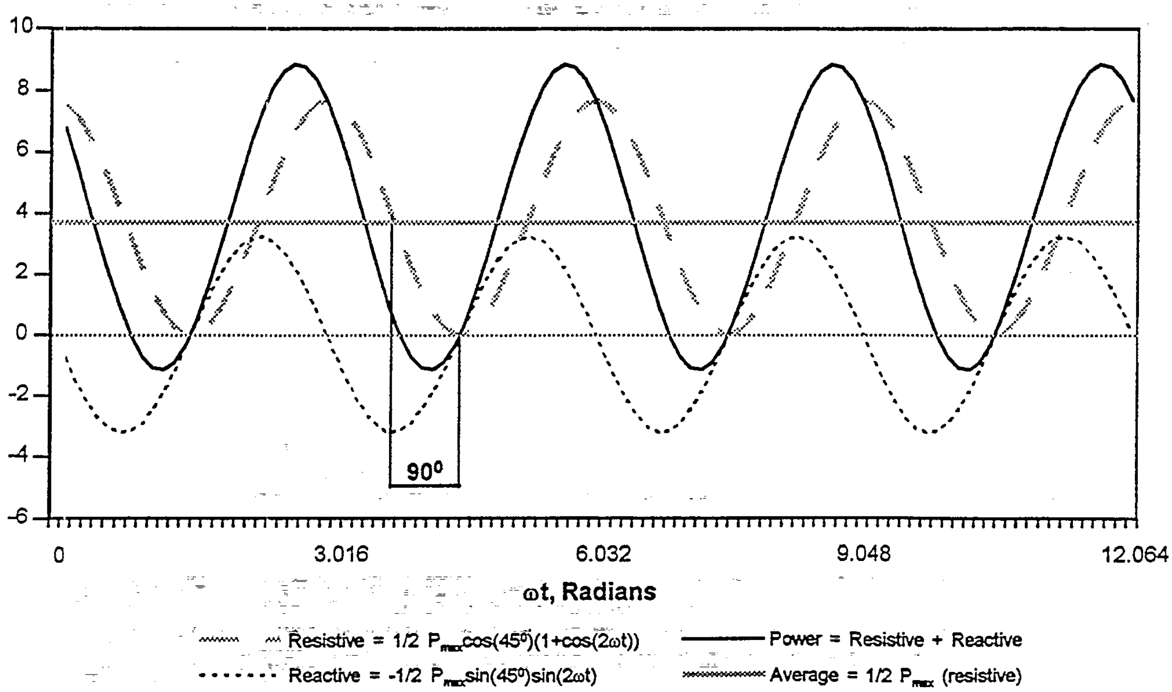


Algebraically, this is equivalent to decomposing the stopped vector of each phasor into two portions, one in phase or in parallel to the reference current or voltage (usually having (= 0) and one 90° out phase with, or perpendicular to, that reference. The parallel components are then added to each other, as are the perpendicular components, to get the combined waveforms phasor.

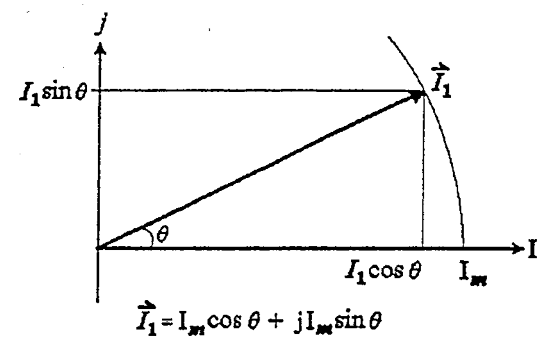
Figure 2.1
Instantaneous AC Power
= Voltage x Current



Instantaneous AC Power
= Resistive + Reactive Components



This algebra is readily performed by applying Euler's Formula,*ei*(=*cos*(+*jsin*(, where j denotes the square root of negative one, to express the horizontal (parallel to the reference vector) and vertical (perpendicular to the reference vector) components of a phasor as the real and imaginary parts of a complex number.[21]



Phasor addition is then just complex addition:

$(x_1+jy_1) + (x_2+jy_2) = (x_1+x_2) + (y_{1+}y_2)$

The second phasor notation is called "polar" or phase-angle form, denoted E_m/((where the sign∠ indicates an angle. This form is very convenient for multiplying and dividing phasors, such as when current and voltage are combined to determine power:

$$(E_1\angle\theta_1)\cdot(E_2\angle\theta_2) = (E_1\cdot E_2)(\angle(\theta_1+\theta_2))$$
$$(E_1\angle\theta_1)/(E_2\angle\theta_2) = (E_1/E_2)(\angle(\theta_1-\theta_2))$$

These are simply the familiar rules for adding exponents when exponentials are multiplied, expressed in phase angle-notation.

---

[21] The construct $j=\sqrt{-1}$ was first regarded as a useful number by Bombelli in the 16^{th} century Its existence allows solutions to algebraic problems like $x^2+k = 0$, where k is greater than zero. The connection between complex numbers, exponentiation, sine and cosine was first noted by Leonard Euler (1707-1783). The easiest way to see that this formula is correct is to expand the sine, cosine and exponential as infinite ,series a derivation given in almost any introductory calculus text. See, for instance, Strang 1991, pp. 360 - 363 or Thomas and Finney 1992, p. 605.

Mathematicians always use the notation $i=\sqrt{-1}$ where the i stands for "imaginary". Electrical engineers use j to avoid confusion with current, i.

It was shown earlier that real power is given by

$P=1/2\ V_m$ (*Im* (*cos*((-()

while reactive power is given by

$Q=1/2\ V_m$(I_m (*sin*((-()

where ((-() is the phase angle difference between v_t *and* l_t. *These* expressions correspond to the real and imaginary components of a complex number given by:

$S = P+jQ = 1/2V_m$(I_m (((-()

using Euler's formula The same *S*, called *complex power*, can be constructed by multiplying the voltage and current phasors together.[22] However, we must first convert the phasor *I* to its *complex conjugate* form *I** by changing the sign on its phase angle, so that *VI** yields the difference in phase angles rather than the sum.[23]

For example, assume that a voltage has an amplitude of V_{max} = 2 volts and a phase angle of 45° ((/4), while the associated current has an amplitude I_{max} = 3 amperes and a phase angle of 15° (π/12). Both voltage and current have the normal frequency of 60 Hz:

v_t = 2*cos*(377*t*+(/4) = 2 ((/4

i_t = 3*cos*(377*t*+(/12) = 3 ((/12.

S= *V1** = (2(((/4)) ((3((-(/12))

= 6(((/6).

The phasors of these waveforms involve only the magnitude and the phase angles:

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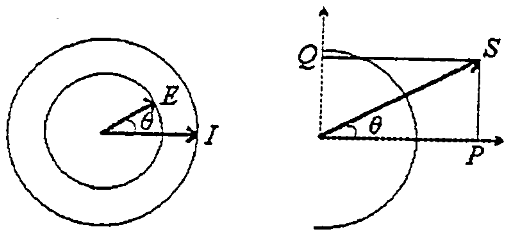
[22] We will use the convention that phasors are denoted by bold italics while their magnitudes are indicated by the same letter in italics alone. Variables that are not phasors, such as *t*, θ, or ω, will also be printed in italics.

[23] *In general, if x = a + jb, then the conjugate x* = a - jb. When x = X*cosθ + *jXsin*θ , *changing* θ *to* θ *gives x* =Xcos*(-θ) + *jXsin*(θ) =*Xcos*θ -*j*sin*θ , *the conjugate of x.*

V= 2(((/4), *I* = 3(((/12. The frequency term, *377t* = (*t* = 2(*ft*, *in* v_t *and* i_t *is* ignored in phasor analysis, since it is common to both components. It affects neither their relative timing (phase-angle difference) nor the level of real or reactive power. (Recall, however, that AC power itself has a frequency twice as great as the voltage or current.)



To find the real and reactive power components of *S* we must convert it back into rectangular notation, *S* = *P* +*jQ*, and then separate the real and imaginary parts:

P = Real{6e* $^{(\pi/6)}$ }

=Real{6[cos(π/6)+jsin(π/6)]}

=6cos(π/6)

$= 3\sqrt{3}\, W$

Q= Image{6[cos((/6)+jsin((/6)]}

= 6sin((/6)

= 3 VAR

$Thus\ S = 3\sqrt{3}\, W + j3VAR$

The magnitude of the complex power (ignoring the angle between *V* and *I*) is called the apparent power, expressed in Volt-Amps:

$$S = \sqrt{(p^2 + Q^2)} = \sqrt{(27+9)} = 6\,VA = 2 \cdot 3 = V \cdot I,$$

where *V* and *I* are the magnitudes of phasors V and I In_this example, only 86.6% of the apparent power is available for useful work, i.e. $3\sqrt{3}\,/\,6 = 86.6\%$ This percentage is equal to the power factor given by cos(π/6).

Complex Impedance

As stated previously, the ratio of voltage to current (both phasors) in an AC circuit is called its impedance, Z. The inverse of impedance is called *admittance*, $Y = 1/Z$. It is analytically useful to express both Z and Y as complex numbers, so that they can be used in phasor calculations. Recall that in a purely resistive circuit, voltage and current stay in phase, i.e. the angle (between the *e* and *i* phasors is zero. Since $I = E/R$ through a resistance, the impedance of a purely resistive circuit is $Z = R + j0$.

In a purely inductive circuit, using $e = Esin((t)=E((($ *as* a reference:

$i = (E/(L)(\ \sin((t - \pi/2) = (E/X_L)(-90^\circ$, so that

$Z = e/i = E/[(E/X_L) \angle -90(= [\,1/[(I/X_L) \angle -90^\circ]] = X_i\cos(90^\circ) + jX_i sin(90^\circ) = jX_L$.

In a purely capacitive circuit:

i = $(CE(\ sin((t + (/2) = (E/X_C)(-90($, so that

$Z = e/i = E/[(E/X_C)\ (90^o] = -jX_{C.}$

Impedances and admittances can be combined with the same rules that apply to resistors:

•• For elements in series, $Z_{total} = Z_1 + Z_2 + ... + Z_n$.

Thus, total *complex impedance* with a series resistor, inductor, and capacitor is:

$Z = R + jX = R + j(X_L - X_c)$.

The corresponding admittance is

$$Y = (1/Z) = \frac{1}{R + jX}\,\frac{R - jX}{R - jX} = \frac{R - j\left(X_L - X_C\right)}{R^2 + \left(X_L - X_C\right)^2}$$

This equation is used extensively in modeling transmission lines, as will be described in Chapter 4.

- For elements in parallel, $1/Z = 1/Z_1 + 1/Z_2 + ... + 1/Z_{n'}$

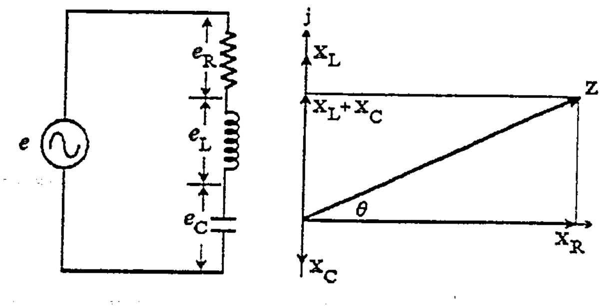
Therefore, a simple RLC circuit with elements in parallel has admittance:

$Y_{total\,=} 1/Z_{total} = 1/R + l/(jX_L) + l/(-jX_C) = 1/R + j(1/X_C - 1/X_L)_,$ or

$Y_{total} = G + j(Bc - B_L)$

where G is conductance and B_C *and* B_L are the inverses of capacitive and inductive reactance, called *susceptances.*

When AC currents, voltages, and impedances are expressed in complex form, the equations for their combination are structurally identical to Ohm's Law, Kirchhoff's Laws, and the definition of power for DC circuits. Using phasors allows simultaneous solving of magnitudes and phase angles that arise from impedances on an AC circuit.



The intuition behind complex impedance can be seen in a simple series RLC circuit, using the current phasor as a reference for comparison with the voltage phasor. First, we decompose the voltage into its constituent voltage drops:

$e(iZ= i[R +j(X_L-X_c)]$

$= iR + jX_Li -jX_ci$

$=e_R + e_L + e_c.$

The rms voltage is the magnitude of *e*:

$E = \left| iZ \right| = I\left| Z \right|.$

The voltage phase angle is:

$\theta = \arctan[(e_L + e_C) / (e_R)]$

$=\arctan[(X_L - X_C)/R] = (_Z.$

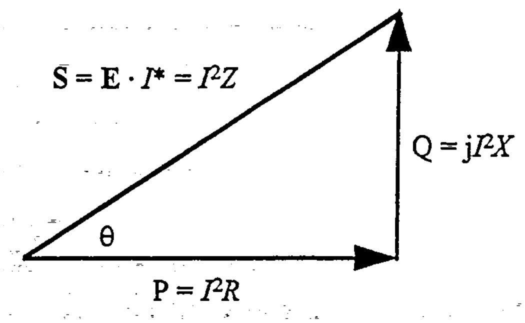
The complex power supplied to each element is given by the phasor product of the current through the element and the voltage drop across the element:

real power in $\text{resistor}_R = e_R(\; i_R = e_R(\; I = I^2R = I^2R(($

reactive $\text{power}_L = e_L(\; i_L = e_L . \; I = jX_LI^2 = I^2X_L(9(($

reactive power$_c = e_c \cdot i_c = e_c \cdot I = jX_C I^2 = I^2 X_C$(-9((

Note that the sign on the reactive power in the capacitor is negative. For this reason capacitors are sometimes said to supply reactive power and inductors to consume it.



Finally, we obtain the complex power by adding the phasors of the complex power supplied to the individual elements:

$S = I^2R + I^2jX_L - I^2jX_C$

$= I^2Z = E$ (I^* ((0

$= E$ (I (cos(+ jE (I (sin(

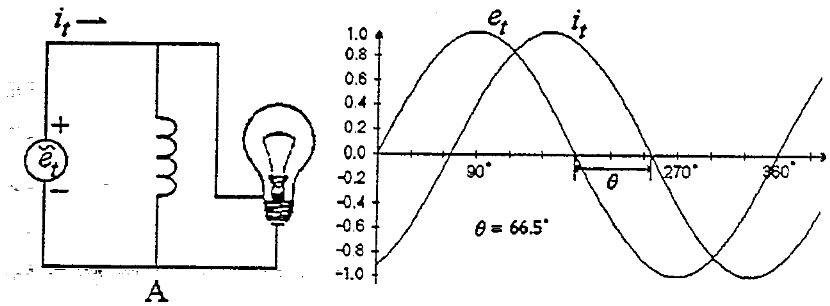
$= S$ (cos(+ jS (sin(

$=P+jQ.$

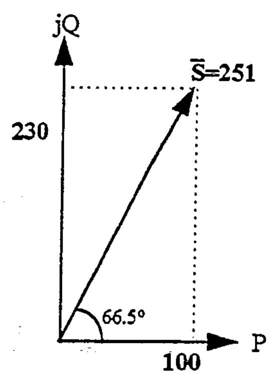
The above completes the tool kit of concepts from physics that is needed to understand power flows on utility systems. Real power is the commodity of ultimate interest in the market, but its delivery entails coping with the effects of reactive aspects of the transmission system itself as well as of the loads. One cannot understand the safe operating limits on power transfer capability, hence the marginal costs of power services, without understanding these concepts. The rest of this chapter presents a detailed numerical example that utilizes nearly all of its concepts. It also demonstrates an important practice of every utility, that of using capacitors to offset the costly effects of inductance on power supply.

An Example of Reactive Power Compensation

This section presents a numerical demonstration of several of the mathematical rules and relationships for AC power developed in this chapter. It examines the real and reactive components of power in a single circuit, expresses the currents, voltage drops and impedances in rectangular and phase angle phasor format, and it shows how currents can be out of phase with each other on parallel paths when their complex impedances differ. Finally, it demonstrates how a shunt capacitor can counter or *compensate* for the undesired reactance of an inductor, thereby improving power supply efficiency considerably.



In the diagram at right, an inductance of unknown magnitude is in parallel to a lightbulb (resistive load). The circuit is powered by a wall outlet (115 V at 60 Hz). It is observed that voltage loads current by 66.5°. The lightbulb dissipates 100 W at 115 V (rms). The first problem is to determine the size of the parallel inductance. When dealing with parallel circuits, it is usually best to use the voltage (typically controllable) as the reference waveform, and assign a phase angle to the current. Since the phase angle and the real power are given, begin by computing the reactive component of power:



- The real power demand is

  P=S (*cos*(= *:0.399S= 100W*, so

  Apparent power *S* = (100 *W*)/(*0.399*)

  = 251 *VA*

- Reactive power *Q* = *S*(sin(= (251 VA)(0.917)

= 230 VAR = 230 VAR

Now both components of complex power are known:

- $S = P + jQ$ = 100 +j230 VA = 251∠66.5°

Next, we can compute the supply voltage and current phasors:

Voltage e =115∠0° = E · cos(+jE · sin(

= 115V + j0

Since the 66.5 degrees phase angle (is stated relative to the voltage phasor, the current a vertical (imaginary) component:

- i_t=(P+jQ)/e= (251∠66.5°)/115∠O°

= 2.18 A∠66.5° = 0.869 +j2

Since the lightbulb dissipates I00 W at 115 volts, the resistance of the lamp must be:

$R = E^2/P$ = (115 V)^2/(100 W) = 132Ω

Which is also the impedance of the lightbulb:

Z_R =R= 132 .Ω.

The unknown impedance of the inductor can be inferred from

1/ Z =1/Z_R + 1/$Z_{L.}$.

- Circuit impedance Z= $e/i = ei^*/ii^*$, where i*= 0.869 -j2 is the complex conjugate of i:

$Z= ei^*/ii^*$ = 115(0.869 -j2)/[(0.869 +j2) · (0.869-j2)]

= *115*(*0.869* - j2)/[0.869^2.-$j^2 2^2$]

= 115(0.869 -j2)/4.755

= 21 -j48.4 Ω

= 52.8 Ω∠ -66.5°

- ReactanceX_L = Inductor impedance Z_L

= $(1/Z-1/Z_R)^{-1}$

$= (Z^*/ZZ^* - 1/Z_R)^{-1}$

$= [(21.0 + j48.4)/(2I^2 + 48.4^2) - 1/132]^{-1}$

$= (j0.0126)^{-1})$

$= -j57.5\ \Omega.$

- Inductance $L = X_L/2(f$

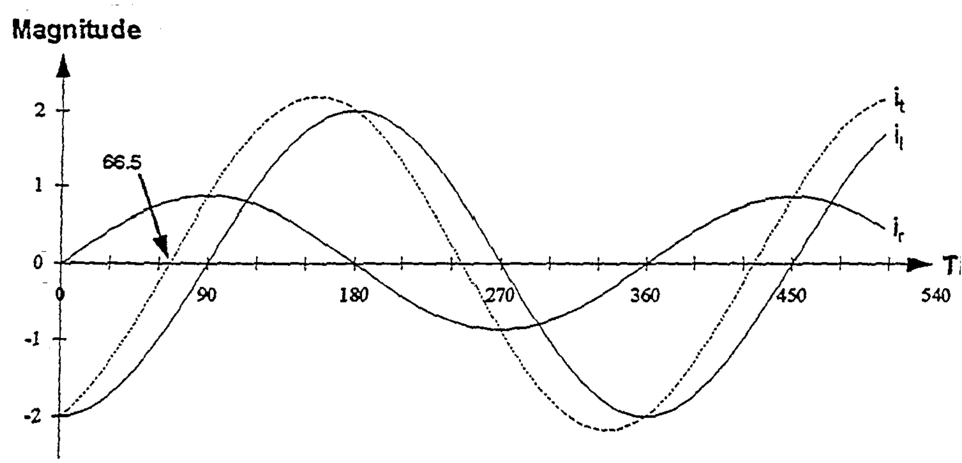
$= (57.5)/[(2\pi)(60)]$

$= 0.153$ H $= 153$ mH

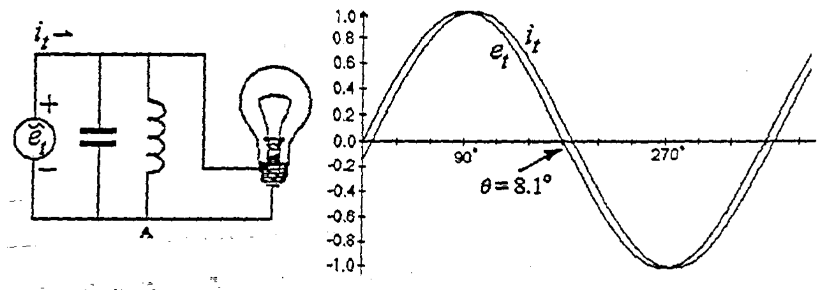
With X_L, X_R and i_r, we can solve for the current in the two parallel paths:

$i_r = e_t/X_R = (115\angle 0°)/1321 = 0.87A\ \angle 0° = 0.87 + 0j$

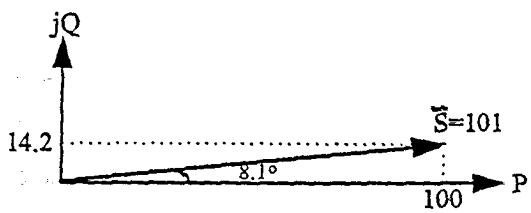
$i_l = e_t/X_L = (115\angle 0°)/(57.5\angle -90°) = 2A\ \angle\ 90° = 0 + j2$



Note that i_l and i_r are 90 degrees out of phase, as expected, and that their sum is $i_t = i_l + i_r = 0.87 + j2$. Since this is equal to $i_t = S/e = (100 + j230)/(115 + j0)$, we have confirmed KCL at the point A where these currents come together. The figure above depicts this addition of currents in sinsoidal form.



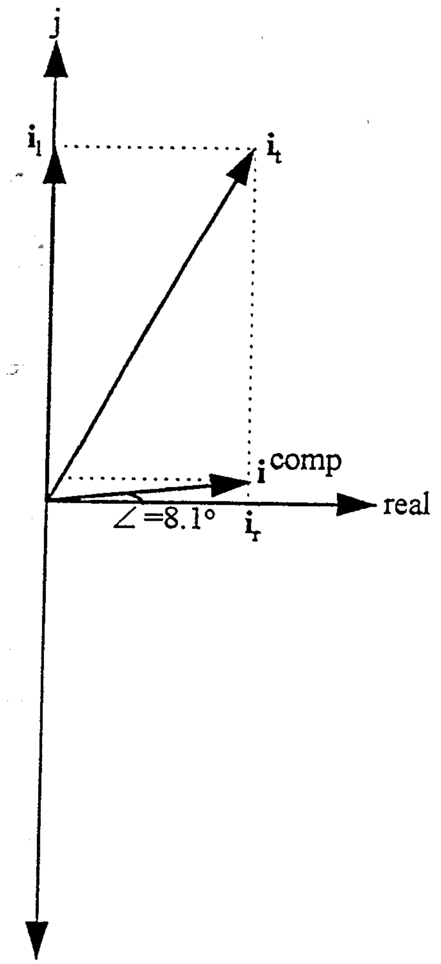
The power factor of this circuit is much too low (cos(=0.398). For maximum efficiency, real power will be close to 100% of apparent power. Achieving this becomes our next problem. In the diagram at the tight, a capacitor has been added in parallel to compensate the reactive power factor of the inductive portion of the load. With the proper capacitance, the apparent power needed to supply I00 W of real power to the lightbulb can be much lower. The required capacitance can be deduced after selecting a target for the power factor. At 99%, the voltage and total current will have a phase angle of.8.1° (cos(8.1°)=0.99 How much capacitance is needed?



- Again, real power dissipated P = 100 W.
  Q/P= tan θ so the new reactive power Q=
  14.2 VAR
- Apparent power
  S= |P + jQ |= |100+j14.2 |=101VA
  versus 251 VA at the previous power factor.

The total current also becomes considerably smaller:

- $i_t = S/e = (101\angle 8.1°)/115 = 0.878\angle 8.1° = .87 + j0.12$
  In a utility transmission system, such reductions in the required input (apparent) power and the line currents become quite important to economical operation.



For the new circuit, $1/Z = 1/Z_R + I/Z_L + 1/Z_C$ because all three elements are in parallel. Only the impedance of the capacitor, Z_c *is* unknown; as before,

R = 132 (,L = 153 mH, and X_L= -j57.5Ω.

- Circuit impedance Z = e/i = 115<0°/(0.878./8.I°) = 131∠-8.1° = 130 -j 18.4 Ω.
- Capacitive reactance X_c = Capacitor impedance $Z_c = (1/Z- I/Z_R -1/Z_L)^{-1}$

  $= (Z^*/ZZ^*- 1/Z_R-Z_L{}^* /Z_LZ_L^*)^{-1}$

  $= [(130 +j18.4)/(131)^2 - 1/132 - 1/(-j57.5)]^{-1}$

  $= [j(0.00108 - 0.0174)]^{-1}$ =j61.3 Ω
- Capacitance $C = [Z_C \cdot (2\pi f)]^{-1} = [(61.3)(2\pi)(60)]^{-1}$ = 43.3 μF.

The current in each of the three parallel branches is now

- •• $i_r = e_t/X_R$ = (115(0°)/132 = 0.87A(0° = 0.87+0j
- •• $i_c = e_t/X_c$ = (115∠0°)/(61.3(90° = 1.88A(-90°

  =0-1.88j
- • $i_l = e_t/X_L$ = (115(0°)/57.5(-90°=2A/90°=0+2j

The currents' sum is the total

$i_t = i_r + i_c + i_i = 0.88\ A + j\ 0.12$

This is equal to *S*/*e* above which again confirms KCL. The figure at right depicts this addition of currents in phasor form, with and without compensation by the parallel capacitor. Note how much smaller the compensated current, i_t^{comp}, is than the uncompensated (prior) current.

3

STRUCTURE OF UTILITY SYSTEMS

In the previous chapter, the behavior of electricity in a variety of devices was explored. A number of rules were introduced, chiefly in the form of equations, to make possible the analysis of any circuit. In this chapter, some of the ways in which electric utility networks differ from the simple networks of the previous chapter are explained.

- Electric utilities produce power in generators that rotate electric magnets within a frame housing three equally spaced conducting coils, resulting in three distinct "phases" of electricity that differ in timing from each other by 120°. This "3-phase generation" results in considerable capital and operating economies in transmitting power by reducing the number of conductors and the power losses in each*. The* algebra for determining the combined effects of the three phases depends on whether they are connected in an arrangement that resembles either the letter Y or a triangle, called a "wye" or "delta" arrangement respectively.
- Numerous transformers are used throughout a utility system to incrcase voltages for efficient transmission and to decrcase them to safe and practical levels for distribution subsystems and end-use devices. For transmission planning and control purposes a local distribution subsystem can be modeled as a single lumped impedance in parallel with the transmission system.
- Transmission lines have their own impedances that must be taken into account by system planners. In particular, the conductors of a transmission line are inherently inductive especially when heavily loaded, which causes them to absorb reactive power. Reactive power must be supplied to the network in order to offset the inductance of the lines and of certain end-use loads such as industrial motors. This can be produced by generators and transmitted, but it is often more economically provided at the site of the reactive load.

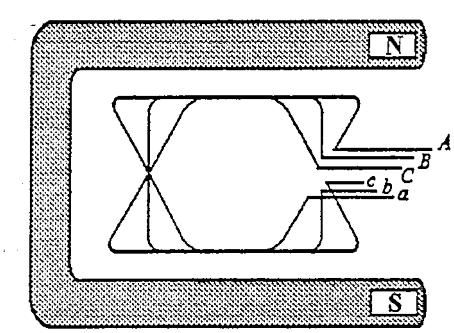
  Such reactive power "compensation" is desirable because it reduces the amount of current required to deliver a given amount of real power, and it improves voltage control. It is provided by reactive elements added in parallel (shunt) or series to the transmission lines. A shunt capacitor supplies reactive power while a series inductor consumes it. Compensation can be either passive (fixed) or active (variable in response to system conditions).
- When power demand changes or system resources are lost due to unplanned disturbances, generator output levels must be adjusted almost instantaneously to keep from over-or under-loading circuits. Conservation of energy dictates that an incrcase in the electric energy delivered to the loads will cause the kinetic energy

(frequency) of the generator turbines to decrcase. Changes in the system frequency as well us in tie-line flows for contractual power exchanges with other utilities can be monitored and used to trigger adjustments to the steam injection into the turbines as needed to restore equilibrium and economical operations. For small changes in load, such adjustments occur in real time under the control of electronic telemetry and feedback mechanisms called Automatic Generation Control. Large changes in load or system status must be anticipated and handled with human intervention.

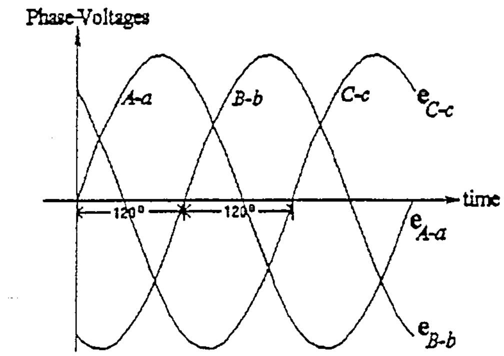
- Several terms and measurements are used to describe a utility system's ability to accommodate changes in power demand or supply availability. "Reliability, is measured by the probability of loss of service to consumers due to unplanned disturbances to system resources, especially unplanned outages of generating units. Reliability is improved by having redundant generation and transmission capacity and by interconnection with other, adjacent utilities. "Security" refers to the ability of the system to accommodate redirected power flows resulting from a sudden loss (e.g. due to lightning) of the transmission circuit between pairs of buses. "Stability" concerns whether generators will remain in synchronization with each other as they jointly respond to changes in system status. Operating transmission and generation resources below their theoretical capacity limits generally improves security and stability.

Three-Phase Generation

In the previous chapter, it was explained that alternating voltage can be produced by rotating a conducting loop in a magnetic field. As a consequence of electromagnetic induction, a sinusoidal wave pattern of electric potential difference occurs across the wires of the rotating loop. A closed AC circuit is created and current flows when an impedance is connected across the ends of this loop, converting the electromagnetic force (emf) into useful work.



If only one current loop is rotated in the magnetic field, the resulting generation is called single-phase power. If three conducting loops are rotated each separated from the other two by 120° within a fixed magnetic field, three alternating currents will be produced simultaneously. Induced emf reaches its maximum when the plane of the enclosed area of a rotating loop is parallel to the magnetic field lines (because it is then cutting magnetic flux at the maximum rate). This position occurs at different times for the three loops, causing the three induced emf's to peak at different times, but to have the same maximum voltage and frequency. The result is three voltage sine waves e_{Aa}, e_{Bb}, *and* e_{Cc} that are identical except for displacement in time:



$e_{Aa} = E_m\, sin(()$

$e_{Bb} = E_m\, sin((-120°);$

$e_{Cc} = E_m\, ((- sin\ 240°).$

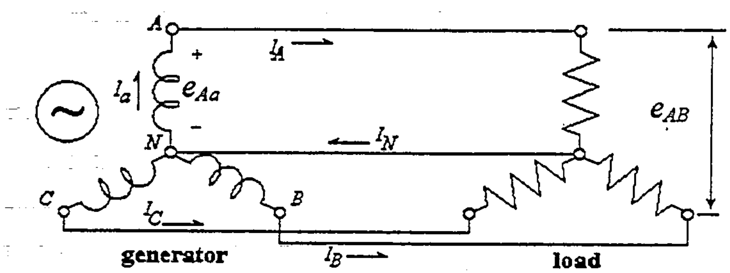
Each induced emf within a loop constitutes a *phase*. This is called three-phase power, and it has many economic advantages that are explained below.

As a mechanical matter, it is usually easier to rotate the magnetic field than the conducting loops, and the physical result is the same. The process is actually accomplished in the following manner: A magnetic field is created by supplying a DC current to *the field windings* around an iron core *rotor*, which rotates within a stationary housing called a *stator. The* stator is wrapped with three *primary* or *armature windings* in each of which an AC emf is induced. The *excitation current* in the field windings can be supplied by an independent DC source or by rectified AC current from the utility system itself. By altering the level of excitation current, a generator can be made to supply reactive power in addition to (partly instead of) real power.[1]

[1] See Carlson and Gissr, 1990, pp. 709 - 750, for the electromechanics of generators and motors. Chapter 5 in this text also explains the P-Q production tradeoff in more detail.

Wye-Connected Generators

A major benefit of three-phase generation is that the number of conductors required to transmit electric energy to end-users can be reduced. Three single-phase generators would require six conducting lines, but the three-phase approach needs only four (or sometimes even three). In order to understand why fewer lines can carry an equivalent amount of power, we must examine how three-phase generators are connected to circuits.[2]



There are two practical methods of connecting the output of a three-phase generator to a circuit. In the *wye-connected generator* shown below, loop ends a, b, and c are joined together at a *neutral* terminal, *N*. (Recall that each loop has two ends: a and, *A*, for example). The other ends supply power to loads that are connected in a similar wye configuration, shown here as resistances. The central points in each wye configuration are then connected to each other in order to complete the path for all three outgoing power flows. The load is said to be *balanced* when it is identical on each line. Under such conditions, the instantaneous current (or voltage) in the return, neutral conducting path will be zero. If it is certain (or controllable) that loads will remain balanced, the neutral line can even be eliminated. This can be seen by examining the sum of the three waveforms for the phase emf's at any arbitrary point in time.

For example, at point t_1 in the figure below, one wave is crossing the x-axis while the other two are equal in magnitude but opposite in sign, so their sum is zero. At point t_2, two waves are both of positive sign but of half magnitude with respect to the third, negative signed wave. Once again the three waves add to zero, as they will do at any other point. The phasor diagram for the three emf's demonstrates the result at time t_1. Recall that phasor addition is performed by aligning one phaser end with the next phasor's starting point. The sum of three vectors of equal length but 120° apart forms a path that returns to the starting point, meaning that their combined magnitude is zero.

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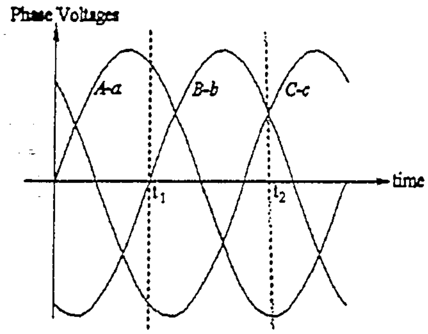
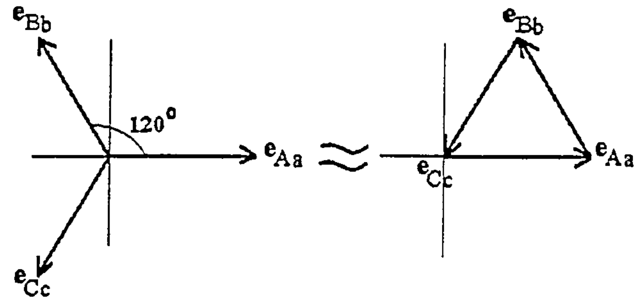
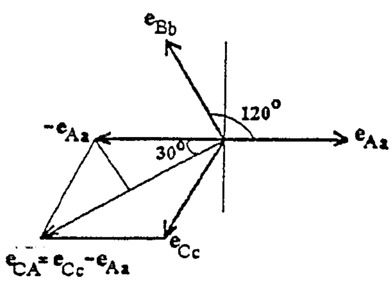
[2] See Henderson, 1980, pp. 175- 194, for a thorough treatment of three-phase power.





By maintaining and taking advantage of a balanced load, a utility can operate with half as many wires as would be needed for an equivalent single-phase system, yielding significant capital cost savings on line construction expenditures. If load is or may become unbalanced, a fourth line becomes necessary (and no longer neutral) in order to carry the net imbalance back to the generator, but one line can accommodate the net imbalance from the three outgoing lines.

Voltage in Three-Phase Generation



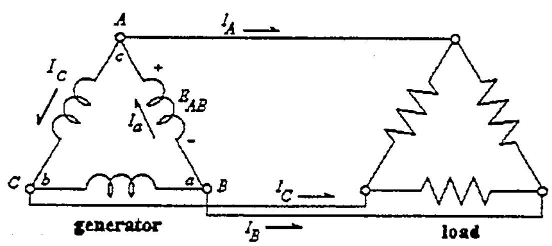
The potential difference between any line and the neutral terminal is called a *phase voltage*, V_p while the potential difference between any two lines is called a *line voltage.* For instance, the line voltage e_{CA} is equal to the phasor sum of the phase voltages $e_{Cc} + e_{aA} = e_{Ce} - e_{Aa..}$ Both the timing (angle) and the magnitude of a line voltage will differ from the associated phase voltages. That is, the line voltage e_{Ca} has a greater magnitude than the phase voltages e_{Ca} *and* $e_{Aa,}$ and it is shifted as well. By rules of trigonometry, the line voltage phasor has a length (magnitude) given by:

$$|e_{ca}| = |-e_{AC}|\cos^{O} + |e_{Cc}|\cos 30^{O}$$

$$= |e_p|\sqrt{3}$$

Where $|e_p| = |e_{Aa}| = |e_{Bb}| = |e_{Cc}|$ is the magnitude of any of the phase voltage.Denoting the magnitude of any line voltage by E_b we have $E_I = \sqrt{3}e_p$.It is evident from inspection of the wye configuration that the current in any line is identical to the current in the corresponding phase of a wye-connected generator, i.e. $I_L = I_p$. Transmission voltages are normally given as rms line voltages rather than phase voltages.For example, a 765 kV transmission line will have an rms phase-to-ground voltage of $675/\sqrt{3} = 424kV$.

Delta-Connected Generators



An alternative to the wye-arrangement of coils is the *delta-connected generator.* In this arrangement, loop end a is joined to *B*, *b* is joined to *C*, and *c to A.* As with a balanced wye-connected generator, this constitutes a three-phase, three-wire supply. The magnitude of a line voltage is now identical to the magnitude of each phase voltage across the ends of a loop: $V_L = V_P$ However, two phases now supply current to each line, so the line current I_A no longer equals the phase current. instead, the line current is given by the phasor sum $I_a - I_c$. Analogous to the previous result on voltages in wye-connected system, $I_L = I_P\sqrt{3}$ for delta-connected generators.

Power in Three-Phase Systems

For both the wye- and delta-connected supplies, power is supplied over three lines in three phases. For the wye-connected generator the per-phase apparent power is

$$S_P = V_P I_P = \left(V_L/\sqrt{3}\right) \cdot I_L$$

For the delta-connected generator,

$$S_P = V_P I_P = V_L.\left(I_L/\sqrt{3}\right),$$

so the power through each line of a three-phase supply is the same whether the supply is wye- or delta-connected.

Instantaneous power in a single-phase system oscillates sinusoidally at 120 cycles per second (see Chapter Two). In contrast the total instantaneous power delivered over three-phase systems is a fixed quantity that has no periodicity whatsoever. This is an important advantage, because it allows three-phase electric motors to operate with no variation in the torque on their rotors. Since the power is constant, the tangential impulse that makes the rotors spin provides the same acceleration throughout the entire cycle.

One way to show that the total three-phase power is constant is to manipulate the sum of three instantaneous phase power equations with trigonometric identities:

$$P_t = P_1 + P_2 + P_3 = V_1 I_1 + V_2 I_2 + V_3 I_3$$

where currents and voltages are phase values. In general, current and voltage may be separated by an angle (so the expression for total instantaneous power is:

$$P_t \quad = \quad V cos((t)\ I\ cos((t + () + V cos((t + 120)\ I\ cos((t +(+ 120) + V cos((t + 240)\ I\ cos((t + (+ 240)$$

$$= \quad VI \{cos((t)\ cos((t +( ) + cos((t\ 120)\ cos((+(+ 120) + ((t + 240)\ cos((t + (+ 240)\}$$

where *V* and *I* are maximum values that are the same in each phase. To simplify, use the trigonometric identity cos(a + b) = cos(a)·cos(b)-sin(a)·sin(b) to separate the fixed angles from these cosine functions, and substitute the values

cos(120) = cos(240) = I/2; and

sin(120) = sin(240) = $\sqrt{3}/2$ to obtain:

P_t = 3/2 V I {cos((t)cos((t + θ) + sin((t)sin((t + θ)}.

Finally, using the trigonometric identity

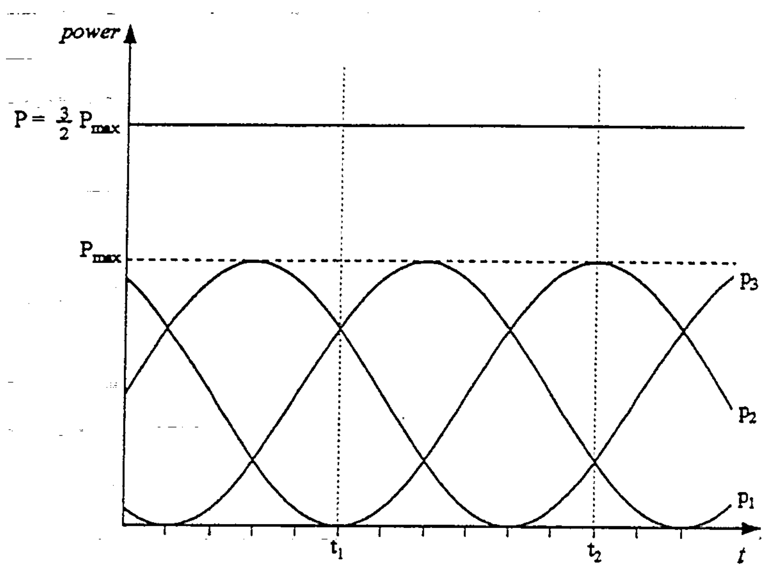
cos(a - b) = cos(a)cos(b·) + sin(a).sin(b)

letting a = (t+ (and b = (t. This yields:

Pt= 3/2 *VI* {cos((t + (- ωt)

= *3/2 VI cos* (.

Thus the time dependency (t drops out of the total instantaneous power. Since total three-phase instantaneous power depends only on constant quantities, it is itself constant. The rearrangement $P_t = 3\ (1/2VI\ cos$ () shows that is just three times the average power in each phase. [3]



The figure at right demonstrates this constancy graphically. In this graph the angle (between the voltage and current is assumed to be zero, causing each of the phase line powers to be always positive. If (were different from zero, each of the line powers would be intermittently negative, i.e. reactive, and the total three-line power would be shifted down by the cosine of the angle between the voltage and the current, as derived above. At any arbitrary point in time, such as the indicated points at t_1 and t_2, the combined heights of the individual functions yield a fixed sum equal to $3/2P_{max} = P_{avg}$

Line Loss Avoidance with Three-Phase Power

We have already seen two significant economic advantages of three-phase power: reduced capital costs and constant instantaneous power for motors. To illustrate an additional benefit of three-phase generation, we compare transmission power losses under single-phase and three-phase generation.

To minimize losses, utilities use transformers to incrcase the voltage and decrcase the current on the line. For instance, suppose that a single-phase transmission circuit were to supply 110 MVA of power. The following shows the required current in a line with resistance of 3Ω at two different transmission voltages:

---

[3] One can also express the total instantaneous power in rms terms and/or in terms of line voltage and line current:

V_L^{rms}	$V_L I_L$	R_L	I_L^{rms}	I_N	$(I_L^2+I_N^2)(R$
138 kV	110.4/MVA	3 Ω	800 A	800 A	3.84 MW
230 kV	110.4 MVA	3 Ω	480 A	480 A	1.38 MW

Stepping up the voltage reduces losses (last column) considerably; note that there are losses in both the sending and returning (neutral) lines. Less current is needed to supply the same total power at the same line voltage on a balanced three-wire transmission system, causing power losses to be much smaller at either voltage level:

V_L^{rms}	$V_L I_L \sqrt{3}$	R_L	I_L^{rms}	I_N	$(3I_L^2+I_N^2)$ (R
138kV	110.4 MVA	3Ω	462A	0 A	1.92 MW
230kV	110.4 MVA	3Ω	277A	0 A	0.69 MW

Note that when the load is balanced no current is carried on the neutral line, contributing further to the reduction in losses. In summary, three-phase generation yields capital and operating cost savings, reduced line losses, and constant torque to inductive motors.

Transmission Line Characteristics

High power transmission lines have a more complex physical structure and electrical behavior than the insulated copper wires familiar to homeowners. Transmission conductors generally are made of many strands of aluminum (less often, copper) that have been twisted around a few strands of steel for strength and flexibility. There may be sixty or more such conducting strands with a combined weight of several thousand pounds per mile of length. Though the conductors resistance is generally only a few hundredths of an ohm per thousand feet, the cumulative effect over several hundred miles of line can cause real power losses of up to a few percent of generation. These losses must be supplied, and the associated heating of the conductors can become a constraint on the feasible power flows. Power supply and transmission planners must also take into consideration several other electrical properties of the lines:

- Inductance - The magnetic field associated with the current in each conductor changes as the alternating voltage varies, causing a changing flux that induces an emf in adjacent transmission lines. There is also inductance between strands within a given conductor and even between adjacent, concentric cylinders in the volume of metal within a given strand. This causes each line to resemble a string of numerous small inductors connected in series. Operationally, it creates a reactive power demand in the line itself that diverts some of the apparent power from the generators to maintaining the induced magnetic fields, and it tends to cause a voltage drop towards the receiving end of the line.[4]

- Capacitance - Transmission lines are generally bare conductors, insulated from each other only at the poles from which they are suspended. Therefore adjacent lines constitute a capacitor, i.e. two parallel conductors separated by a dielectric, air. As the voltage difference between adjacent lines changes, the amount and distribution of electrostatic charge between the two lines must adjust. Some of the flow of current within the lines serves this purpose, diverting a portion of the apparent power to maintenance of these electric fields between conductors. In this regard, a power line behaves like a perfect conductor with many small capacitors in parallel between it and the neutral line or ground. (Recall that capacitance adds in parallel, while inductance adds in series.) This capacitive reactance can offset some or all of the line's inductive reactance, reducing the reactive power demand and restoring the line voltage.
- Shunt Conductance - Current can flow between conductors at the insulators (especially if the latter are damaged) or between a conductor and the earth. This effect, called a corona, occurs when the electric field strength of the conductor exceeds the dielectric breakdown strength of air. Such leakage is usually very minor, so shunt conductance is often neglected in power planning studies.

For a given type and spatial arrangement of conductors, line impedance can be determined. As a general rule, they do not vary significantly with the level of utilization of the line. The table below provides some representative values for impedance *line constants* per unit length for conductors, of different voltage and power capacities. [5] Note that the reactance levels are generally much higher than the resistance, by a factor of as much as ten or more:

Line Voltage Rating	Number of conducting Strands	Resistance (r) per mile	Current Capacity Rating	Inductive Reactance X_L per mile	Capacitive Reactance x_C per mile

p_t(rms per phase) = $3/2\ V_{p,max}\ I_{p,max} \cos I_{p,max} \cos\theta$

$$= 3\left(V_{p,amx}/\sqrt{2}\right)\left(I_{p,\max}/\sqrt{2}\right)\cos\theta$$

$$= 3\ V_{p,max}\ I_{p,max} \cos\theta = 3P_p$$

or, p_t (line) $= 3/2\left(V_{p,\max}/\sqrt{3}\right)\left(I_{p,\max}\right)\cos\theta = \sqrt{3}V_{t,rms}\ I_{t,rms}\cos\theta$

[4] Note that line inductance problems can be avoided completely with DC transmission lines. This offers economy in some respects, but the costs of converting AC power into DC and back at the ends of the line can be prohibitive except on very long, very high voltage lines.

[5] In this example, it is assumed that the high voltage transmission lines are in a 3-phase configuration and that the spacing between conductors per phase is eighteen feet. The 138 kV and 345 kV lines refer to conductors 50 feet above ground, while the 765 kV line has values for conductors 110 feet above ground. Parameters were obtained from Bergen, 1986. p. 77.

138 kV	54	0.1688 Ω	770 A	0.789 Ω	0.186 Ω
345 kV	45	0.0565 Ω	1010 A	*0.596* (	0.142 Ω
765 kV	54	0.0201 Ω	1250 A	*0.535* (	0.129 Ω

These line impedances are a factor in the sizing of transmission line power flows. The total reactance in a line to its length l times the per-mile reactance, $X_{L^=} \approx I \cdot x_L$ and $X_C \approx I \cdot x_C$[6]. Over that length, the amount of reactive power absorbed by the series inductance is I^2X_LWhile the amount "created" (offset) by the shunt (parallel) capacitance is V^2/X_C When $I^2 X_L = V^2/X_C$ the amount of reactive power consumed by the flow of current through the series inductance equal the amount created by the voltage across the shunt capacitance at all points along the line. Therefore, the line does not require additional reactive power compensation- it is said to be *naturally loaded.* condition is achieved when the per phase load on the system is equal to $P = V^2/Z_0$, where $Z_0 = \sqrt{(X_L \cdot X_C)}$ and V is the rated phase voltage (so total power is 3P).[7] This level of load P is called the per phase *surge impedance load* (SIL) and Z_0 is called the surge *impedance*. (Note that this generally will not equal the line's actual impedance.) The SIL for a typical 345 kV line is around 400 MW. Generators supply additional power by producing more current while keeping voltages nearly constant. Therefore, for power transfers below the *SIL*, line charging by the shunt capacitance (the V^2/X_C term) exceed the reactive power absorbed by the line inductance (the $I^2 X_C$erm). Above the SIL, reactive power losses exceed the line charging, since I grows but V does not.

Reactive power can be supplied by generators by altering the amount of exicitation current supplied to the field windings in the generator. An underexcited field produces leading armature current that can be transmitted to a lagging (reactive) load. However, utilities strive to keep the need to generate reactive power as low as possible, because any amount of generator-supplied reactive power demand Q will require the injection of a higher level of current for a given amount of real power to be delivered. That is, since real power demand *P* is equal to *VI co*s(, required current is given by *I* = *P*/(*Vcos*(). Sinc*e* cos(decreases with increasing (as the line or load becomes more reactive *I* must increase with *Q* for a given amount of real power *P* to be delivered at a specified voltage. Moreover, both real and reactive power losses are directly proportional to the square of the current, hence they incrcase rapidly with larger phase angles (between voltage and current. The farther reactive power must be transmitted, the more of it will be consumed by the transmission lines themselves. Consequently, utilities try to minimize reactive power flows by compensating for reactive demands

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[7] If $I^2X_L = V^2/X_C$. then $V^2 I2XLX_C = V^4$. Taking the square root of both sides and letting *P* = *VI*, we have $P\sqrt{X_L X_C} = V^2$.

on-site, with a mixture of controllable and fixed capacitors and inductors that offset locally the unwanted reactances of the line or loads.

Transmission and Distribution

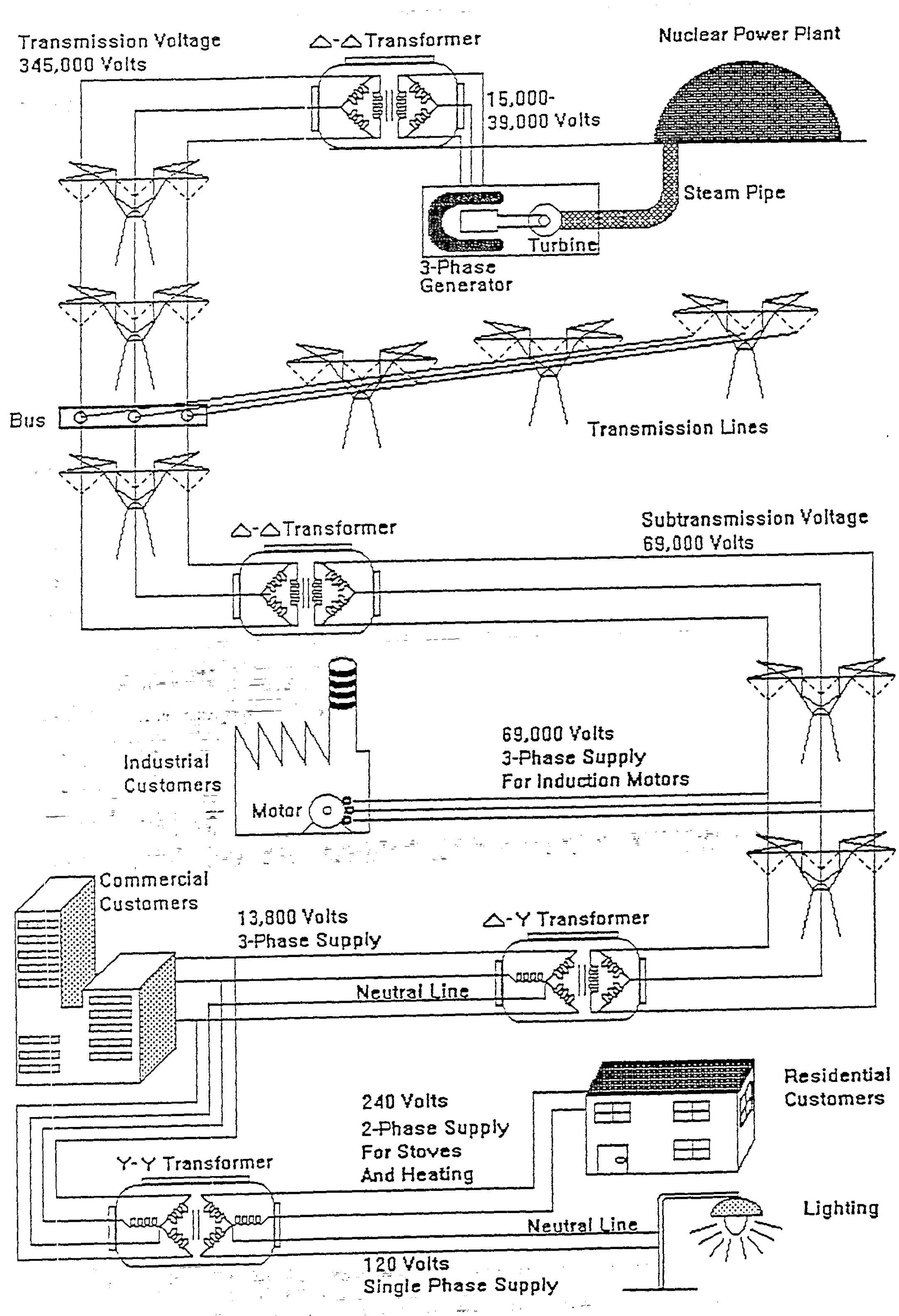
Figure 3.1 on the next page is a schematic representation of how generation, transmission and distribution resources are configured in a utility system. At the source of electric power, different kinds of energy (kinetic, chemical, nuclear) are converted into electrical energy in a three-phase generator producing power at around 15,000 volts. Transformers then step up this voltage to a few hundred thousand volts at a *bus* to the transmission line. A bus is any location where electrical devices have interconnected inputs and outputs. There are buses at the junctions of transmission Lines with generators, transformers, other transmission lines and distribution subsystems.

Three-phase transmission lines carry low current, high voltage power through isolated areas, using three or four cables (depending on whether the neutral line is present or not). Typically, voltages of 69,000 Volts or above are deemed transmission level, while conductors operating at or below 13,000 Volts are usually accounted for as part of the distribution system. Near the distribution areas a step-down transformer (substation) decrcases the voltage and adds the neutral line that is used in the provision of the single-phase power to consumers. Different types of loads use a different number of phases to meet their power and voltage needs. Three-phase motors take advantage of the constant instantaneous three-phase power, while resistive devices like stoves and light bulbs use line or phase voltage, according to the size of their requirements.

The transmission system is electrically isolated from the structure and performance of the distribution systems by step-down substation transformers. For transmission planning purposes this means that regional distribution-level loads can be aggregated and regarded as the effects of a single impedance. Operationally, this electrical isolation makes each distribution region relatively immune to the effects of disturbances in other regions, e.g. when lightning knocks out a distribution line. Changes in the end-use load requirements are conveyed back to the transmission system by the substation transformer through changes in current. An incrcase in power demand at the customer loads causes an incrcase in the distribution level current that induces a corresponding change in the current level on the transmission side of the transformer. These current variations will cause the generators to deliver more or less power as needed, through feedback mechanisms explained below.

Not shown in Figure 3.1 are the important components for transmission system monitoring and protection, such as circuit breakers, capacitor banks, and the system operating center. In addition, this figure depicts only a single path from one generator to the load(s). Any real utility includes several generators interconnected to loads in more of a web-like geometry, with multiple paths on which the power will flow. By virtue of Kirchhoff's Laws, power arriving at any bus will distribute itself in part to each of the other circuit paths going out of that bus, in inverse proportion to their respective impedances. This phenomenon is referred to as *loop flow;* it has the consequence that power cannot be directed to flow along specific circuits, e.g. just those owned by a particular utility or those that happen to be operating at below their rated capacity. Loop flows can become a financial and operating problem when power spreads out over lines not owned by the parties scheduling a power transfer.

Figure 3.1


Transmission Voltage
345,000 Volts
△-△Transformer
Nuclear Power Plant
15,000-
39,000 Volts
Steam Pipe
Turbine
3-Phase
Generator
Bus
Transmission Lines
△-△Transformer
Subtransmission Voltage
69,000 Volts
69,000 Volts
3-Phase Supply
For Induction Motors
Industrial
Customers
Motor
Commercial
Customers
13,800 Volts
3-Phase Supply
△-Y Transformer
Neutral Line
Residential
Customers
240 Volts
2-Phase Supply
For Stoves
And Heating
Y-Y Transformer
Lighting
Neutral Line
120 Volts
Single Phase Supply

Control of Generators

It is critical that generation facilities be able to respond essentially instantaneously to the shifting patterns of electric load and that the transmission system have enough flow capacity and controllable voltage support (reactive power compensation) to serve a wide range of demand requirements. Much of this control must be automatic, relying on feedback from the system itself to guide the corrective actions. Two indicators of system status are primarily relied upon for much of this *Automatic Generation Control* (AGC): system frequency and tie-line flows.[8]

Contract terms and regulatory requirements, and practical engineering considerations require a utility to provide electric power within a very narrow range of frequencies. Even modest deviation can cause severe damage to generators and to electric equipment designed specifically for 60 cycles per second. Steam turbines in particular are vulnerable to fatigue and cracking if operated away from 60 Hz by as little as ±0.05 Hz. for even a few minutes. A fully loaded steam turbine may be able to sustain 58 Hz for no more than ten minutes cumulatively over its entire operating life.[9]

When the mechanical torque on the turbines (due to the steam or other prime mover) is exactly equal to the opposing electrical torque (arising from the counter-emf in the loops of the generator), the frequency of rotation will be a constant. However, when the power demand changes so that it no longer equals power supplied, the rate of rotation of the generator(s) will automatically incrcase or decrcase as dictated by conservation of energy. For instance, when the load increases, the electrical torque exceeds the mechanical torque, causing the frequency of rotation (hence of the supplied alternating current) throughout the system to decline. More prime mover energy (water flow, steam, etc.) must be supplied to one or more generators to balance the incrcased power demand. A mechanism on each load-following generator called the *governor* manages these changes in energy input whenever the frequency change exceeds a narrow "deadband" range. A gas-fired peaking plant will have a narrower deadband and a faster response than a baseload coal plant, so the peaking plant will respond first to any short-term variations in load. By adjusting the responsiveness of these generators (called the *droop*), *the* desired sharing of responsibility across several generators for load changes can be pre-programmed into the system. As will be explained further below, this adjustment actually involves two phases. First the plants' mechanical power is matched to the electrical power demanded, usually in a matter of two to ten seconds.

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[8] Voltages are also closely monitored in order to protect system stability, but frequency and tie-line changes are the primary barometers for maintaining the economic operation of the system. See Chapter 5 for a fuller discussion of stability.

[9] To protect the system and its customers, load shedding is scheduled to occur in the MAIN NERC region in 10% blocks whenever frequency drops to 59.3, 59.0, and 58.7 Hz. See Stambach and Ewart, EPRI EL-6360-L, 1989 for a discussion of frequency tolerances and for operating procedures to manage transmission system dynamics.

This will not occur at 60 Hz but, at a slightly lower frequency when meeting incrcased demand. Second, the governor *setpoints are* gradually adjusted over a few minutes to move the system back to 60 Hz. Within certain limits, these adjustments can be done in a fashion that achieves an economic dispatch order among the participating plants.

One very important concern for transmission planners and system operators is maintaining system *stability* at or very near 60 Hz as multiple generators adjust to accommodate a new demand or supply condition. The governor mechanisms cannot usually cause an immediate, direct restoration of the system frequency. Instead, the momentum built up in the turbines when they initially respond to incrcased loading causes them to overshoot the 60 Hz target a bit. Provided they do not overshoot by too much, the electric torque will act as a brake on the generators and induce a reverse correction. Ideally, these oscillations quickly dampen out and converge on 60 Hz for all generators on the system. If they do not, and the generators fall out of synchronization with each other, voltage variations throughout the system can become erratic and extreme , creating unintended power flows that trigger rapid protective shutdown of overloaded circuits. Again by conservation of energy, this sends very strong feedback signals to the generators, occasionally creating a very complex problem that results in widespread *voltage collapse* [10] Thus it is critical that system frequency be tightly controlled.

Most utilities are interconnected with several of their neighboring utilities. This linkage affords considerable capital and operating economies, largely because of economies of scope and the related benefits of risk diversification. With a larger network, more alternative paths are available for moving power to each demand node. With more generators interconnected, the probability of unplanned plant outages that comprise a significant portion of total load is greatly reduced, so generation capacity reserve margins per MW of peak load can be lower for a given target level *of reliability.* {Reliability is often measured by the probability that plant outages will make it infeasible to serve the entire power demand. Utilities usually maintain sufficient generation and transmission capacity reserves that this is a very unlikely possibility). The total load of an interconnected system itself is somewhat more stable than that of an individual utility because of imperfect correlation of demand across the large pool of end-users. This allows better utilization of baseload plants to capture their operating economies of scale. All of these pooling benefits improve system reliability and

[10] While protracted metropolitan blackouts due to voltage collapse are fairly rare, collapses lasting a few second to a minute have occurred roughly once annually throughout the developed world every year since 1970. Most recently (12/14/94), an outage at a 345 kV substation in Idaho (and possibly other events) triggered a cascading outage that affected ten states and British Columbia, causing the WSCC into-connected system to separate into four islands. The power was restored in most areas within a few minutes,, though some were out for a few hours. See NERC, August 199 I, for a survey of the causes of and planning procedures for coping with this risk, and Electric Utility Week (12/19194) for details on the WSCC event.

decrcase capital and operating costs. In order to manage how such benefits are shared, the interconnected utilities must control *the tie-line flows* between themselves at their borders. Tie-line flow measurement and management are the second important means of achieving economic performance goals for the transmission and generation system.

Operationally, interconnected or "pooled" utilities are usually managed as several *control areas*, within each of which there is a sufficient number of generators to follow load and to control system frequency and tie-line exchanges. The tie-lines between control areas are used for contractual power flows under pooled dispatch (economy interchange), wheeling arrangements, bulk power sales from one utility to another, and unplanned flows that occur when one utility suddenly needs the slack generation capacity of another for rectifying an emergency condition. Any change from scheduled tie-line flows becomes the basis for centrally controlled (pool level) corrective action (primarily to the generators) in order to assure that one utility does not inadvertently supply replacement power to another without compensation. The adjustments to restore planned tie-line flows are not necessarily the same as those that would restore the system frequency. Such changes in flows or system frequency are called *area control errors* (ACE). The ACE is measured every few seconds at each control area. It is given by:

$$\begin{aligned} ACE_j &= \text{average frequency in control area}_j \cdot \text{bias}_j \\ &\quad + \text{net unscheduled tieline flows into or out of area } j \end{aligned}$$

The *bias* term captures how many MWs of local generation are typically required to offset a 0.01 Hz frequency shift. This ACE is measured in MW, and it will roughly equal the amount of generation shortfall in a given control area. Because of the governor controls described above, an imbalance in area j may be first met by a load-following unit outside of area j. This will be reflected in ACE_j as a frequency decline and a net scheduled import, while the area(s) supplying the new demand (or replacing the lost supply) in j will measure a frequency decline and a net export. Exports have the opposite sign of imports, so these ACE measurements will be approximately zero outside of area j. This reveals which area needs to make adjustments, without having to evaluate the state of each and every supply or demand bus within each control area. The AGC system adjusts the setpoints of the governors on plants that are participating in load-following and frequency regulation to restore each ACE to zero. This also brings frequency back to 60 Hz.[11]

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[11] In fact, restoring ACE to zero may not cause frequency to equal precisely Hz, in part because of inaccuracies in estimating control area biases. As a result, utilities may find that a system clock gains or loses a small amount of time over the course of a day, indicating that some inadvertent, interchange has occurred. This is generally corrected in kind with subsequent MWs rather than monetary payments. See Wood and Wallenberg, 1984, pp. 291 - 354, for a discussion of AGC and interchange management

Computers and electronic telemetry equipment are used to make these kinds of system observations and to help the system controller to maintain economic dispatch of the generators for modest changes in load. These computers are based at control area centers, of which there were 154 in the U.S. as of 1993.

For large changes in load, such as might arise unexpectedly due to a forced outage or a lighting strike that disables a portion of the transmission network, human anticipation and astute, timely operator intervention are required. The primary defense against such contingencies is to maintain buffers of reserve generation and transmission capacities sufficient to absorb the most severe potential disruptions to the system. A system capable of automatically adapting to any single contingent disruption, i.e. of adjusting without further loss of lines or loads to a new supply-demand balance at 60 Hz, is said to be *secure.*

The design of unbundled electric power services and open access protocols must be sensitive to the fact that generation operations and transmission control are intimately related. The boundary between the two functions is not a precise one. Generators sometimes have to be adjusted to protect the transmission system, and transmission transactions such as wheeling obligations may alter the security of network operations. Analytically, determining how many MW of transmission flow capacity reserves are required for security and how generation and wheeling can be scheduled to achieve a pattern of power flows consistent with the needed transmission reserve margins requires modeling techniques like those described in the next chapter.

4

POWER FLOW MODELING

Overview

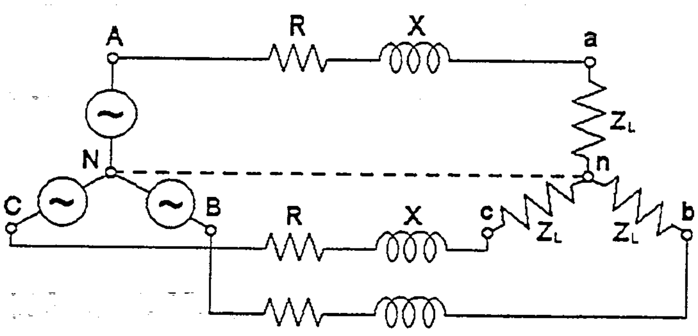
This chapter deals with how to predict the pattern of real and reactive power flows that will occur on a utility power network when a given set of plants is used to meet a given load. This is primarily an engineering question, not an economic one, though load flow results can be used as inputs to the design of wheeling tariffs and economic studies. The next chapter will deal with how to incorporate load flow equations into a planning problem that includes economic objectives and capacity constraints. The first step in any planning problem is to simplify, and transmission flow analysis usually involves several simplifications:

- It is convenient to represent the three-phase power transmission that actually occurs as a single phase, i.e. as occurring over one conductor rather than three (with appropriately reduced voltages and power levels). This is reasonable because system managers are generally able to keep the system operating in a "balanced" fashion, such that the currents over the three lines differ from each other only in timing, by angles of ±120 degrees. As long as this is the case, each of the three lines per generator behaves identically, and analysis of any one (called "per phase" analysis) suffices to describe the behavior of all three.
- Another simplification is to state all measurements as multiples of reference levels of voltage, power, current, etc., rather than as the actual levels of those factors. This is a significant algebraic convenience, called "per unit" analysis. It is particularly helpful when a system involves many transformers. An economist turning to engineering studies for inputs to a system model will see only per unit notation.
- In the simple circuits of Chapter 2, the electrical properties of the conductors could be ignored. Even though all wires are resistive, their effect is usually inconsequential compared to the resistance of the working load components on the circuit. But as was described in the last chapter, transmission lines are of such length and physical structure that their internal resistance, inductance, and capacitance can significantly affect power delivery. We show in this chapter that line impedance causes the voltage at one end to be out of phase with the voltage at the other (possibly in addition to being out of phase with the current). This timing difference between adjacent bus voltages is called the "power angle". It is an important determinant of power flow capacity. Indeed, the power angle difference between buses is essential for power to flow at all.

- To be strictly accurate, transmission line impedances must be modeled as being continuously distributed along the length of each line. We will describe the equations used for this purpose and their solution in this chapter, because they shed some light on what is physically happening in the lines. However, it is often unnecessary to solve such detailed equations, since one is usually concerned only with how power enters and leaves the system at the supply or demand buses; the continuous variation along the lines is not of interest. When this is the case, it is possible to treat the line impedances as though they were lumped at the ends and/or midpoint of idealized lines having no other impedances. For short lines, up to around 200 miles long, one can merely use the cumulative impedance over the line length for this purpose, but if the line is very long, a correction factor must be applied. One of the most commonly applied representations of a transmission line with lumped impedances is called the "pi-model".
- In the examples of Chapter 2, source voltages were given, i.e. known to have fixed AC or DC magnitudes. This made it possible to find the currents in the network by solving simple linear equations (albeit in complex variables). In contrast, power is the variable of interest in utility planning analyses, and both voltages and current are allowed to vary to achieve the necessary power flow (though voltages do not vary nearly as much). The equations for power flow therefore involve nonlinear terms like $V \cdot I$ or V^2/X. Moreover, there are generally several sources (generators) of power, so that many such equations must be solved simultaneously, each of which describes either the real or the reactive power flowing between a pair of adjacent buses. By exploiting per-phase, per-unit, and pi-model simplifications, this "AC load flow" problem can be solved for a given set of input (generation) and output (demand) levels. The solution yields the real and reactive power flows on each line as well as all the bus voltages, power angles, and line losses throughout the network.
- Power supply operators usually manage their systems in a fashion that minimizes reactive power flows, thereby maintaining fairly small power angles and nearly uniform voltage levels. Assuming this will be the case, the flow planning problem can be made much simpler by ignoring reactive power, which reduces the number of simultaneous equations needed to describe the network by half. More importantly, the real power flow equations become linear, allowing much faster (and more familiar) solution procedures. This "DC load flow" problem, so called because it is analytically comparable to predicting current flows on a DC circuit, yields the real power flows between all buses assuming line losses will be zero. By reiterating the analysis, a first-order estimate of the real flows with losses can be obtained.

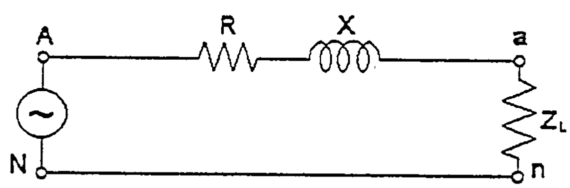
The most important simplifying assumption underlying the power flow modeling techniques described in this chapter is that the utility system will be operating in a known, fixed state, i.e. under precisely the conditions being simulated with the power flow model. Such "steady-state" planning does not reveal how the system can or will make a transition from one state to another. This information is critical to knowing how to plan for an unexpected, sudden disruption such as a line or transformer being lost to lightning. The problem of analyzing and controlling system transitions has a large bearing on how much slack capacity must be kept in reserve on the transmission network, i.e. on how much is not available for wheeling transactions. That problem is solved with scenario analyses that evaluate how the system would perform under a variety of disruptive contingencies.

Per Phase and Per Unit Modeling



It was explained in Chapter 3 that utility power plants typically produce balanced three-phase power: Three conductors carry power from the generator, with the voltages on each line all having the same magnitude and frequency but with a shift of ±:120° relative to each other. A fourth, neutral wire may be used to carry the return current (if any) from the three outgoing phases. The figure at right schematically depicts such a generator connected to a three-phase load in the "wye" configuration. Applying Kirchhoff's current law to the neutral node n, we have

$I_{nN} + I_{na} + I_{nb} + I_{nc} = 0.$



Since the circuit is balanced, the phasor sum $I_{na} + I_{nb} + I_{nc}$ is equal to zero, so I_{nN} must be equal to zero as well. This means there is no voltage drop between n and N and that the loop NAan will behave electrically in the same manner as the single-phase loop shown in the figure below. The other two phases are also equivalent to this simplified circuit albeit with a phase shift. Consequently, the behavior of the entire circuit can be analyzed with just a line-to-neutral single phase, converting the results back to three-phase via the formulas derived in Chapter 3:

Line Voltage: $V_{line} \approx \sqrt{3}\, V_{phase}$ (for wye configuration)

Total Apparent Power=S_{tot}= 3 S_{phase}

This representation of a transmission network is called per-phase *analysis.*[1]

We will see below that the capacitance between the conductors of a three-phase system must sometimes be recognized, but this does not undermine the ability to model the system on a per-phase basis. It is only necessary to add the appropriate shunt capacitances between each line and neutral (ground).

Another convenience for power flow modeling is obtained by expressing all voltages, currents, and impedances inter unit (p.u.) terms. This is simply expressing all electrical quantities as proportions of reference levels chosen or derived for each type of measure. For instance, generators are rated in terms of their design capacity to produce power and voltage, stated in terms of maximum volt-amperage (or wattage at a specified power factor) and voltage output. These two rated VA and V capacities on any generator on a system could be chosen as reference levels. All other generators' capacities would be expressed as percentages of the reference levels. Reference quantifies for current and impedance measurements can also be calculated from these two references:

$I_{ref}=VA_{ref}/V_{ref}$

$Z_{ref}=V_{ref}/I_{ref}$

For example, if the reference generators rated power is 400 MVA at 69 kV, then

I_{ref}=400x10^6/69x10^3=5797.1 A and Z_{ref}= 69x10^3 V/5797.1 A= 11.9Ω. A load on that circuit with an impedance of 20 Ω would have a p.u. impedance of 20/11.9= 1.68 p.u.

There are several advantages to per unit notation. Foremost of these is that it allows considerable simplification of how loads at different voltages (across transformers) are represented in network models - important because transformers can be very numerous on a large power system.. The p.u. impedances of both the high and low voltage coils of an idealized transformer turn out to be identical, allowing two circuits linked by a

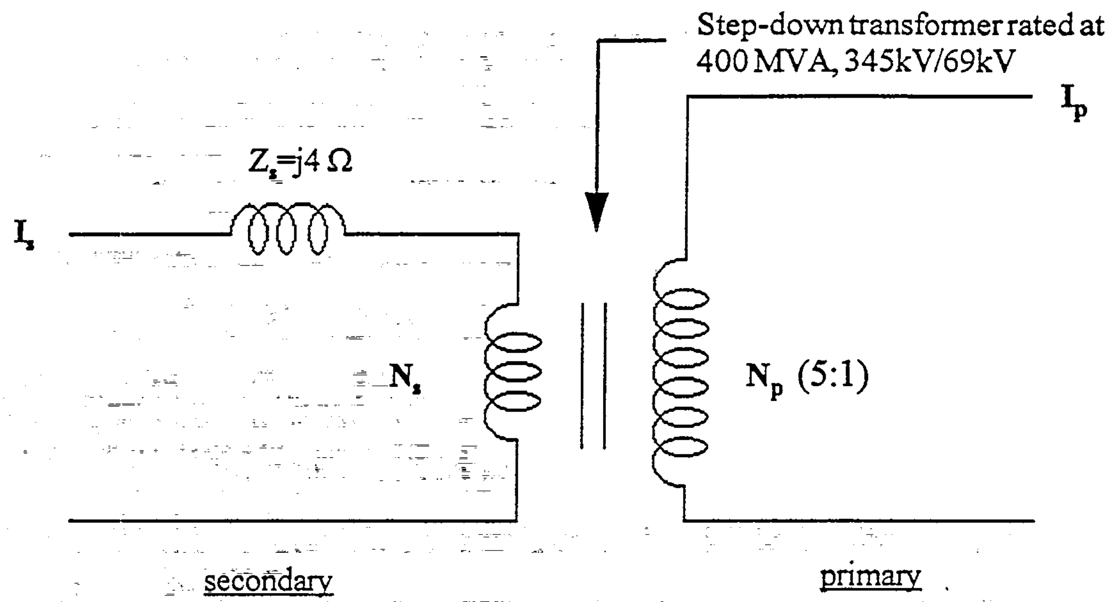
[1] It is also possible to use the line-to-line voltages in a per-phase representation of a 3-phase system, with corresponding adjustments to the conversion rules for going from per-phase back to 3-phase results.

transformer to be modeled as though the secondary load were a single impedance in the primary circuit.[2] This conversion is shown in Figure 4.1 The secondary (low voltage) side of the circuit is expressed in p.u. terms relative to the ratings on that side of the transformer. The effective primary-side impedance Z_p is then equal to Z_s times the square of the ratio of the number of turns. (This relation is derived in Figure 4.1.) Thus a 4 Ω load appears to be a 100 Ω load to the 345 kV system. (That is, a 100 Ω load at 345 kV would consume the same power as a 4 Ω load at 69 KV. Note that $100 = 5^2 \times 4$, and the step-down transformer has a turn ration of 5 = *345/69* kV). Moreover, in per unit terms (scaling by the reference impedance on the primary side) both sides of the transformer have the same size p.u. impedance,allowing the equivalent circuit to be redrawn without the transformer present as in the bottom portion of Figure 4.1.[3]

[2] An idealized transformer is one having no magnetic flux leakage or internal resistance, which would require a slightly different circuit representation from that in Figure 4.1.

[3] See Eaton and Cohen, 1983, pp. 95-106, or Pete, 1992, pp. 95-103 for good treatments and numerical examples of p.u. conversion.

Figure 4.1
Per-Unit Reresention of a Transformer



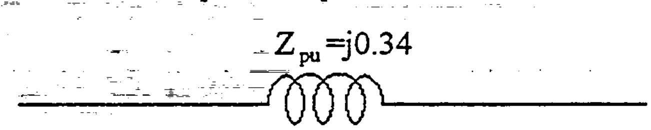
Base	VA	400	MVA	= transformer rating	400	MVA	= transformer rating
Base	V	69	kV	= transformer V_s	345	kV	= transformer V_p
Base	I	5797	A	$= 400\cdot10^6/69\cdot10^3$	1159	A	$= 400\cdot10^6/345\cdot10^3$
Base	Z	11.9	Ω	$= 69\cdot10^3/5797$	297.6	Ω	$= 345\cdot10^3/1159$

$$
\begin{aligned}
Z_s^{pu} &= Z_s/Z^s_{base} \\
&= j4/11.9 \\
&= j0.34
\end{aligned}
$$

$$
\begin{aligned}
Z_p &= E_s/I_s \\
E_p &= E_s(N_p/N_s) \\
I_p &= I_s\,(N_s/N_p) \\
Z_p &= (E_s/I_s)\,/\,(N_s/N_p)^2 \\
&= Z_s\,(N_s/V_p)^2 \\
&= Z_s\,(V_s/V_p)^2 \\
&= Z_s\,(345/69)^2 \\
&= j4\times25 \\
&= j100
\end{aligned}
$$

$$
\begin{aligned}
Z_p^{pu} &= Z_p/Z^p_{base} \\
&= j100/297.6 \\
&= j0.34 \text{ p.u.}
\end{aligned}
$$

Equivalent p.u. circuit:



Another advantage is that the p.u. ranges for operation of many kinds of electrical devices and components are fairly narrow, even though their absolute sizes may differ considerably. Planners familiar with these typical ranges can tell at a glance whether p.u. values in a flow model are normal or critical. For instance, voltages are not generally allowed to deviate more than a few percent from target levels anywhere on a utility system. The normal range for p.u. voltages should be around .95 - 1.05.

Pi -Model of a Transmission Line

Chapter 3 described the impedances arising from the internal structure and parallel physical paths of high voltage transmission lines. Because of these line impedances, a model of power flows in a power transmission system cannot treat current and voltage as being merely time-dependent (unlike the way such flows were described in Chapter 2). Both will also vary with position along the line: i = i(x,t) and v = v(x,t). As a result, two equations are needed to describe the flows on a transmission line:

$$\partial v(x,t)/\partial x = r^{x} i(x,t) - I^{x} \partial i(x,t)/\partial t$$

$$\partial i(x,t)/\partial x = g^{x} v(x,t) + c^{x} \partial v(x,t)/\partial t$$

The symbol $\partial f(x,y,z)/\partial y$, called the "partial derivative of f with respect to y," stands for the derivative of the function f(x,y,z) that would apply if x and z were held constant while only y (or whatever variable is in the "denominator") varied infinitesimally.

In words, the first of these two "partial differential equations" says that the change in voltage along the length of a transmission line depends on the line resistance r per unit length times the current at that location (the familiar $i^{x}r$ voltage drop), minus the inductance I per unit length times the rate of change over time in the current at that location. (Recall from Chapter 2 that the counter-emf in an inductor is given by $v = - I^{x}$ di/dt.) The second equation describes a similar relation for the change in current along the length, which depends on the local voltage times the shunt conductance g per unit length, plus the unit capacitance c times the rate of change in the voltage (c.f. from Chapter 2, that current in a capacitor is given by $i = c^{x}$ dv/dt). Note that this g does not equal I/r. The unit resistance r in the first equation is along the length of a conductor, while conductance g in the second equation is between the conductors.

The solution to these equations is presented in most power engineering textbooks. It reveals that both the current and voltage transmitted along the conductor take the form of traveling sinusoidal waves that propagate along the line with velocity almost equal to the speed of light in a vacuum but whose maximum amplitudes attenuate with distance. By analogy, v(x,t) is rather like the series of waves that children can send down a long jump-rope by making sharp, rhythmic pulses at one end. At any instant, a photograph of the entire rope would capture a collection of sine curves (the x-dependency) whose heights would decline somewhat with distance from the child (the attenuation). A moving picture with a narrow field of view would show that any point on the rope moves up and down in a sinusoidal rhythm (the time-dependency). A movie with a wide field of view would show that the whole pattern of pulses moves down the line from the child to the other end at the propagation velocity.[4]

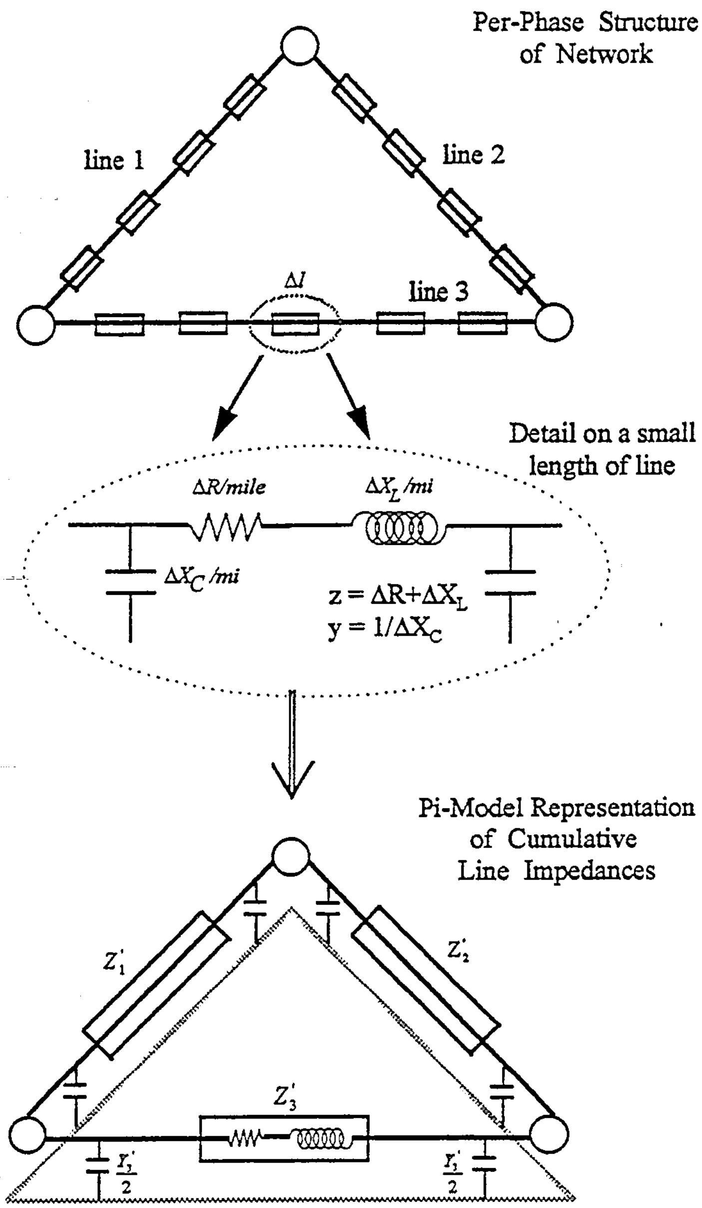
The solution to these equations also involves a traveling wave moving in the opposite direction. In the rope analogy, this would be a reflected wave moving back toward the child. Its phase relative to the incident or incoming waves depends on how the rope is anchored at the "receiving" end, e.g. whether it is tethered, free, or driven by another child's arm. The analogous concept on a power line is the nature of the load connected at the receiving end (e.g. short-circuited, open-circuited, or connected to an end-use impedance). Two sinusoidal current waves i(x,t) also propagate along the line, one in each direction, as traveling waves.

Because of the x-dependency of these waves, the voltages at two different locations along the line will be oscillating slightly out of phase with each other. This timing gap is called the *power angle* or *voltage angle*, and it should not be confused with the phase angle between voltage and current at any given point along the line. The power angle actually facilitates the flow of power, since it represents a potential difference between the two ends of a transmission line.

However, the power angle cannot become too extreme becuse it also determines a "stability limit" on feasible power transfer over long distances (explained in Chapter 5).

[4] See Skilling, 1951, for a clear though mathematically sophisticated discussion of these traveling waves.

Pi-Model Representation



$$Z' = (\Delta R^{/mi} + \Delta X_L^{/mi}) \times length \times [f(X_L, l) \approx 1.0\angle\varepsilon°]$$
$$Y' = (1/\Delta X_C^{/mi}) \times length \times [g(X_C, l) \approx 1.0\angle\delta°]$$

Often, it is not necessary to use the exact, traveling wave equations, because it is not necessary to understand what is happening to the electromagnetic waves at every point along the line. Instead, it is sufficient to predict what will happen to power and voltage only at the ends of each line. For this purpose, it is possible to solve a simpler problem, in which the per mile series impedances (z) and parallel admittances (y) are "lumped" and treated as discrete loads Z' and Y'/2 at the midpoint and ends, respectively, of an ideal (lossless) line. Because the resulting diagram for each line looks like the Greek letter II, the simplified representation on is called the *pi model.* The figure at right shows how the per-mile impedances are aggregated up to the equivalent lumped parameters by multiplying by the line length and a correction factor.

The correction factor for the equivalent lumped impedance depends on how long the line is. If the transmission line being evaluated is short, its shunt capacitance can be ignored entirely. That is, Y' is assumed to be equal to zero, while Z' is equal to zl, the per mile series reactance times total line length. A power line less than 50 miles long would be deemed short. A line of intermediate length, e.g. 50-200 miles, may have enough shunt capacitance to be worth modeling; one-half of the cumulative per mile line admittance is placed at each end, i.e· Y'/2 where Y' = yl. (Recall that capacitances in parallel are additive.) Z' can again be set equal to zl. For a line exceeding 200 miles in length, the lumped parameters Y' and Z' that are equivalent to the continuous line parameters involve slight correction factors which are phasors. The correction for Y' tends to have a magnitude a bit bigger than 1.0 and an angle just slightly less than zero, while the correction phasor for Z' is usually a bit less than 1.0 with a barely positive angle.[5]

Power Flow in the Pi Model

In the circuits discussed in Chapter 2, we were interested in solving for voltage and current. When one of these was known, the other could be found using linear functions that involved the impedances of the circuit elements. In electric utility power applications, the flow of power is of more interest, and the driving voltages are not specified a priori except within certain engineering bounds. As we will see below, this means that the analytic problem is a *nonlinear* one, involving many equations that must be solved simultaneously.

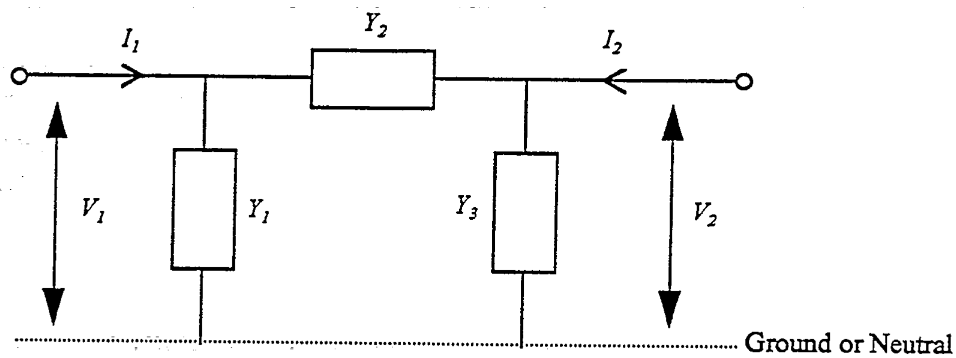
Most of the essential elements of this problem can be seen by looking at the flow of power along a single line that has been represented as a pi-model. Let the pi-model's lumped impedances be expressed as admittances denoted as Y_j where j = 1 or 3 for the shunt (capacitive) elements at each end and j = 2 for the series (resistive and inductive) element in the middle. Then the currents at the junctions at either end must simultaneously satisfy Kirchhoff's Current Law, which can be expressed in terms of these admittances in the manner shown in Figure 4.2. For instance, the first equation says that the current at node 1 will branch into two components: one through Y_1 that is proportional to V_1 (the potential difference with respect to ground) and one through the series admittance Y_2 that is proportional to the voltage difference across that lumped element. A similar equation describes the current at node 2. Rearranging terms so that the V_1 and V_2 components are separated, we see that the currents can be expressed as a

[5] The medium and long-line adjustments are sometimes referred to as the nominal pi and the equivalent pi models, respectively.

product of a *bus admittance matrix* and a voltage matrix: $\underline{I} = \underline{Y} \times \underline{V}$ (where the underlined bold capitals denote matrices).

FIGURE 4.2
Bus Admittance Matrix for Pi-Model

Pi-Model



KCL

$$I_1 = Y_1V_1 + Y_2(V_1-V_2)$$
$$= (Y_1+Y_2)V_1 - Y_2V_2$$

$$I_2 = Y_3V_2 + Y_2(V_2-V_1)$$
$$= Y_3V_2 - Y_2(V_1-V_2)$$
$$= -Y_2V_1 + (Y_3+Y_2)V_2$$

Admittance Matrix*

$$|I| = \begin{vmatrix} Y_1+Y_2 & -Y_2 \\ -Y_2 & Y_3+Y_2 \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \end{vmatrix}$$

$$\underline{\mathbf{I}} = \underline{\mathbf{Y}} \times \underline{\mathbf{V}}$$

Recall also from Chapter 2 that complex power is given by $S = VI^*$ where I^* is the conjugate of I (needed to capture the phase angle difference between V and I). Applying this principle to the pi-model matrices, we have $\underline{S} = \underline{V} \times \underline{I}^* = \underline{V} \times (\underline{Y} \times \underline{V})^*$, i.e. S is a quadratic function of the voltages.[6] This algebra holds for power networks

---

[6] The complex conjugate of a matrix is just the matrix composed of the conjugate of each of its elements, and the conjugate of a product is the product of the conjugates. (The conjugate of a + jb is a - jb.)

consisting of many linked pi-model segments, provided the $\underline{Y}$. matrix is constructed appropriately:

- the y_{ii} elements of $\underline{Y}$ are equal to the sum of all admittances connected to the ith node, and
- the y_{ii} elements, $i \neq j$, are equal to the negative of the admittance connecting nodes i and j

The matrix in Figure 4.2 illustrates these rules. For instance $y_{11} = Y_1 + Y_2$ while $y_{12} = -Y_2$. If bus 1 of this π-segment were connected to additional lines, there would be more admittance terms in the equation for $y_{1i,}$ corresponding to the additional branches into which the bus 1 current will flow. The matrices would also have to be larger to reflect the KCL equations at the other buses.

This quadratic dependence of power flow on voltages is perhaps easier to see in conventional algebraic form with no phasors or complex numbers, though the matrix notation is essential for solving large network problems. Figure 4.3 derives the power flows at each end of the pi-model, assuming that the shunt admittances are zero. The key result is:

$$P_{12} = [I/(R^2 + X^2)]([RV_1^2 - RV_1V_2 \cos (_{12} + XV_1V_2 \sin(_{12}] \qquad \text{(Eq. 4.1)}$$

where R and X are the lumped equivalent magnitudes of the line's series impedances *and* $(_{12} = (_1, - (_2$ is the power angle between the bus voltages V_1 and V_2 at each end.

On a two-bus network (one generator and one load), there will be little uncertainty about the direction of power flow, but this is not the case if there are many generators and many load buses. Thus one can solve for P_{ji} just as easily as for P_{ij} and the above matrix multiplications in fact solve for both. On the pi-model, the solution for P_{21} is very similar to that for P_{12} (see Figure 4.3), but it has relabled subscripts (1 ↔ 2) and a change of sign on the last term (because $\sin(_{21} = -\sin(_{12})$. This sign change indicates that the magnitude of P_{21} is not simply the opposite of P_{12}. Of course, real power is only flowing in one or the other direction, so what does this mean? The actual direction of flow becomes apparent in that, flows away from a node will have a positive sign. while flows into a node will be negative: If the flow is from 1 to 2, the*n* P_{12} will be positive and P_{21} will be negative with a slightly different magnitude. The difference in magnitudes will equal the real power (I^2R) losses over that line segment:

$$P_{12}^{\text{losses}} = P_{12} + P_{21}$$

FIGURE 4.3
AC Power Flow in Pi-Model Without Shunt Elements

$$\underline{I} = \underline{V} \times (\underline{Y} \times \underline{V})^* = \begin{vmatrix} V_1 \\ V_2 \end{vmatrix} \begin{vmatrix} Y_2^* & -Y_2^* \\ -Y_2^* & Y_2^* \end{vmatrix} \begin{vmatrix} V_1^* \\ V_1^* \end{vmatrix}$$

$$S_{12} = V_1 Y_2^* (V_1^* - V_2^*) = V_1 \times \left(\frac{V_1^* - V_2^*}{Z^*} \right)$$

$$= (V_1^2 - V_1 V_2 e^{j(\theta_1 - \theta_2)}) / (R - jX_L)$$

Converting from phasor to trigonometric notation:

$$S_{12} = \left(\frac{R + jX_L}{R^2 + X_L^2} \right) \left(V_1^2 - V_1 V_2 \left(\cos(\theta_1 - \theta_2) + j \sin(\theta_1 - \theta_2) \right) \right)$$

Breaking this into real and imaginary parts, and substituting θ_{12} for $\theta_1 - \theta_2$:

$$S_{12} = \left(\frac{1}{R^2 + X_L^2} \right) \left[RV_1^2 - RV_1 V_2 \cos\theta_{12} + XV_1 V_2 \sin\theta_{12} \right]$$

$$+ j \left(\frac{1}{R^2 + X_L^2} \right) \left[XV_1^2 - XV_1 V_2 \cos\theta_{12} - RV_1 V_2 \sin\theta_{12} \right]$$

$$= P_{12} + jQ_{12}$$

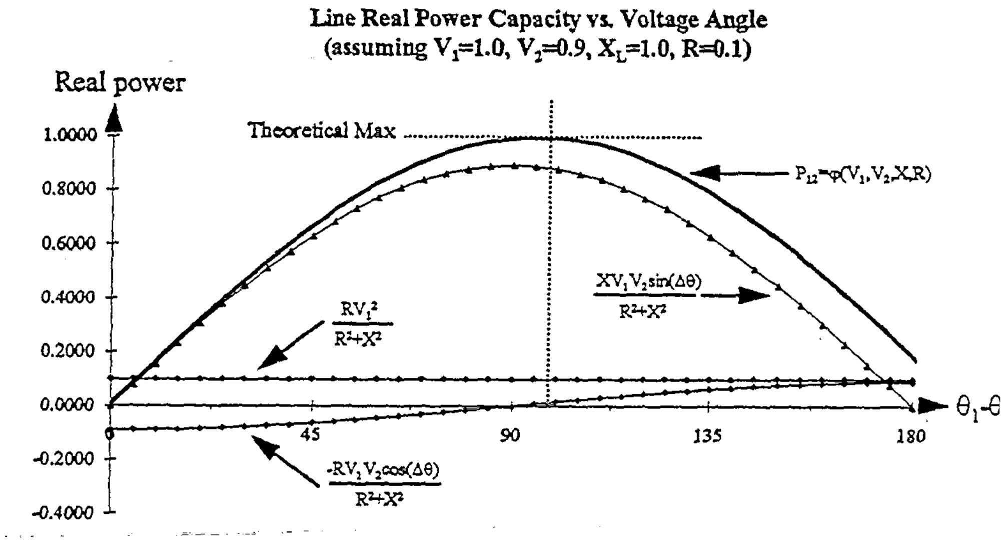
where $P_{12} = \left(\frac{1}{R^2 + X_L^2} \right) \left[RV_1^2 - RV_1 V_2 \cos\theta_{12} + XV_1 V_2 \sin\theta_{12} \right]$

At the other end of the line:

$$P_{21} = \left(\frac{1}{R^2 + X_L^2} \right) \left[RV_2^2 - RV_1 V_2 \cos\theta_{12} - XV_1 V_2 \sin\theta_{12} \right]$$

$$P_{loss} = P_{12} + P_{21}$$
$$Q_{loss} = Q_{12} + Q_{21}$$

Equation 4.1 for P_{12} indicates that at any given power angle ($_{12,}$ power flow is proportional to the square of the voltage. For reasons explained in Chapter *5*, V_1 *and* V_2 are controlled by system operators such that both stay quite close to the rated capacity of the line. Consequently, a 345 kV line will have more than twice the power transfer capacity of a line rated at 230 kV, every thing else being equal, since $(345/230)^2 = 2.25$ (ignoring any other changes in conductor design that might impose constraints).



The figure on the right displays P_{12} as a function of the angle ($_{12'}$ along with its three components in Eq. 4.1. Note that at small voltage angle differences the cosine term in RV_1V_2 non-sinusoidal term in RV_1^2 cancel each other almost perfectly, so the real power flow is almost equal to the X V_1V_2sine term. Since at small angles the sine function is approximately equal to the angle (in radians), the real power flow increases almost linearly with the power angle itself up to about $\Delta(_{12}=30°$. At larger angles, P_{12} is no longer linear but still is dominated by the sin(term, reaching a maximum at roughly 90°. Recall from Chapter 3 that the series reactive impedance in a transmission line is usually many times bigger than the resistance, i.e. $X \geq 10.R$. If we ignore terms in *R* altogether, the expression for P_{12} simplifies to V_1V_2sin ($_{12}$ /*X*, which clearly is largest at ($_{12}$ = 90°. The actual maximum will occur at ($_{12}$= 180°-(*Z*, where Z is the series impedance at angle (Z with respect to the current. Since Z is mostly inductance,(Z is close to 90° and (180°-(Z is just a bit more than 90°.

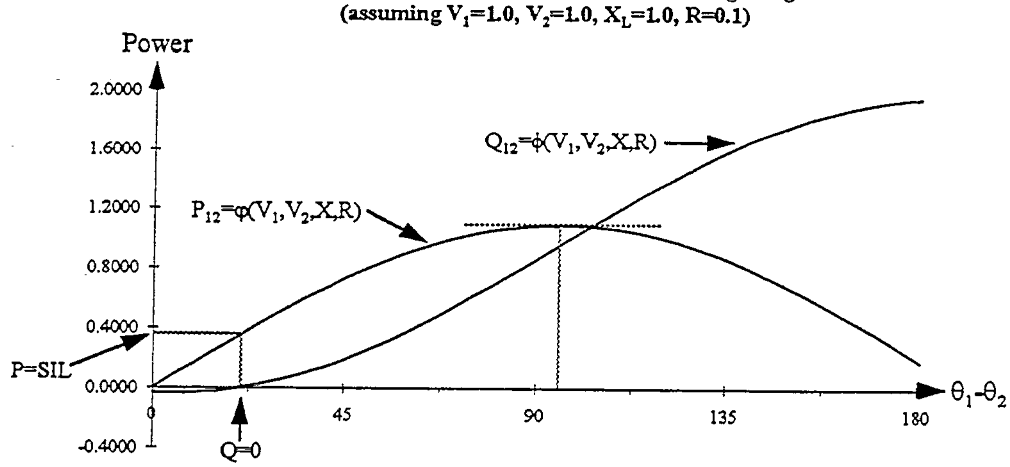
In practice, it is not safe to let power angles become this large, because the generators become unstable at this *theoretical stability limit* (further explained in the next chapter). The practical stability limit usually involves a power angle of no more than 40-50° - depending on system characteristics such as the size and location of the largest generators — and angles this large are very seldom used. During normal operations, power angles tend to be less than 10°, so MW flows are often in the range of 1/6 to 1/3 of the lines' theoretical stability maximums.

There are similar equations for the flow of reactive power in a pi-model segment. For instance, in the 1→2 direction.[7]:

$$Q_{12} = [1/(R^2+X^2)] \times [XV_1^{\,2} - XV_1V_2cos(_{12} - RV_1V_2sin(_{12}]$$

Q_{21} again differs from Q_{12} with the sum equal *to* reactive power line losses, I^2X. It may occur that reactive power does not flow in the same direction as real power, because Q can be either supplied or consumed by both the lines and/or the loads, depending on their operating state. Reactive power will tend to flow from a load with a loading (capacitive) power factor into the adjacent transmission lines. A line that is heavily loaded with real power will be carrying a high current and have substantial I^2X consumption, requiring VAR injections into the line at both ends. Conversely, a lightly loaded line may supply VARs to the buses at each end. The figure at right shows how Q_{12} varies with $(_{12}$ Note that it starts negative, when the line is mostly capacitive, and becomes significantly positive as the real power, hence current, increases.

Line Real and Reactive Power Flow vs. Voltage Angle
(assuming V_1=1.0, V_2=1.0, X_L=1.0, R=0.1)



It is not immediately evident from the equations for P_{12} or Q_{12} that power transfer capacity declines with line length, but it does. Primarily this is due to the denominator in Equation 4.1, because cumulative impedance increases with length. An uncompensated *345* kV line might have 1200-1300 MW of capacity for 50 miles, but as little as 250 MW over 600 miles.[8] Reactive power compensation can incrcase a heavily

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[7] If shunt capacitance of Y'= Xc were recognized for this line, the equation for Q_{12} would be as above plus a term equal to $-^1/^2\ V_2Y'$

[8] See Kelly, 1987, p. 52.

utilized line's transfer capacity over a long distance by reducing its net impedance *X*, which also reduces the *IX* voltage drop so that V_2 stays close to V_1. This is one way of upgrading the capacity of a transmission system without having to add new lines.

As mentioned above, it is desirable to keep the voltage angles to no more than a few degrees. For small angles, cos(Δ() is approximately 1.0 and not very sensitive to changes in angle, while sin(Δ() is approximately equal to Δ((in radians) and increases almost one-for-one in percentage terms with changes in the angle. Under these conditions, changes in P_{12} will tend to be driven predominantly by changes in ($_{12}$, while Q_{12} will tend to be most sensitive to changes in V_1 or V_2. The intuition for this *P*-(sensitivity is that Δ(creates all of the sinusoidal voltage difference between the ends of the line that drives real AC power along it. For instance, a 3° difference in timing between V_1 and V_2 involves a potential difference between the ends of the line that has a magnitude of .052 V_1 (for angles smaller than 30°, a 1.00 percentage change in timing causes about 1.75 percentage change in potential difference). This is enough emf to drive a large current through the minor line impedances. The *Q*-*V* interdependency arises because reactive elements induce a counter-emf 180° out of phase with the supplied voltage, which tends to alter local voltages.

Notwithstanding this "decoupling" of real and reactive power flows, they are by no means fully independent. For instance, an incrcase in real power line loads, e.g. from a wheeling contract, will tend to incrcase reactive losses over all the network lines that are carrying incrcased current for the wheel. Everything else being equal, this will reduce voltages at load buses as well. Thus wheeling contracts may require reactive power "compensation" of either the physical or monetary kind, if they are not to affect existing customers adversely.

An AC Load Flow Example

One problem of interest to utilities is predicting the pattern of real and reactive power flows that will occur for a given set of demands and planned dispatch of the power plants. For instance, this problem could arise when a wheeling contract is proposed, due to concern over whether the contract path will be similar to the actual path or whether unaccounted-for *loop flows* will tend to carry most of the third-party power. The *AC load flow* method provides the answer to this question.

In an AC load flow analysis, each line in the network is represented in per phase terms and in pi model form. The problem is then formulated as $\underline{\mathbf{S}} = \underline{\mathbf{V}} \times (\underline{\mathbf{Y}} \times \underline{\mathbf{V}})^*$ where $\underline{\mathbf{Y}}$ is the bus admittance matrix like that described in Figure 4.2 and $\underline{\mathbf{S}}$ and .$\underline{\mathbf{V}}$ are matrices of the complex power and voltage phasors that must be determined simultaneously. At each demand node, P and Q (i.e. complex S) are given. At each supply node but one, called the *swing bus*, *the* planned P and V (or P and Q) are given. A swing bus is needed because the system line losses will depend on the pattern of actual flows — on how much current flows through what line at what impedance. This pattern is not known *a priori*, so at least one generator must be left free to make up for line *loss* requirements as well as any end-use demand not served by the other, committed units. At the swing bus, a voltage magnitude V and ∠V, a reference angle for all other voltages, are given. The problem is to find the P and Q requirements on the swing bus and the resulting flows, losses, reactive supplies, and voltage angles throughout the entire network.

Notice that at each bus, two out of four of the needed parameters (P, Q, V, or θ) are given The equations must be solved iteratively until the other two are known at each node. Conceptually, this procedure is fairly straightforward. Its mechanics can be understood by considering a simpler problem involving only real power flows, not all four variables. That problem could be solved by first assuming that there would be no line losses and that the swing bus simply had to supply any unserved end-use demand. The flows required by Kirchhoff's laws could be solved and the associated I^2R losses in each line calculated. Adding these losses back to the end-use demand on each line, the model could be run again. The swing bus would pick up the extra load, resulting in slightly revised flows and losses again, and so on. After a few iterations, the incremental improvement in the solution would become insignificant.

The full AC problem is solved in much the same fashion, but it involves making some additional assumptions about the initial values for all the missing supply and demand variables. With such guesses in hand, one can solve for all of the line flows and determine if their impact on the angles and delivered power flows are consistent with the original assumptions. To the extent these differ, the assumptions must be revised to be more in accord with the most recent set of simulated flows. The process is repeated until the latest results no longer diverge materially from the prior iteration's results.[9]

Figures 4.4 through 4.7 provide an example of AC flows on a small network. The assumptions about its configuration are given in the figure below, in conventional (per-phase) terms. It is intended to demonstrate numerically some of the relationships between P, Q, V_1 and θ that have been described above. It is not intended to depict an ideally configured or typical electric utility.

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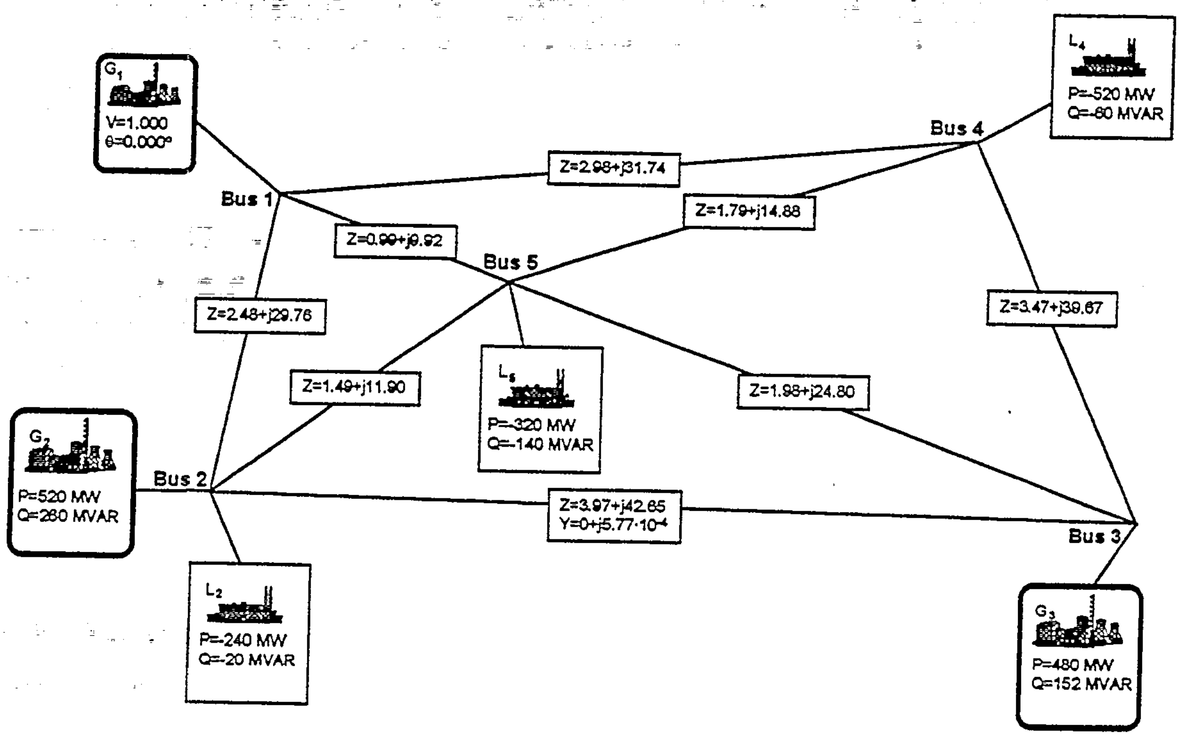
[9] A commonly used numerical solution technique is the Newton-Raphson method. It uses the partial derivatives to the nonlinear power equations to set up some linear equations that are solved, e.g. by Gaussian elimination. This approximate solution is the next estimate of the solution for the nonlinear equations, at which location the partial derivatives are recalculated, repeating the process until convergence. Other algorithm are possible and may be preferred in certain circumstances, because they are faster or require less computer resources. Because the AC load flow problem can be quite large, involving many power plants and a huge but sparse bus admittance matrix (i.e., having only a few non-zero values per row or column), efficiency of solution is very important and has been researched extensively. See Glover, 1987, pp. 234-274, for an introduction to some basic solution techniques.



This example involves an X-shaped network with its perimeter connected, i.e. 8 lines and 5 buses. Three of the buses are generation buses, and three are load buses. As an arbitrary convention, buses have been numbered counterclockwise from the upper left, with the center being bus 5. G_1 is the swing bus. One of the load buses requires a lot of reactive power such as could arise when serving a group of industrial loads (i.e. load 5 consumes 320 MW and 140 MVARs lagging, giving a power factor of 91.6%.) Consistent with the pi-model conventions described above, the lines are treated as having no shunt impedances, except for the longest one (line 2-3, 150 miles). The calculations for the per unit parameters are shown in detail for that transmission line in Figure 4.4. (Stability limits to MW flows calculated in the bottom rows of this figure are ignored by the AC flow model, as are any other operating constraints. Relative dispatch costs are also not taken into account, so AC flows may not be in any sense economically optimal. They are merely consistent with planned scheduling of the plants to meet a given demand. For an explanation of the capacity limits on the transmission lines and buses, refer to the next chapter.)

Figure 4.4 Calculations of Per Unit Parameters for Line 2-3

Base values:

Three-phase base voltage, $V_B^{3\phi} = 345.00$ kV
Three-phase base power, $S_B^{3\phi} = 1200$ MW
Per phase base voltage, $V_B^{1\phi} = V_B^{3\phi}/\sqrt{3} = 199.2$ kV
Per phase base power, $S_B^{1\phi} = S_B^{3\phi}/3 = 400$MW
Base impedance, $Z_B = (V_B)^2/S_B = 99.1875\ \Omega$
Base admittance, $Y_B = 1/Z_B = 0.01008$ Mhos

Line characteristics:

Length, $l = 150$ mi (240 km)
Series resistance, $r = 0.0264\ \Omega$/mi
Series inductive reactance, $x_L = 0.2843\ \Omega$/mi
Series capacitive reactance, $x_C \approx 0\ \Omega$/mi
Shunt conductance, $g \approx 0$ Mhos/mi
Shunt capacitance, $C = 1.02 \cdot 10^{-8}$ Farads/mi

Π-model conversion:

Resistance, $R = 0.0264 \times 150 = 3.97\ \Omega$
Per unit resistence, $R_{pu} = R/Z_B = 0.04$
Inductive reactance, $X_L = 0.2843 \times 150 = 42.65\ \Omega$
Per unit inductive reactance, $X_{L,pu} = X_L/Z_B = 0.4$
Per unit impedance, $Z_{pu} = R_{pu} + jX_{L,pu} = 0.04 + j0.43$
Shunt capacitance, $C = 1.02 \cdot 10^{-8} \times 150 = 1.53 \cdot 10^{-6}$ Farads
Capacitive reactance, $X_C = 1/(\omega C) = 1/(377 \times 1.53 \cdot 10^{-6}) = 1733.7\ \Omega$
Shunt susceptance, $B = 1/X_C = 5.768 \cdot 10^{-4}$ Mhos
Per unit shunt susceptance, $B_{pu} = B/Y_{pu} = 0.0574$
Per unit admittance, $Y_{pu} = G_{pu} + jB_{pu} = 0 + j0.0574$

Surge impedance calculations:

Per phase surge impedance, $Z_C = \sqrt{L/C} = 271.52\ \Omega$
Per phase surge impedance loading, $SIL = (V_B)^2/Z_C = 146.12$ MW
Voltage wavelength, $\lambda \cong c/f = 3 \cdot 10^8/60 = 5000$ km $= 3100$ mi
Theoretical steady state stability limit, $P_{max} = SIL/\sin(2\pi l/\lambda) = 489.17$ MW
20° practical stability limit, $P_{20^\circ} = P_{max}/\sin(20^\circ) = 167.31$ MW

The resulting real power flows in per unit quantities are shown on the upper part of Figure 4.5, along with the real power losses (in parentheses), voltage magnitudes and angles. Note that the angles tend to be fairly small, all within a range of -5.200° to +6.069°. Total real power losses in per phase actual quantities are 13.6 MW, or around 1.3% of the total real power consumed at the load buses. However, the losses are sometimes larger or smaller on individual lines, ranging from 0.7 to 1.9% of the received power.

The corresponding reactive power flows, losses, voltage magnitudes and angles are shown on the lower part of Figure 4.5. Voltage magnitudes vary by only a few percent, from a low of 0.984 p.u. to a high of 1.059 p.u. Note that voltages at all buses decline in the direction of the reactive power flow, and the voltage angles decrcase in the direction of real power flow. To accommodate the scheduling of the other generators, the swing bus G_1 is forced to consume reactive power, i.e. to produce lagging VARs. The reactive power consumed by the swing bus is fairly substantial, 100 MVARs vs. 94 MWs of real power (94 = .235 p.u. x 400 MW, the base power in Figure 4.4). This gives this generator a power factor of 0.685, or a power angle of 46.8°, a bit larger an angle than many generators could sustain over any long period of time. The shunt capacitance of line 2-3 supplies reactive power with the effect being a net gain in reactive flow along the line. (The shunt capacitance in line 2-3, shown as two small vertical lines, really occurs all along the length.) This is because that line carries so little real power that its capacitive effects exceed its inductive effects[10] It carries only 0.141 p.u. x 400 = 56.40 MW, well below its per phase SIL, of 146.12 MW. (See Figure 4.3). Note that over line 1-5 real power flows in the opposite direction to reactive power. Overall, reactive losses are much larger than real losses; in total, Q_{loss} equals 35% of total reactive generation.

Figure 4.6 presents two tables of sensitivity cases that reveal what happens when the real or reactive power demands are changed at the demand nodes (buses 2, 4 and 5). We see in the upper table that an incrcase in real power demands (of about 50% at each of the loads) tends to alter the voltage angles by as much as 6.5°, but it does not cause much change in either the reactive power flows or voltage magnitudes. (This table is presented in per unit terms, because it is easier to see percentage changes in p.u. terms.) In contrast, the lower table shows that significantly increasing the reactive power bus demands (by more than 100%) causes voltage magnitudes to decline considerably at the demand buses (up to 10.6% change on bus 4), while real power flows and voltage angles remain relatively unchanged (less than 1.7° of change). This demonstrates the sensitivity of P to θ and Q to V, and the non-sensitivity of P to V and Q to θ discussed above. Note that the swing bus provides more real power in the high reactive load scenario (0.015 p.u. x 400 = 6 MW). Because of the voltage drops associated with reactive loads, much more current must be transmitted in order to serve a given real power demand. This results in incrcased line losses, which must be made up by the swing bus. Thus reactive demand has a cost even though it does not entail any average energy consumption itself.

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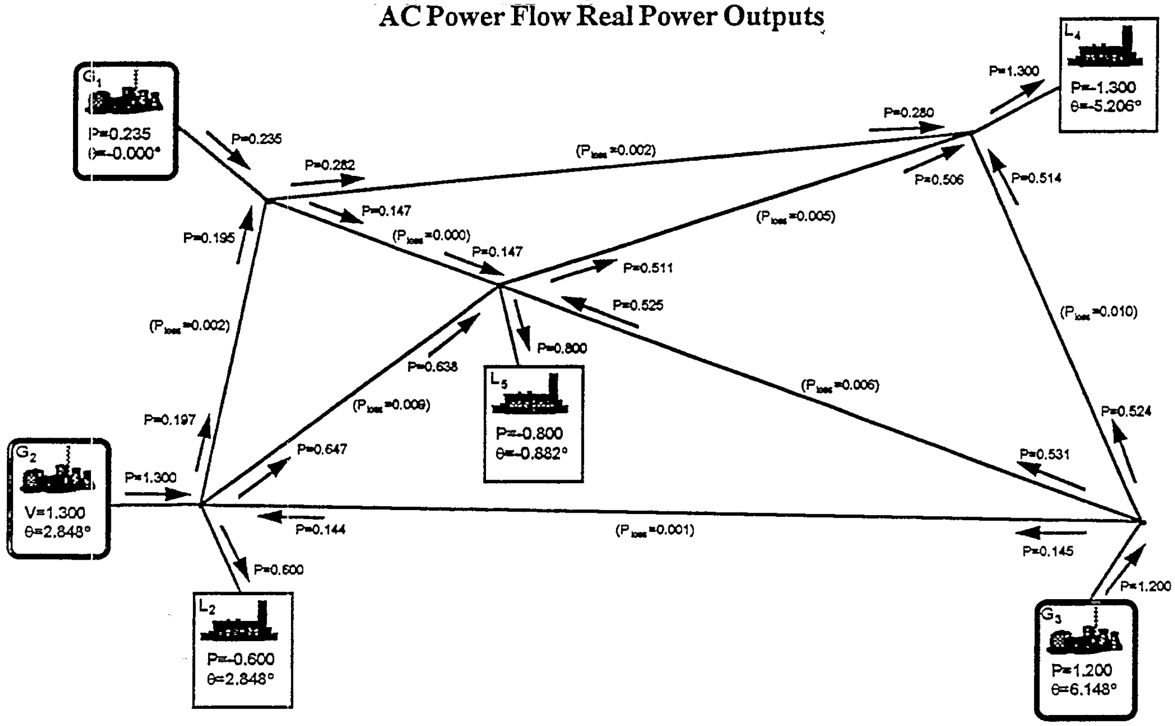
[10] This low utilization of line 2-3 is used in Chapter 5 to schedule a contract path wheeling load, which is evaluated with an economically optimized power flow model.

Figure 4.5

AC Power Flow Real Power Outputs



AC Power Flow Reactive Power Outputs

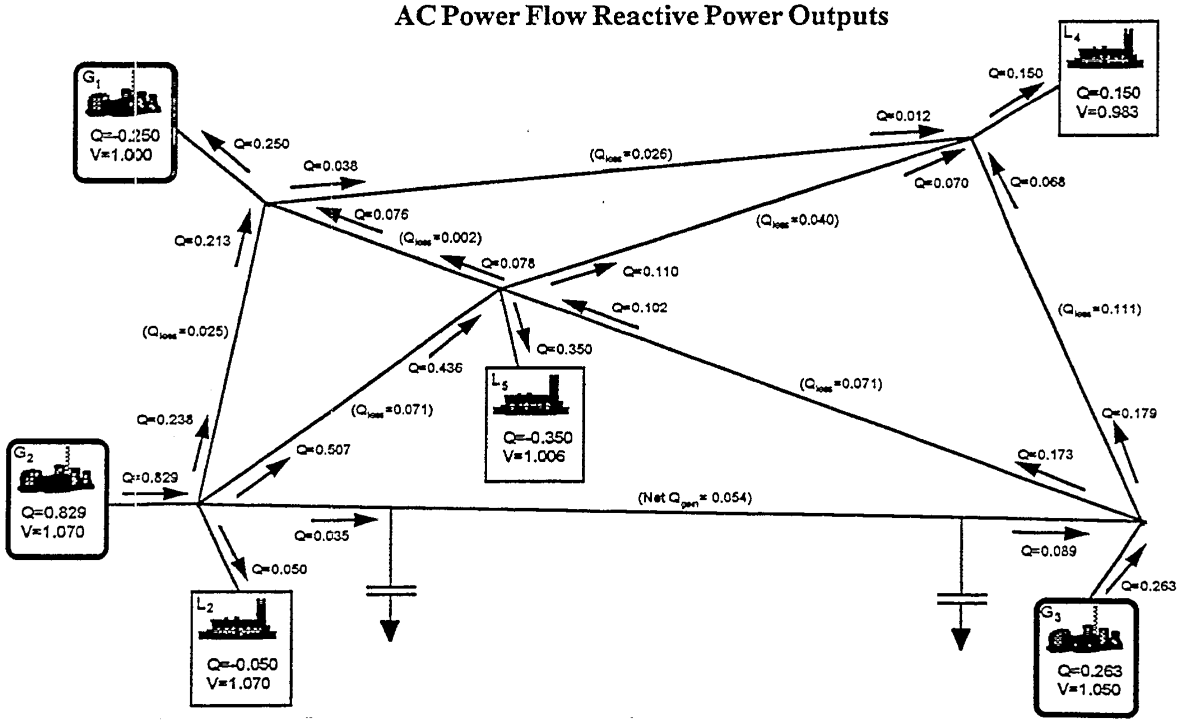


Figure 4.6
P-θ and Q-V Sensitivities
(Per Unit Values)

Comparison of AC Power Flows with Incrcased Real Power Demands

	Real Power		*Reactive ,Power*		*Voltage Magnitude*		*Voltage Angle*	
Bus#	**Base Case**	**ΔP**	**Base Case**	**ΔP**	**Base Case**	**ΔP**	**Base Case**	**ΔP**
G1	0.235	1.657	-0.250	0.038	1.000	1.000	0.000	0.000
G2	1.300	1.300	0.829	0.650	1.070	1.045	2.848	-2.029
G3	1.200	1.200	0.263	0.380	1.050	1.041	6.148	1.170
L2	-0.600	**-1.000**	-0.050	-0.050	1.070	1.041	2.848	-2.029
L4	-1.300	**-1.900**	-0.150	-0.150	0.983	0.959	-5.206	-11.789
L5	-0.800	**-1.200**	-0.350	-0.350	1.006	0.989	-0.882	-5.314
Losses	0.034	0.057	0.282	0.518				

Comparison of AC Power Flows with Incrcased Reactive Power Demands

	Real Power		*Reactive ,Power*		*Voltage Magnitude*		*Voltage Angle*	
Bus#	**Base Case**	**ΔQ**	**Base Case**	**ΔQ**	**Base Case**	**ΔQ**	**Base Case**	**ΔQ**
G1	0.235	0.249	-0.250	1.222	1.000	1.000	0.000	0.000
G2	1.300	1.300	0.829	0.650	1.070	0.980	2.848	3.897
G3	1.200	1.200	0.263	0.380	1.050	0.973	6.148	7.799
L2	-0.600	-0.600	-0.050	-0.300	1.070	0.980	2.848	3.897
L4	-1.300	-1.300	-0.150	-0.550	0.983	0.880	-5.206	-5.610
L5	-0.800	-0.800	-0.350	-0.950	1.006	0.921	-0.882	-0.522
Losses	0.034	0.049	0.282	0.452				

DC Load Flow

In the base case AC load flow above, none of the voltages departed much from the reference (swing bus) voltage, i.e. the p.u. voltages were close to 1.0. Also, none of the power angles was at all large. The biggest was 6.07° =. 1059 radians, for which the sine is .1057, almost the same value as the angle itself. These narrow ranges of variation are typical of how utilities try to run their systems: Voltage drops are usually kept to within ±5% of the reference voltage, and voltage angles are held to within ±10° along each line. In addition, the series inductance on a transmission line tends to be so much larger than the resistance, 10 or more times greater, that the resistance has minimal relative effect on real power flows. These features of *steady-state* operations can be exploited to develop a simpler model of real power flows. Starting with:

$P_{12} = [1/(R^2 + X^2)\}([RV_1^2 - RV_1V_2cos(_{12} + XV_1V_2sin(_{12}],$

substitute the assumptions that

$R = 0$ and $V_1 = V_2 = 1.0$ (in p.u terms), and

$\cos(_{12} = 1.0$ and $\sin(_{12} = \Delta($,

obtaining

$P_{12} = \Delta(/X,$

Similar simplifications, but without assuming R = 0, yield

$P_{12}^{loss} = R' P_{12}^2.$

Such equations are referred to as the *DC load flow* approximations for each line, because they are structurally similar to the equations for Ohm's Law and real power losses in a DC network, with *P* analogous to *I*, Δ(, analogous to Δ*V* and *X* playing the role of resistance (or R, in the loss formula).

The comparative simplicity of the DC flow equations makes it easier to understand how the relative power angles between buses are determined simultaneously with the flows on a network: By Kirchhoff's current law, the real power entering or leaving each bus must equal the sum of the other real powers flowing into or out of that bus. Thus the matrix of supply and demand bus powers $\underline{P}_{bus}$ can be expressed as a sum of elements in a matrix of line flows $\underline{P}_{line}$ that reflects which pairs of buses are interconnected and which are not. These line flows P_{ij} in turn depend on the matrix of relative angles $\underline{\theta}$ by way of $\underline{P}_{ij} = \Delta\underline{\theta}_{ij}/\underline{X}_{ij}$, as derived above. This chain of relations can be inverted to find the angles as a function of bus powers that satisfy Kirchhoff's laws.

Many utilities rely on DC load flow analyses rather than AC load flow, because they anticipate being able to keep their systems in tolerance with the required assumptions, and because the system modeling is much simpler. The above DC equations are linear in *P* as a function of ((, and only one-half as many simultaneous equations are involved. (All reactive and voltage variables are simply dropped.) The equation for losses is not part of the simultaneous equations, so the model must be iterated if it is important to one's analysis that the swing bus provides enough power to cover the losses. When this is done, this model predicts real flows fairly well. (Recall that losses are usually only 1-2% of total real power demand, which may be ignored for certain purposes.) The upper part of Figure 4.7 provides the DC load flow approximation to the base case AC load flow problem in Figure 4.5, and compares the two estimates of real flows. Agreement is within a few percent on every line. However, the swing bus provides 0.034 p.u.= 13.6 MW less real power than in the DC case, since it ignores line losses.

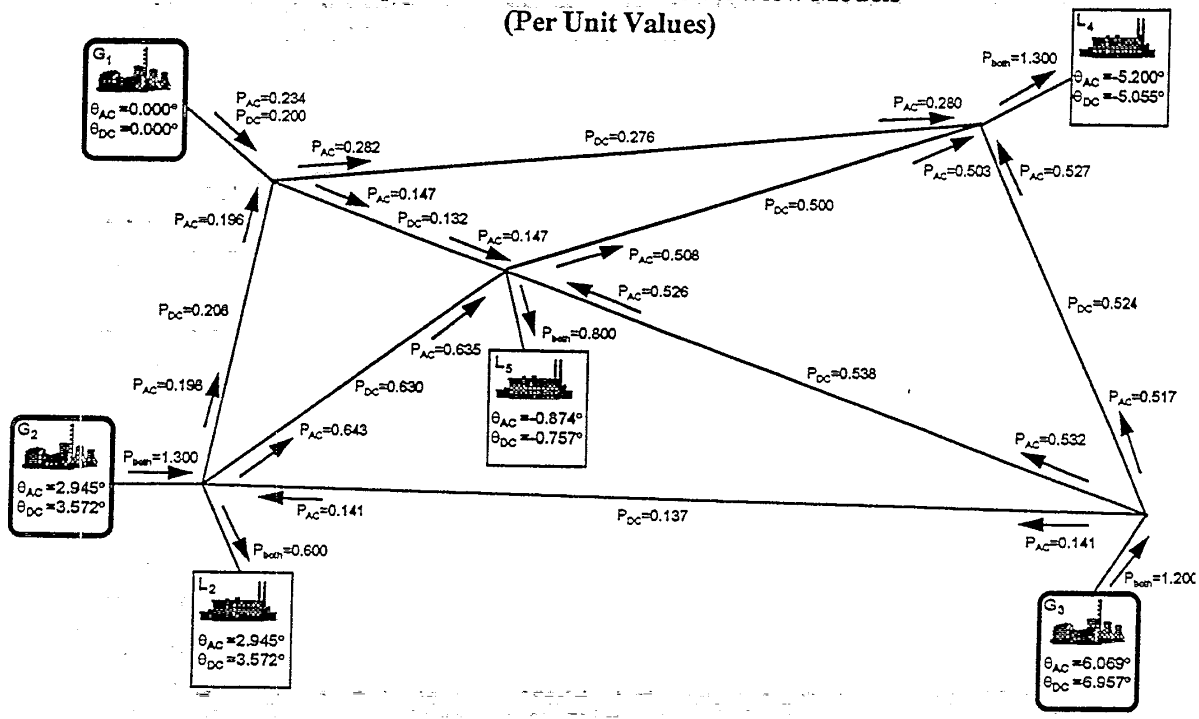
The lower half of Figure 4.7 also presents the per phase values for the real power and currents carried on each line, which are on the order of a few hundred amperes. These are typical values for 345 kV lines. Current levels are interesting to transmission planners and system managers because of the *thermal limits* on feasible power flows. Real power losses, proportional to I^2R, are manifest as heat in the conductors. If line losses become large for too long a period of time, the lines may overheat and become damaged. To avoid this, operators attend to the maximum tolerable power flows - which depend on duration in addition to MW level, since heat build-up is partly cumulative over time. Typically, thermal limits expressed in MW are about three times the SIL. This limit is rarely a problem for long lines, which tend to be constrained by their stability limits (explained in the next chapter). Short lines are more likely to be thermally limited. Interestingly, seasonal changes in temperature may affect thermal limits in some lines by a few percent.

Neither the AC or the DC load flow models are economic models. In particular, they do not take into consideration the relative cost of producing power at each of the generators, so they may not produce the cheapest solution. Nonetheless, they have many economic applications. For instance, one could determine which lines will be utilized on peak by a proposed wheeling contract under planned plant dispatch. The incrcased use of certain lines could be the basis for a MW-mile assignment of costs to the wheeling customer. Both AC and DC load flow models are also used in *contingency studies to* determine what would happen to line loadings if there were a system disruption given the prevailing plant dispatch. The system is simulated under normal operations, and then re-evaluated with one line (or generation station, or transformer) removed at a time. If any lines would have to carry more power than their design limits, or if any bus voltages would become too low or high after the natural flow re-routing, system operations can be altered to leave more slack capacity in place to absorb the potential disturbances and prevent a system security problem.[11] These slack capacity reserves become inputs to the Optimal Power Flow problem for economic dispatch subject to network constraints, described in the next chapter.

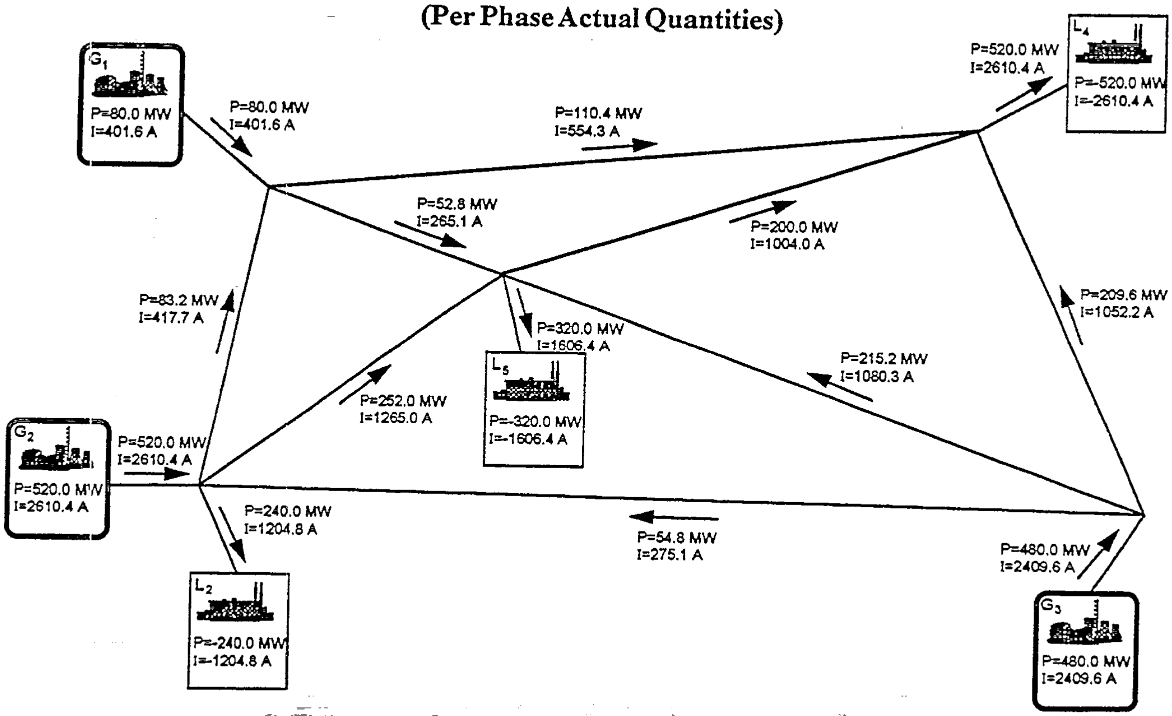
Transfer capability between two points on a network, e.g. for a wheeling transaction, can be assessed with a load flow model. There is adequate, reliable capacity for a proposed transaction if it would not violate any security (voltage or line loading) constraints in a worst-case contingency simulation, in which the most critical line or lines on the system were assumed to be lost during a period of typical or normal generation dispatch. (It is important that this analysis be performed for a very large transmission area, to assure that loop flows onto otherwise adjacent systems are not overlooked.) This would be a sufficient, indeed very conservative, condition for feasibility, since it treats the security concern as an absolute constraint. If a party inquiring about access were willing to pay the shadow prices on any constrained resources (in addition to its share of fixed costs) and/or to be interrupted when marginal costs becane high, such a strict capacity availability standard might be unnecessary. Network marginal costs are also explained in the next chapter.

[11] Note that these load flow models reveal what could happen in a contingency but not how to either fix (react) or prevent (anticipate) any potential overloads in the most efficient manner (i.e. the fastest, or the cheapest, or best by some other criteria). Other models, often based on the AC or DC load method, but allowing rapid solution and some degree of optimization of the response, can be used for this purpose. See Ilic and Phadke, 1986, or Stott and Hobson, 1978, for some examples.

Figure 4.7
Comparison of AC and DC Power Flow Models
(Per Unit Values)



DC Power and Current Flows
(Per Phase Actual Quantities)



5

OPTIMAL POWER FLOW ANALYSIS

The economic implications of the relations between real and reactive flows, voltages and power angles, and so on, arise from how efficient utilization and expansion of a utility's power production and transmission resources are dictated by the capabilities and geometry of the transmission network. The problem of how to dispatch generation facilities to meet demand in least cost fashion without exceeding the safety, reliability, and capacity limits on generators and transmission is called "optimal power flow", or OPF. This chapter explains and demonstrates OPF modeling, and it discusses the value (as well as limitations) of such modeling in understanding short run marginal costs and the economic impacts of wheeling obligations.

- The chapter begins with a specification of a common form of the OPF objective function and its constraints: Minimize the operating (short run variable) costs of generation subject to such constraints as generator limits on real and reactive power, transmission line limits based on thermal, voltage, and stability considerations, Kirchhoff's Laws, and the obligation to serve total demand including losses and any contractually obligated flows by priority. Mathematically, this problem is an example of nonlinear constrained optimization problems, requiring some computationally-intensive solution techniques (a few of which are described in very general terms).
- OPF modeling yields several kinds of outputs: the recommended power production at each generating station, the resulting line flows, identification of binding constraints, and the short run marginal costs of electric power at each bus. From an economic perspective, the marginal cost information is probably the most valuable of these. It can be an input (along with other considerations) to such problems as how to price wheeling services, when and where to expand the transmission system, how to identify and penalize inconvenient locations for proposed NUG facilities, and how to justify a transmission capacity reserve margin policy.
- Importantly, marginal costs generally will not bear any meaningful or consistent relation to embedded costs, because marginal costs reflect the costs to the system of being constrained, not the costs <u>of</u> the constrained resources themselves.[1] As a result, they tend to vary by location and time of use in a manner that can appear

[1] "Embedded costs" are based on shares of costs as they are recognized in financial accounting statements, e.g. based on the depreciated net book value of the utility's assets. Traditional "cost of service" pricing of utility services utilizes embedded costs. For a detailed discussion of the contract between embedded cost accounting and pricing, see CSA Energy, 1994

somewhat idiosyncratic and non-intuitive. They will often, but not always, be similar to the cost of an adjacent generating unit plus a few percent for line losses. They can be either lower or much higher than this, because finding an economic solution which satisfies Kirchhoff's laws and other network constraints may require substantial departures from strict economic dispatch order. Most of this chapter is devoted to understanding how electric network marginal costs can behave, using several numerical examples.

- Since marginal costs arise from constrained facilities, it is critical that the basis for assumed OPF constraints be clearly understood before any OPF results are used in utility or regulatory policies. Some constraints are related to physical limits on the individual components of the system. Some are more like operating guidelines than strictly inviolate physical limits. Though somewhat subjective, these may be the most important constraints of all. In particular, some constraints involve setting aside reserves of transmission line capacity for the sake of being able to accommodate an unplanned potential disruption or disturbance to the system. These reserves may give the appearance that some portions of the system are being underutilized when they are actually protecting the system, somewhat like insurance may appear underutilized prior to a calamity.

Like any modeling technique, OPF has its limitations. An important one to bear in mind is that OPF is a steady state method of analysis that identifies optimal generation only under a given set of constraints and supply/demand conditions. Since actual operating conditions change frequently on an electric network (e.g. demands and unit availabilities vary considerably throughout the day and/or year), one OPF analysis will rarely suffice to understand what is feasible or attractive. OPF marginal costs must be evaluated in many scenarios to appreciate how they will vary over the range of foreseeable operating states whenever decisions having long term consequences are to be made. In general, transmission expansion planning and network reliability management cannot be fully evaluated using just OPF, though OPF can be a helpful input to these problems. Alternatives and complements to OPF include the load flow contingency modeling discussed in Chapter 4 coupled with off line economic and financial analyses. In addition, some techniques for combining cost accounting with transmission control algorithms are emerging that should prove valuable in marginal analyses of transmission systems.

Statement of the Optimization Problem

To express the *optimal power flow* problem mathematically, one must first specify a system performance measure and how it is to be optimized. There are many possibilities, such as minimizing real power losses, minimizing pollution emissions, or minimizing the operating costs of generation. The latter is perhaps the most common application and it is certainly the most relevant to transmission operating economics, so it will be used here.[2] That is, we will take the *objective function to be* minimizing the costs of generation of real power, given by the sum of the variable operating cost per kWh, c_j, of each unit times the kWh output P_j selected by the model as optimal:

OPF Objective Function:

$$\text{Minimize Total Generator Costs} = \sum_{j=1}^{n} c_j \times P_l$$

Unlike AC or DC load flow analysis, there is no prior designation in this function of how much power or voltage will be supplied by any power supply limits. In essence, all generators are swing buses, whose outputs will be chosen simultaneously by the optimization algorithm.[3-4]

Notice that reactive power terms are not included in the objective function. Of course, reactive power must also be supplied, and doing so may involve setup costs and capital costs, such as the cost of capacitor banks or adjustments to the generators. But recall from Chapter 2 that reactive loads only borrow and return power within each AC cycle, so the average amount of energy required to serve them is zero. Consequently, generating VARs involves no fuel costs.

Instead, the operating costs of supplying VARs will arise indirectly, because of how their provision displaces real power production, interferes with the ideal economic order of dispatch for the power plants, and alters the amount of current, hence real

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[2] Some economic treatments of OPF maximize a welfare function, such as total customer benefits minus costs. This approach requires that a benefits function be specified, such as willingness-to-pay (area under demand curves) over the full range of potential service prices. Electricity prices must then be calculated internally based on the costs of the simulated operations and the resource mix. Demand constraints must have "soft" boundaries, i.e. levels that are determined by the proposed prices by way of elasticities. This is an elegant approach-if one has confidence in all the required data about customer price sensitivitybut it is a much more common practice for utility planners to solve only the cost-minimizing problem described above. Cost minimization implicitly treats the benefits of power provision as fixed and insensitive to the cost at which power is supplied (i.e. very low demand elasticity), so minimizing costs is economically and operationally equivalent to maximizing net benefits. This is a good assumption over the very short run, such as the time horizon for OPF, though it may not suffice for long range capacity planning.

[3] Constant cost/kWh functions c_j are sufficient for illustrating how the transmission network constrains the economic dispatch. For real planning studies, more realistic production cost functions would be necessary to capture the sensitivity of marginal heat rate to level of output, e.g. cost curves that are quadratic or piecewise linear in real power output between generator loading points.

[4] As a modeling practice, it is often useful to include a pseudo-swing generator in one's OPF analysis, where that unit has very expensive but unlimited production capacity. This improves the likelihood that a feasible solution will be found while the model is being developed and tuned.

power line losses, that must supplied. All of these indirect costs are reflected in OPF results, by virtue of how the constraints are specified.

The other component of an OPF formulation, besides the objective function, is a set of mathematical expressions for the constraints under which the system can be operated. These can be extremely numerous, as they describe all of the relevant physical limits and flow conservation rules at each bus and on each line. They include the maximum feasible outputs of real and reactive power at the generators, and the sizes of the demands to be met at the load buses. There may also be voltage drop and voltage angle limits between the buses. For reasons explained below, these cannot deviate more than a few percent across the system.

Line limits include constraints on the maximum real power flows the individual transmission lines can tolerate. For instance, it was indicated in Chapter 4 that there is a thermal limit on how much current can flow before the lines overheat to unsafe levels. Since current is nearly proportional to real power, this is expressed in OPF as a maximum real power flow on each line. There is also a maximum theoretical power transfer capability for each line that is based on the impedance of the line and the voltage angle between its ends. As demonstrated in Chapter 4, this maximum occurs at an angle close to 90°, but the system will be unstable at that extreme. Most utilities adopt a conservative margin of error in setting this limit, tolerating voltage angles no larger than a few degrees. In the OPF examples in this chapter, the practical stability limit has been expressed as each line's MW flow capability calculated at a 20° voltage angle.[5]

Some constraints embody the network geometry (which lines are connected to each other), assure that Kirchhoff's laws are respected, and insist that both real and reactive bus demands be served inclusive of line losses for both real and reactive power. These considerations are satisfied by including the matrix of AC load flow equations $\underline{\mathbf{S}} = \underline{\mathbf{V}} \times (\underline{\mathbf{Y}} \times \underline{\mathbf{V}})^*$ in the OPF constraints. (See page 4-9 for AC load flow. In the formulation shown below, those equations are broken out in a more familiar form.)

The time frame for operations simulated by OPF analysis is necessarily very short, since it is a steady or *static analysis* in which operating conditions are assumed to be known and fixed with certainty. Thus OPF simulation is usually used to evaluate only the coming few minutes of operations, e.g. as an aid to adjusting governor setpoints to achieve economic dispatch. It can also be used as a longer-range planning tool, to evaluate how to run the system economically under hypothesized demand and supply

[5] This is the only explicit constraint on line-by-line real power flows that has been included in the examples in this chapter. Adding an additional set of thermal constraints would not qualitatively alter the results.

conditions that are expected to be typical or important for system operations at intervals in the future.

OPF Algebraic Specification

Minimize: Total Real Power Supply Cost

$= \Sigma c_j P_j^G$

Subject To:

Physical Operating Constraints

Generation VA Limit	$0 \le S_j^G \le Sj,max$
P & Q Tradeoffs	$S_j = \sqrt{P_j^2 + Q_j^2}$
Real power Limit	$0 \le P_j^G \le P_j^{G,max}$
Reactive Power Limit	$0 \le Q_j^G \le Q_j^{G,max}$
Stability Limit	$P_{ij} \le P_{ij}^{max}$ @ $\Delta\theta = 20°$
Voltage Limits	$\Delta V_j = V_j^{rated} \pm 10\%$

AC Load Flow Constraints

Real Power Flow *	$P_{ij} = \psi(V,\theta,X,R)$
Reactive Power Flow	$Q_{ij} = \varphi(V,\theta,X,R)$
Real Power Demand	$\Sigma p_{ij} = P_j^L$
Reactive Power Demand	$\Sigma q_{ij} = Q_j^L$
Real Power Supply	$\Sigma P_j^G = \Sigma P_j^L + \Sigma P_{ij}^{loss}$
Reactive Power Supply	$\Sigma Q_j^G = \Sigma Q_j^L + \Sigma Q_{ij}^{loss}$

* $P_{ij} = [1/(R^2 + X^2)] \times [RV_i^2 - RV_iV_j\cos(\Delta\theta_{ij}) + RV_iV_j\sin(\Delta\theta_{ij})]$

$Q_{ij} = [1/(R^2 + X^2)] \times [XV_i^2 - XV_iV_j\cos(\Delta\theta_{ij}) - RV_iV_j\sin(\Delta\theta_{ij})]$

The equations to the right are a general algebraic formulation of the OPF problem.[6] With the OPF specified in this form, standard solution algorithms for *nonlinear*

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[6] It must be stressed again that this is not the only possible OPF specification. The appropriate form depends on the purposes and complexity of the situation being evaluated. For instance, this specification includes neither decision variables nor constraints to reflect wheeling loads or interruptible services. Also, as is explained subsequently, the engineering constraints on real vs. reactive power output at generators can be rather elaborate, involving many different kinds of limits on voltages and currents throughout the generator and its auxiliary systems. Those may not all be critical to certain planning questions. In this specification, we have

constrained optimization problems can be brought to bear. In essence, these routines search for the best combination of solution variables, here real power generation levels, that are consistent with the constraints. Like the AC load flow problem by itself; finding an OPF solution is an iterative process of improving on prior trial solutions. The trick is to make that search efficient, since there are an immense number of possible combinations of flows that are feasible (not violating any constraints) to be investigated.

Nearly all solution procedures for nonlinear optimization start with a trial solution that is merely feasible though by no means optimal. They then try to improve upon this combination of solution variables by moving in the direction which improves the objective function the fastest. (Such techniques are called *gradient search* methods.) They gradually converge upon a solution, recognized by the fact that at the optimal point the direction in which the objective function would improve fastest (the *gradient*) is perpendicular to the surface created by the constraint equations. Any departures from the particular combination of generation decisions at this point would undermine performance.[7]

In the process of determining that a solution is at hand, optimization algorithms must evaluate how much better or worse the solution would become if it were moved in each possible direction, i.e. if production was shifted from one currently utilized generator in favor of production from one or more other units. Of course, it may be unfeasible to incrcase the output of a given unit beyond a certain point because of a production, voltage, or line flow constraint. Such a constraint is said to be *binding.* In that case, the foregone opportunity to improve the solution is imputed to the binding constraint as a *shadow price* or *short run marginal cost* of that resource limit. Its value is equal to how much the total system cost would decrcase if that constraint limit could be relaxed by

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used only fixed limits on P, Q, and S, which are sufficient to demonstrate the basic economic tradeoffs involved between P and Q production. Finally, this formulation does not include any controllable sources of reactive power other than generators, such as static VAR. compactors (SVCs).

[7] This is called the *first-order condition* for a maximum. Recall that in ordinary single variable calculus a local maximum occurs where the first derivative is zero and the second derivative is negative. The second derivative test *(* or *second-order condition)* rules out that the point is a minimum rather that a maximum. The second-order conditions for a constrained optimization are more elaborate, since (1) there are many variables and (2) the optimal point might be at an interior point or at the intersection of several constraints. Assuming the objective function is concave and the constraint functions are convex, then the *Kuhn-Tucker conditions* are necessary and sufficient for a local maximum. See Wismer and Chattergy, 1979, for a very accessible introduction to nonlinear optimization. See El-Abiad for a brief treatment of these concepts as applied to transmission planning.

an increment of additional capacity. Note that this is not the cost of the constrained facility, but the system cost elsewhere of being constrained by the facility (i.e. the cost of working around this constraint). Hence shadow prices will not bear any necessary relation to embedded (accounting) costs.

From an economic perspective, these shadow prices are the most useful part of the OPF output, even more important than the generation solution itself; because they reveal how valuable it would be to relax the constraints. Whenever a binding constraint is preventing a potential decrcase in costs of $x per kW (its shadow price), then it would be worth spending up to $x per kW to relax that constraint - though one must be careful about how far to go in relaxing the constraint. Some other constraint could be binding at a level of operation only slightly greater than the current level, so fully eliminating a binding constraint might not be worthwhile. Another economic application of shadow prices is in efficient pricing. Generally, the most efficient use of resources occurs when prices are equal to marginal costs.[8] Since marginal costs are so central to the economics of power flow management, most of the discussion that follows concerns why they can behave in unexpected ways. Since marginal costs arise from binding network constraints, we begin with a more detailed discussion of how these capacity and operating limits are determined.

Operating Constraints on Generation and Power Flow

As indicated algebraically in the box on page 5-5, OPF constraints are upper or lower limits on how much utilization of a given system component is allowable or feasible. Such limits may exist because of engineering properties of a network component, e.g. if it cannot safely produce or carry more than a certain quantity of power, or they may be imposed by system managers as operating guidelines intended to preserve the system's ability to accommodate unplanned potential changes in supply or demand. Setting the latter limits is somewhat subjective, involving a great deal of contingency analysis and expert judgment. These kinds of limits may be sources of contention, because they can create the appearance that a component is being underutilized and might be available to those seeking access to the system. It is particularly important that policy makers, planners, rate managers, and shippers understand these reasons for the constraints.

Generator Power Constraints

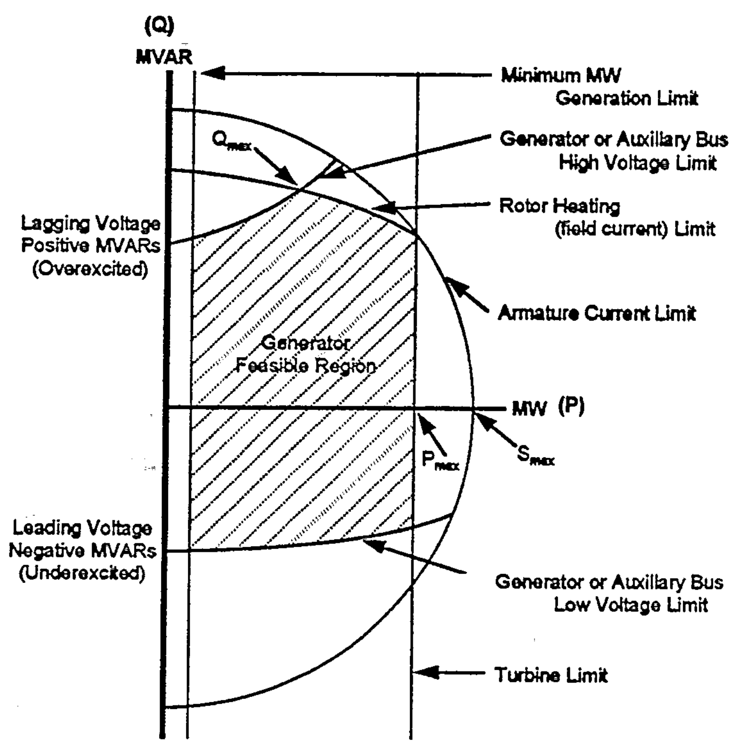
A generator can produce either MWs or MVARs, but there are several kinds of limits on how readily one form of power production can be substituted for the other. The first is that all generators have a maximum apparent power rating that restricts P and Q to

[8] The last part of this chapter includes a brief discussion of the possible benefits as well as the limitations of relying on OPF marginal costs in pricing and planning.

combinations that lie within the apparent power circle $P^2 + Q^2 \leq S^2_{max}$. S_{max} usually exceeds the maximum amounts P_{max} or Q_{max} of either real and reactive power that a generator can produce, because of limits on currents and voltages in auxiliary components of the generator. P_{max} tends to be based on the each turbine's physical limits, while Q_{max} is often determined by rotor heating tolerances. Reflecting these limits, generator ratings are often stated as a maximum MVA (apparent power) at a power factor less than I00%.

There also can be a minimum MW generation level as well as a maximum. The range of feasible P-Q output combinations lies in the box-like shaded area of the figure on the next page. Even though it is possible to operate along the maximum Q perimeter, most utility managers will not do so except under extreme conditions, because of wanting to preserve some Q production slack for responding to sudden changes in system voltages.[9]

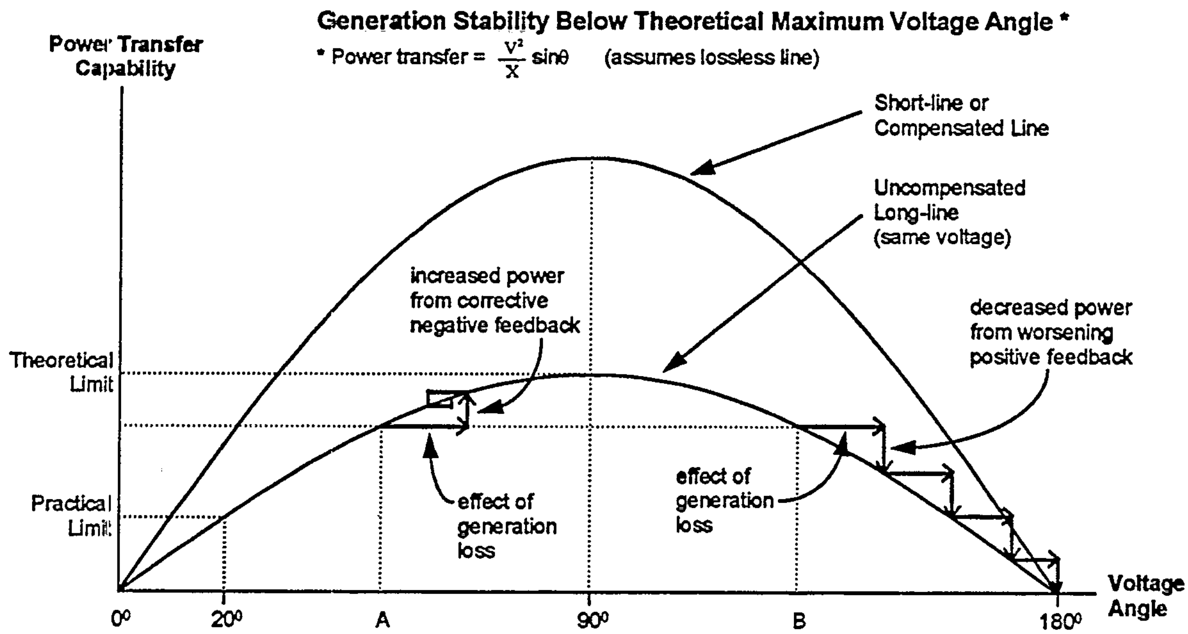
Line Stability Limits



[9] Not all types of generators are equally flexible in producing Q. If a generator is expected to produce significant amounts of reactive power, it can be built with higher rotor heating and related operating tolerances. Such enhancements will increase the capital cost of the generator, but they are primarily undertaken to benefit the transmission system. Thus the boundary between transmission and generation assets or operations is not extremely well-defined.

As was shown in Chapter 4, the maximum real power transfer along a line depends on the voltage angle between the two ends. It was also stated that the theoretical maximum at roughly 90° is not a stable point of operation, and so most lines are utilized with a much smaller voltage angle difference. 20° to 40° was identified as a more typical angle for a "practical stability limit". The reason a voltage angle at 90° or above is unstable is because of the feedback linkages between generators and loads that were described in Chapter 3. Recall that conservation of energy causes a change in power requirements (such as would occur if a distribution subsystem were suddenly disconnected by lightning) to alter the system frequency. A decrcased supply or incrcased load causes the frequency to fall, while a net excess supply causes frequency to incrcase. This in turn causes the governors to adjust the generator speeds and produce correspondingly more or less power. Ideally, the turbine adjustments would be precisely those required to restore equilibrium, but in practice it is not usually possible to go directly to the required new power level without some minor overshooting of the target. That overshooting causes instability when operating at a voltage angle greater than 90°.

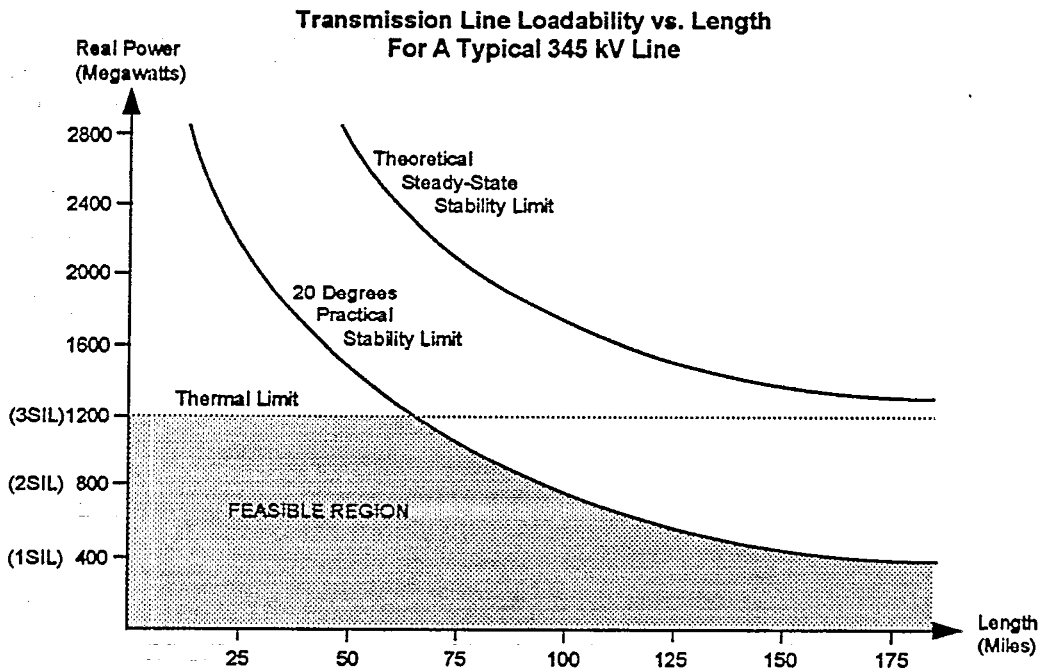
The nature of this instability is shown in the figure on the next page. Suppose a power plant is connected radially to the rest of the system, and that a generator elsewhere on the system is lost at a time when the local plant is operating at either point A (below 90°) or point B (above 90°), both of which have the same real power output. The loss of supply would cause the system frequency to decrcase and the voltage angle along the line from the local plant to incrcase. When operating at point A, these changes induce an incrcase in power output, as desired. But this increase in P causes a small frequency incrcase itself meaning that the initial response is just a bit too large. This induces a much smaller opposite adjustment that decrcases power output, and so on, with dampening oscillations that equilibrate at a point somewhere beyond the old point A.


Generation Stability Below Theoretical Maximum Voltage Angle *
* Power transfer = V²/X sinθ (assumes lossless line)
Power Transfer Capability
Short-line or Compensated Line
Uncompensated Long-line (same voltage)
increased power from corrective negative feedback
decreased power from worsening positive feedback
Theoretical Limit
Practical Limit
effect of generation loss
effect of generation loss
0°
20°
A
90°
B
180°
Voltage Angle

If the initial voltage angle happened to be above 90°, e.g. at point B, the story would not have such a happy ending. The initial loss of supply would decrcase the frequency and incrcase the voltage angle as before, but now this will cause a reduction in power transfer from the local plant to the rest of the system. This in turn decrcases the system frequency even more, which further increases the voltage angle and induces another reduction in power transfer, and so on, loading to a precipitous loss of supply.

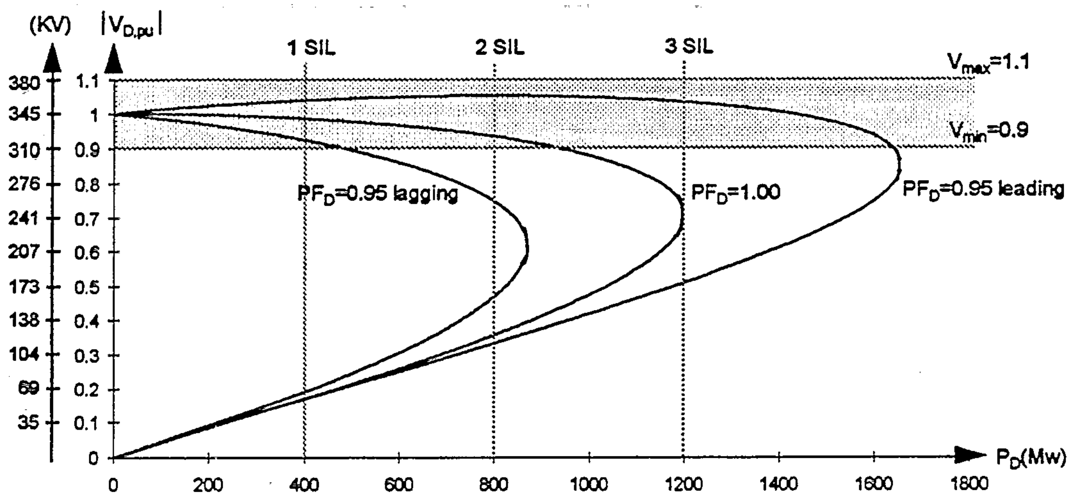
There are two ways to help avoid this problem. First is to stay well away from operating at 90° voltage angles on any line, where even a minor incrcase in demand would be destabilizing, (For this reason, this point is called the "steady-state stability limit. One must back off by enough to absorb the largest oscillations in frequency that could arise on any line due to any plausible, important potential disturbance (thereby achieving "transient stability" for at least those contingencies that have been anticipated). This loads to the 20-40° rule of thumb for setting practical line loading limits; 20° is the angle utilized for the line capacity constraints in the examples in this chapter. In fact, another sometimes stronger requirement on voltage angles is needed for stability: the *system voltage angle*, or largest angle between any pair of generators on the system, must also be less than 90°. On systems with many generators, this may necessitate that all of the individual line angles be quite small, e.g. less than 5-10°.

The second way to help alleviate instability exposure is to incrcase the power transfer capacity of critical lines, e.g. by compensating their intrinsic line reactances with capacitors. A reduction in line inductance will incrcase the height of the power transfer curve at any voltage angle. This increases the ability to accommodate a sudden disturbance, in essence because it becomes harder for the line to be "pushed" over the 90° limit. For the same reason, a shorter line will tend to have a higher stability limit than a long line, as depicted by the higher sine curve in the preceding figure. The thermal limit is therefore more likely to constrain a short line than its stability limits, while the reverse is true for a long line:



On the other hand, fixed shunt capacitor compensation of lines can be a double-edged sword for stability management, because the line compensation tends to mask how closely the system is operating to the limit of instability. This situation is shown in the figure on the next page, which shows how voltage declines (again on a single radial line) when a larger or a more inductive real power load is served. There is a point where the slope of each of these power-voltage curves becomes vertical (corresponding to the theoretical stability limits of the previous figures) at which voltage collapse occurs. These *P-V curves* are flatter and sharper when the load, P_D, has a loading power factor, PF_D or equivalently, when the line is compensated. That is, there is very little change in voltage along the line until the critical limit is reached, at which point there may be too little slack to correct the situation in response to any further incrcase in requirements.

Effects of Load on Receiving-end Voltage of a Radial Line*



* Assumes uncompensated line with series inductance only (no resistance, no shunt capacitance)

$$|V_D^2| = \frac{1-\beta P_D \pm \sqrt{1-P_D(P_D+2\beta)}}{2} \quad ; \beta=\tan\theta$$

It might appear that this situation could be avoided by simply monitoring voltage angles everywhere on the system. Until very recently, that has not been possible, because of the need for extremely high synchronicity of measurements taken at each bus. Absent such measurements, the *network state* could only be estimated statistically, using critical voltages and generator outputs as predictive variables. Obviously voltage does not explain or predict as much when lines are heavily compensated. In the future, the use of satellites with stationary orbits may allow very accurate simultaneous real-time measurement of voltage angles at critical points on a network.[10]

The stability problem is actually much more complicated than has been described, largely because the above discussion takes the point of view of a single plant and line. On an integrated network, with multiple power plants, circuits, and demand centers, several positive feedback mechanisms must be recognized:

- Fixed shunt capacitances provide line compensation in proportion to V^2. If voltages start to drop, reactive compensation falls off as well, making lines more inductive and voltage reductions over the lines even greater.
- Several generating units will be adjusting automatically and concurrently to changes in frequency, and they may induce feedbacks among themselves that produce slow oscillations in system voltage or frequency (called "dynamic instability"). System managers may also intervene to adjust the system.

[10] See Pflinger, Enge and Clement, 1992.

Inappropriate corrective action can trigger flows that overload previously secure circuits and worsen the situation.

- Some demands are highly voltage sensitive. Inductive motors have an incrcased demand for reactive power when voltage dips. They may also stall and drop entirely off the system, decreasing load diversity and concentrating the effects of the system imbalance on remaining loads.
- Line impedances are constant only at a fixed frequency, e.g. $X_L = 2\pi f L$. If frequency varies by much, the reactive power requirements of the lines can change, again possibly exacerbating the voltage maintenance problem.[11]

To anticipate these problems, a significant amount of contingency and security analysis must be performed at frequent intervals. The result will be management decisions about how much generation and line transmission capacity to set aside as reserves for unplanned disturbances. Considerable judgment is involved in these decisions, and there will often be more than one way to deal with a potential problem. Consequently, there will be no unambiguous reason for why a particular capacity constraint has been set, even if it is unambiguously necessary to set some reserves aside to avoid such problems. Moreover, contingency planning analyses can be so computationally elaborate and numerous that it is difficult to take many economic factors into consideration. Thus capacity reserves for network security are a potential source of disagreement regarding when a transmission service is or is not feasible. It is essential that policymakers appreciate the system complexities of this issue, and that open access policies not be set without consideration of network security.

System vs. Component Capacity

Once physical limits and operating guidelines for reserve component capacities have been specified, it is possible to begin OPF modeling. As will be demonstrated in the examples below, the capacity of a utility system to accommodate a request for transmission service cannot be taken for granted simply because there appear to be components that are not fully utilized. The primary reason for this is loop flows, which are manifestations of Kirchhoff's Laws. Since power cannot be directed to flow through a particular line or lines, any transmission service will generally use some portion of many components of a network. The capacity limit for a proposed transaction is determined by the "weakest link" in the chain of affected components. In general, there is no a *priori* way to predict what this link or limit will be, absent some sort of power flow modeling. Relatedly, one cannot make a good estimate of the marginal cost of a transaction based on accounting or contractual cost allocations. System modeling is required to identify the affected components and their incremental costs.

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[11] NERC, 1991, provides a survey of the causes of voltage collapse and the panning and operational adjustment that can help reduce this risk.

Because of these system interdependencies, transmission capacity is not a fixed or static property of the network components. A system may have wheeling capacity at certain times of day or year while it does not have such slack at other times, because of differences in the then-prevailing pattern of power flows. A change in the location of demands can alter which incremental flows are feasible, even if there has been no change in which generating stations are dispatched. Similarly, if flow analyses reveal that there is wheeling capacity of X MW from points A to B or Y available from A to C, there will not generally be X+Y available from A to B and C simultaneously, even if B and C are on different lines proceeding away from A, because of potential loop flows or voltage problems from the joint service that exceed design limits elsewhere on the network. That question must be evaluated separately[12] Moreover, it will not necessarily be the case that transmission capacity limits are experienced only during periods of peak power demand. Periods of lighter demand may entail significant use of generating stations that are remote from demand centers, thereby causing heavy line loading that in turn causes voltage problems or pushes parts of the system up against their contingency limits.[13]

OPF Examples

The network on which we will demonstrate optimal power flows has the same configuration (line parameters, generators, and demands) as the system that was used in the AC load flow examples of Chapter 4. Recall that this is an enclosed X-shaped system with 345 kV lines, 3 generators, and 3 loads. Buses are numbered counter-clockwise from the upper left and ending with 5 in the center. One line, from bus 2 to bus 3, is long enough that its shunt capacitance must be recognized, while the other lines are assumed to have only uncompensated series impedance. Two of the loads (L_4 and L_5) are fairly reactive (lagging), so some generators must supply VARs.

To this framework have been added assumptions about generator operating costs per kWh and real and reactive capacity limits on each generation facility and transmission line. These assumptions are summarized in per phase terms in the figure below.

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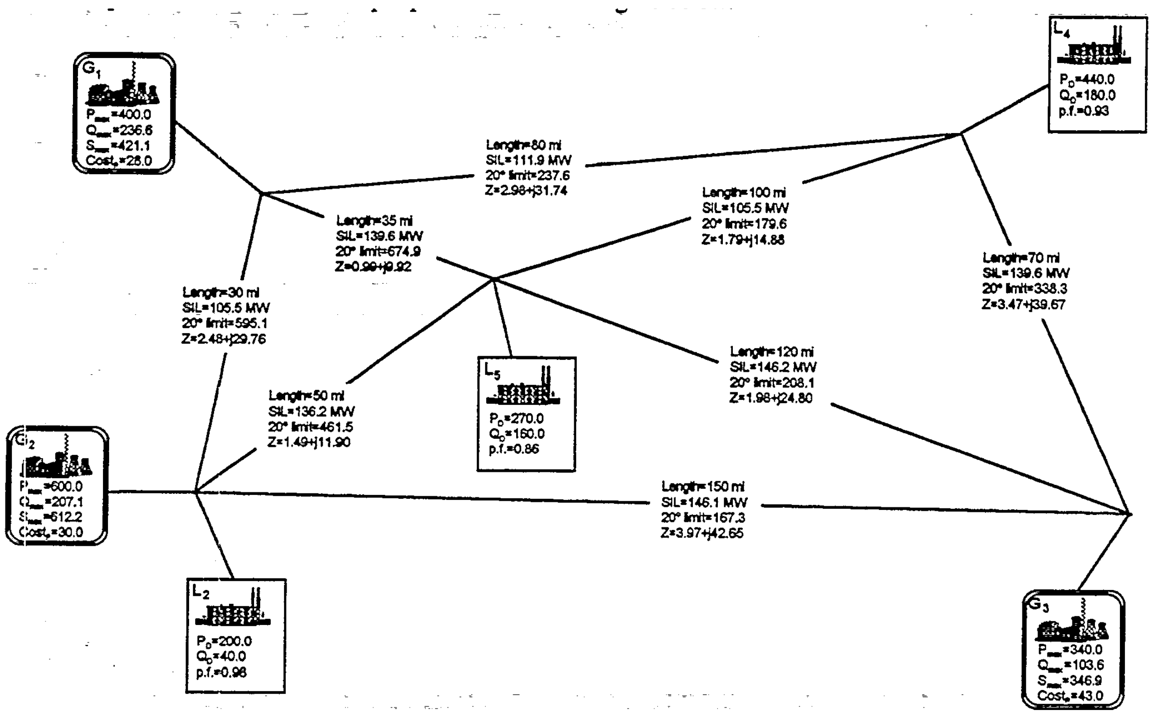
[12] See NERC, 1980, for a precise statement of how transfer capability between utility system is defined and measured.

[13] See CSA Energy, pp. 9-32 through 9-35, for a description of this phenomenon on the UK grid.



These parameters will be used in all of the examples in this chapter, unless otherwise noted. The assumed cost and capacity magnitudes are typical of actual utility facilities. For instance, generator operating costs range from 28 to 43 dollars per MWh, and these units' P and Q maximums cannot both be produced simultaneously ($S^2_{max} < P^2_{max} + Q^2_{max}$). Line impedances are realistic for 345 kV lines. There is a bit of excess capacity (any two of the three plants could cover the entire load, and transmission line 2-3 is utilized well below its SIL), but some network facilities are used to their production or flow limits in different scenarios. These examples should not be construed as depicting norms or ideals for utility configuration or operation. Indeed, this network includes some constraints that probably could be alleviated economically, but they are assumed to obtain here for the sake of demonstrating more clearly certain properties of power flow economics. We present two cases with several sensitivity variations:

- A base case - which is examined in detail to understand its shadow prices at each bus for real and reactive power. These values span a considerable range, and it is not always immediately evident or intuitive as to how they depend on plant operating cost - a complexity typical of marginal costs on even a small network. For instance, one bus has a shadow price for real power that is over two times as large as any generating plant's operating cost, yet average system losses are only one percent. Sensitivity cases are used to decompose the bus marginal costs into the underlying changes in use of different system components. There is also a shifted demand case in which total demand is unchanged from the base case but its location is different, causing significantly different capacity constraints to be binding and correspondingly different marginal costs. This illustrates the need for repeated OPF analysis, whenever conditions change or if a long planning horizon must be anticipated.

- Wheeling cases **-** in which slack line capacities from the base case are used to ship MW to either of two different buses. One case results in a very costly (roughly 5 cents/kWh) wheel, while the other has a negative marginal cost (i.e., it reduces costs for the wheeling utility). Both scenarios illustrate that wheeling marginal costs are equal to the difference in bus marginal costs between injection and withdrawal points.

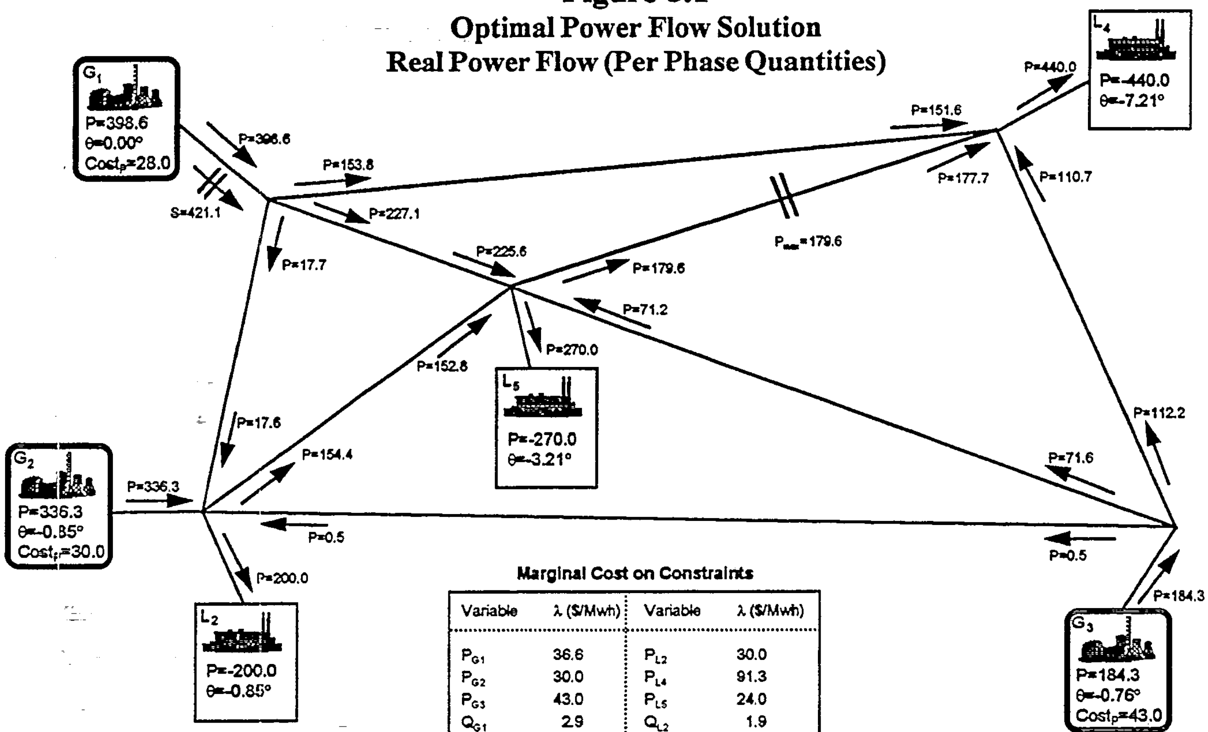
The best way to understand what is happening in these cases is to use schematic maps of the production and flows. As an unavoidable consequence, there is a great deal of visual similarity between the following figures, though the numerical results differ in very interesting ways.

Base Case

The base case has been designed to have a line constraint (from bus 5 to bus 4) be binding. Figure 5.1 shows the resulting per-phase flows for real (upper half of the page) and reactive power (lower half). First note how these flows conform to the general relations between *P*, (, *Q* and *V* that have been described earlier in this primer. For instance, the *P*-(and *Q*-*V* interdependencies are observable, and V tends to decrcase along lines in which *Q* and *P* flow in the same direction. Also notice that the plants are not dispatched in the ideal economic order. Absent network constraints, the entire 910 MW load (ignoring line losses) could be served by G_1 and G_2, the two lowest cost units. (They also have enough VAR capacity to meet the total Q demand.) Total cost would be $26,500 = 400x28 + 510x30. Instead, G_3 produces 184.3 MW at $43 /MWh, raising the actual total cost by roughly 10% to $29,175.

Whenever any constraint is binding, the resource that is operating at its limit is marked with a double slash through (or sometimes adjacent to) the numerical limit. In this case, there are four variables that are at their limits: The line from Load 5 to Load 4 is against its practical stability limit at P_{45} = 179.6 MW. Generator 1 is against its apparent power limit when S = 421.1 MVA, and Generators 2 and 3 are against their per phase reactive power output limits at Q_2 = 207.1 MVAR and Q_3 = 103.6 MVAR. Because these resources are operating at capacity limits, there is a non-zero shadow price associated with each. Since each demand is precisely met, these constraints are also at their limits, resulting in a bus marginal price for real and reactive power at each demand node. These shadow prices are summarized in the small box in the middle of the page, along with costs for any generators that are not constrained. They display several interesting patterns that can be found from time to time on almost any utility network:

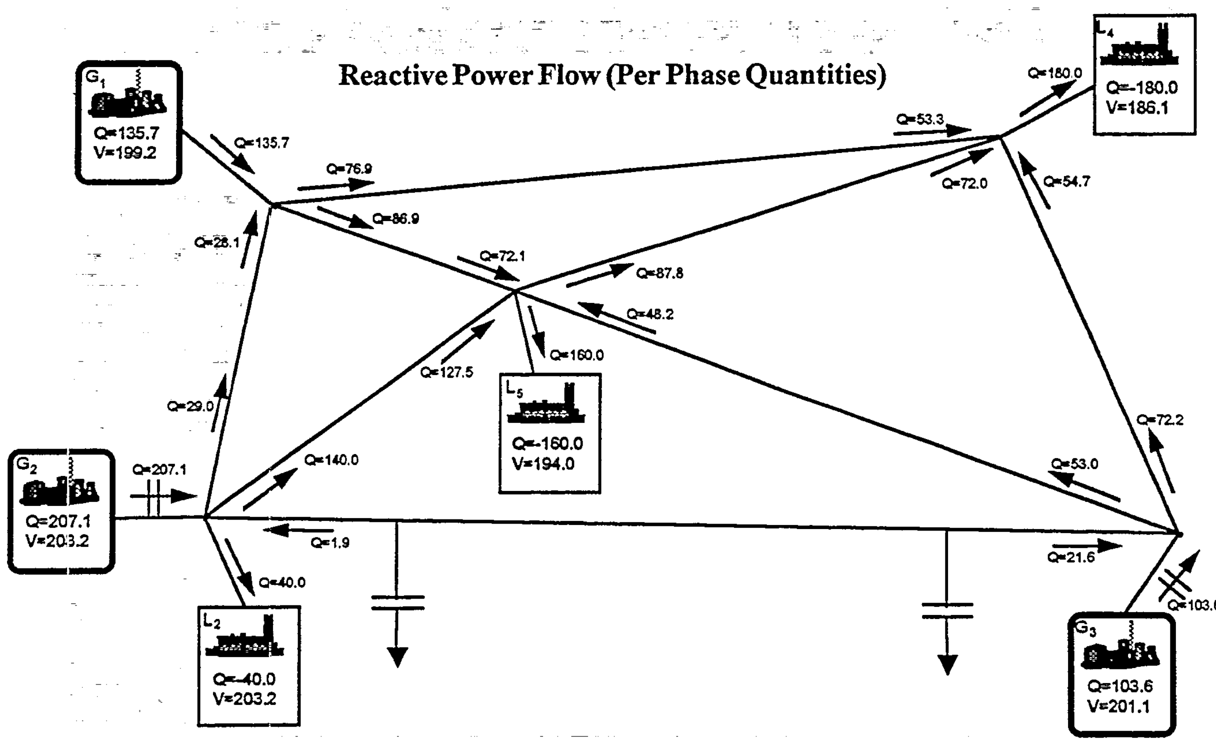
Figure 5.1
Optimal Power Flow Solution
Real Power Flow (Per Phase Quantities)



Marginal Cost on Constraints

Variable	λ ($/Mwh)	Variable	λ ($/Mwh)
P_{G1}	36.6	P_{L2}	30.0
P_{G2}	30.0	P_{L4}	91.3
P_{G3}	43.0	P_{L5}	24.0
Q_{G1}	2.9	Q_{L2}	1.9
Q_{G2} ||	1.9	Q_{L4}	5.9
Q_{G3} ||	3.8	Q_{L5}	2.1
S_{G1} ||	9.0	L_{P4-5} ||	107.2

Reactive Power Flow (Per Phase Quantities)



- *Real power marginal costs very nearly equal to an adjacent plant's operating cost* - In this example, I_2 is marginally supplied by the immediately adjacent G_2 at its operating cost of $30/MWh. More commonly, bus MW marginal costs will be equal to the operating cost of a nearby unconstrained generator plus 1-2% for transmission losses.
- *Marginal costs much larger than any plant's operating cost plus line losses* - At L_4 the marginal cost of P is $91.3/MWh, well over twice the cost of the most expensive plant (G_3 at $43/MWh). Real power losses are only 1.01% on the whole network and only trivially different from this average on the lines into bus 4, so the problem is obviously not losses. It is due to the fact that the line between bus 5 and bus 4 is operating at its practical stability limit, expressed as a MW flow limit. Note that the angle between buses 5 and 4 is <u>not</u> 20°, yet this was the critical angle used for determining the stability limit. The discrepancy arises because the 20° MW flow limit was calculated as a MW flow limit, assuming that voltage magnitudes would be identical at each end of the line. In this case a considerable relative voltage drop occurs at L_4 because it has a fairly reactive load, so a 20° angle is not needed to induce the maximum MW flow over that line. Specifically, V_4 = 186.1 kV = .9343 p.u., while V_5 = .974 p.u.

  While the Line 4-5 constraint is the cause of the high shadow price at bus 4, it is not the explanation for why that marginal coat is so high. That is explained by how Kirchhoff's Laws distribute responsibility among the generators for the supply that must be delivered over the remaining lines, while trying to preserve an economic dispatch. The figure on the next page shows how this is achieved, by simulating what would happen to generation and real power flows if demand at L_4 were incrcased by 1 MW.[14]

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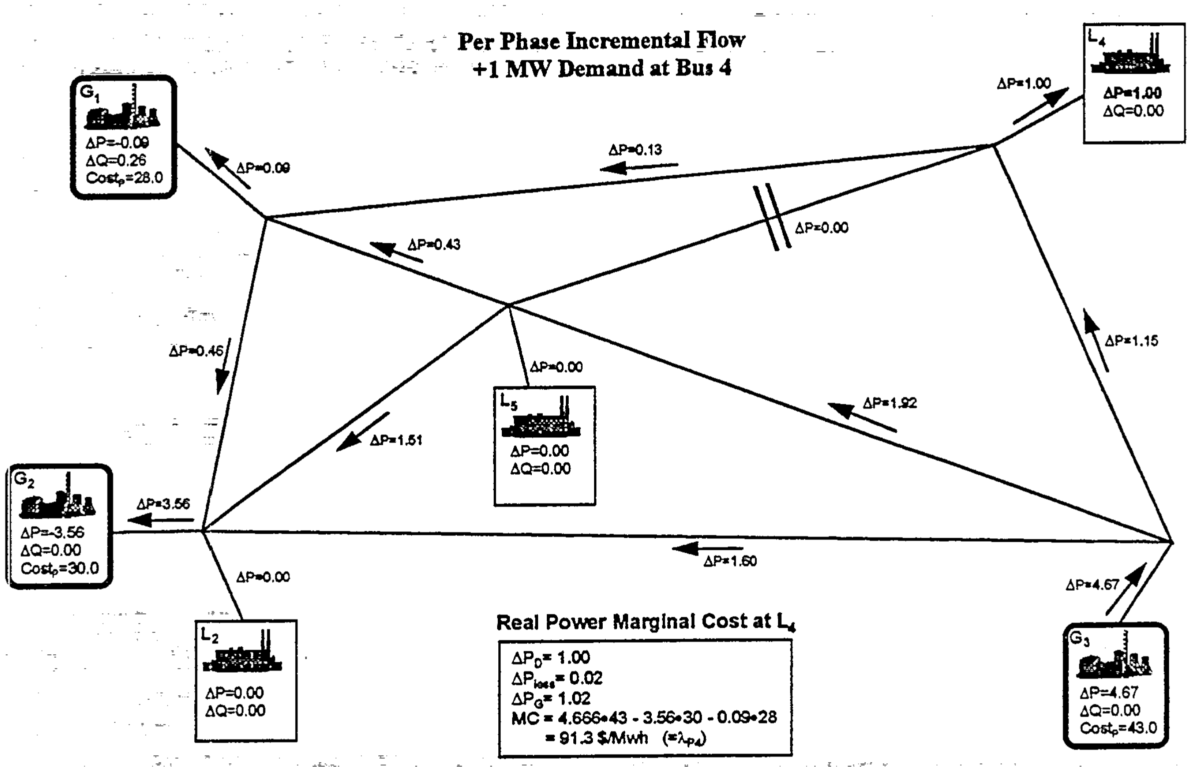
[14] Often it is not possible to interpret network marginal costs without simulating exactly what would happen if a marginal change actually occurred. There are just too many interacting factors to have an intuitive grasp of what is driving the costs. Note that in this figure, only one value for ΔP has been shown on each line. In reality, there is a value for ΔP_{ij} and for ΔP_{ji} on each line ij, but these values are so close to each other that the distinction is not important in these marginal sensitivity simulations.



The most striking feature of this marginal supply pattern is that no single plant simply produces an extra MW and ships it over to L_4. Instead, each plant makes a change, sometimes by much more and sometimes by much less than 1 MW. Specifically, G_3 produces 4.67 more MW while G_2 backs off by 3.56 MW and G_1 by .09 MW. These changes flow over all of the lines except the constrained one, in a manner consistent with Kirchhoff's laws. Indeed, Kirchhoff's laws are one of the main reasons why several generators provide odd-sized blocks of additional MWs that add up to 1.0, net of losses. (P-Q tradeoff constraints at the various generators also influence this pattern.) Since there is incrcased reliance on expensive unit G_3 and decrcased reliance on lower cost units G_1 and G_2, the net marginal cost is much greater than G_3's 43 $/MWh.[15]

•• *Bus marginal costs below the operating cost at any adjacent power plant* - Which occurs at L_5 in the center of the network. Here it would cost only $24/MWh to supply more real power, yet the cheapest generator on the system costs $28 to run. As shown in the incremental flow pattern above, this occurs because an incrcased load at Bus 5 would require the $30/MW ($G_2$ to be used more heavily. Some of this incrcased output must flow on each line away from Bus 2, so the production at higher cost (G_3

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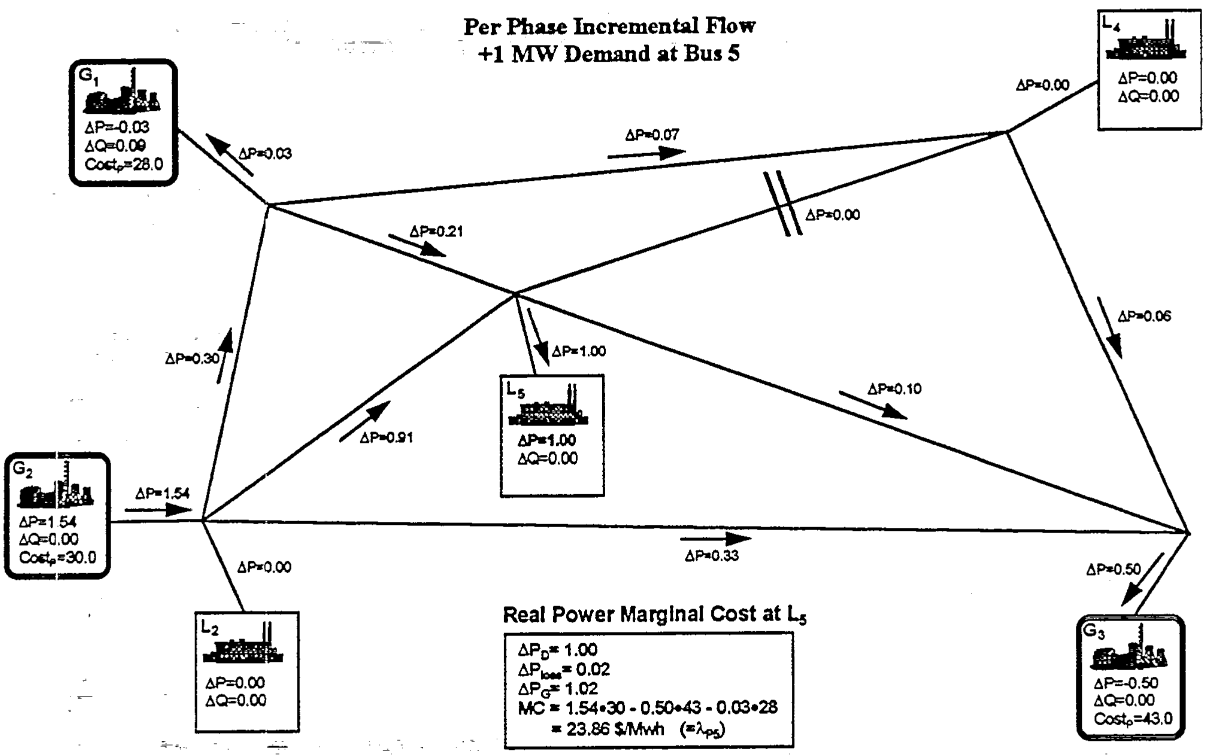
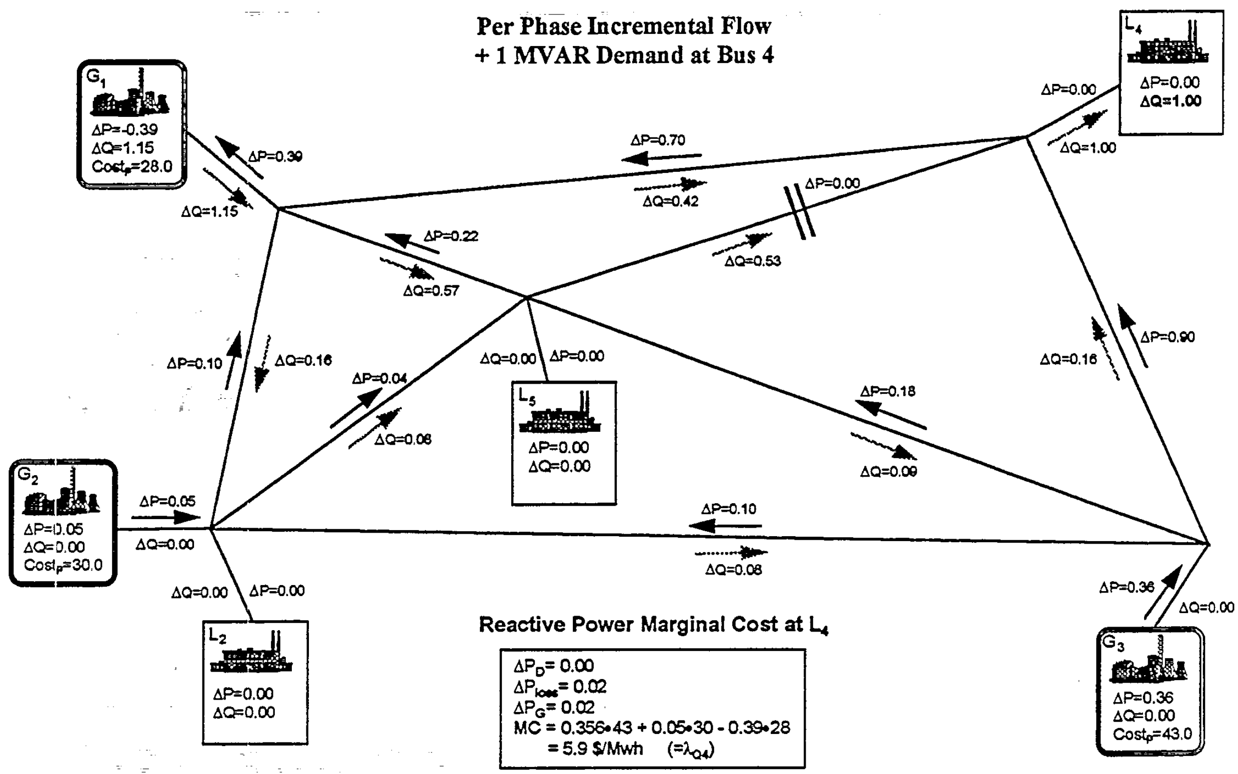
[15] Note that in actual operation, a utility (or its control system) might not make so many plant adjustments to accommodate a marginal change in load. In general, optimization models like OPF recommend numerous changes to improve the objective function only slightly. But any smoother or more simplified response will have higher marginal operating costs than tile OPF solution.

(at $43/MW) would have to be reduced. The net effect is a marginal cost below the cheapest plant's variable cost!

- *Marginal costs at some generator that exceed the direct costs of operating that plant itself-* This happens at G_1 which costs $28/MWh to run but has a real power marginal cost of $36.6/MWh. It occurs because G_1 is operating against its apparent power constraint, though not against either of its P or Q limits. Supplying more P would require it to supply less Q, which would put more of a burden for Q supplies on other, higher cost units.

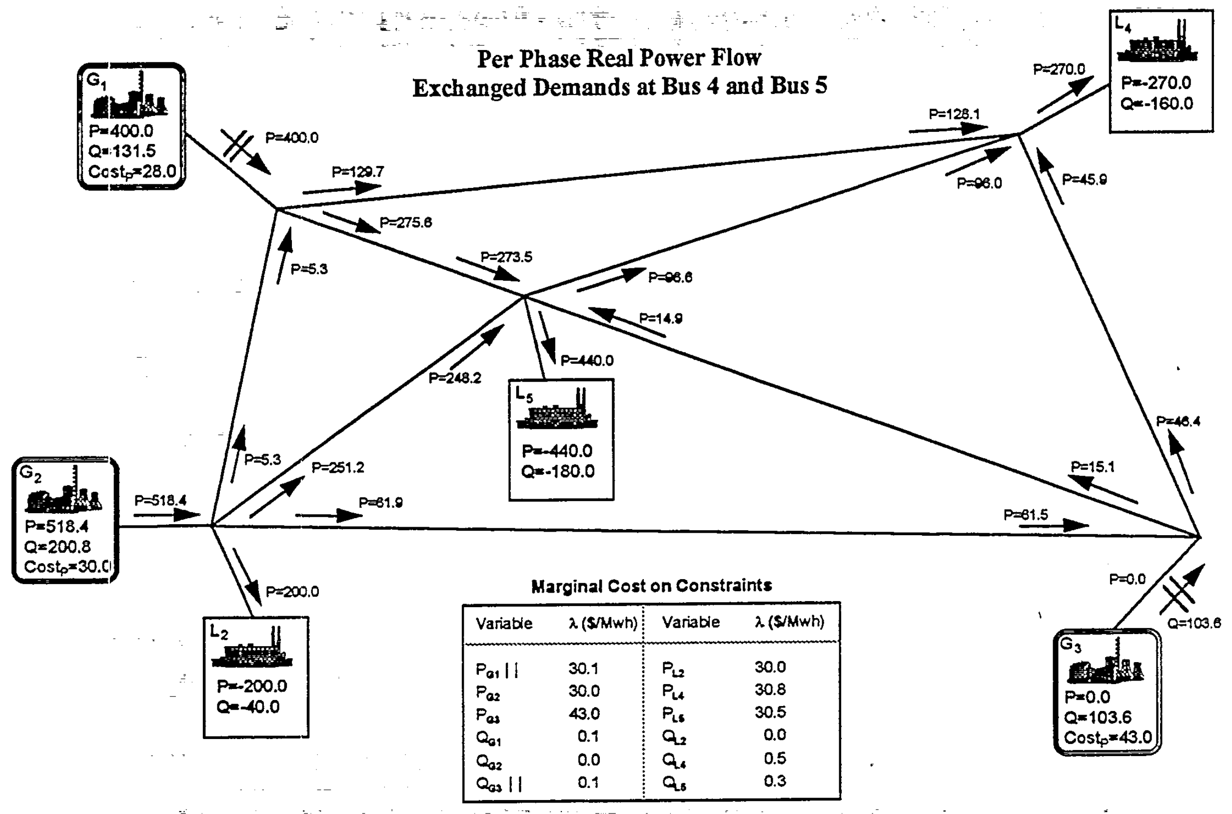


- *Reactive power marginal costs 10-20 times smaller than real power marginal costs* - This is true throughout this network and is typical of most utility operations much of the time, because incremental Q is generally supplied by backing off of a small amount of P at one or more generators and replacing it with more P produced elsewhere. For instance, the following figure demonstrates how an extra MVAR of demand at L_4 would be supplied by changes in output at each of the three generating stations. (However, recall from the figure on page 5-8 that there are parts of the feasible P-Q generating region that are bounded by nearly horizontal Q limits. When it is necessary to operate against these limits, Q marginal costs will be much higher because lots of P must be foregone, and produced elsewhere, to get just a bit more Q.)



An interesting change in the base case occurs if demands at Bus 4 are exchanged with those at Bus 5, leaving the same total load and the same production and transmission resources. The effect is shown below. (Compare to real power flows in the top halt of Figure 5.1). With this pattern of demand, it is feasible to use a strict economic dispatch; G_3 produces no real power whatsoever, though it is producing VARs.[16] The bus shadow prices arc all nearly equal to each other and to the cost of the marginal supply plant, G_2. The differences are due to line losses that affect some buses more than others.

[16] In the OPF specification used throughout these examples, there is no minimum MW output at any plant, so it is feasible to produce VARs only. In industry practice, this would be an unusual, though not unheard of, mode of operation.



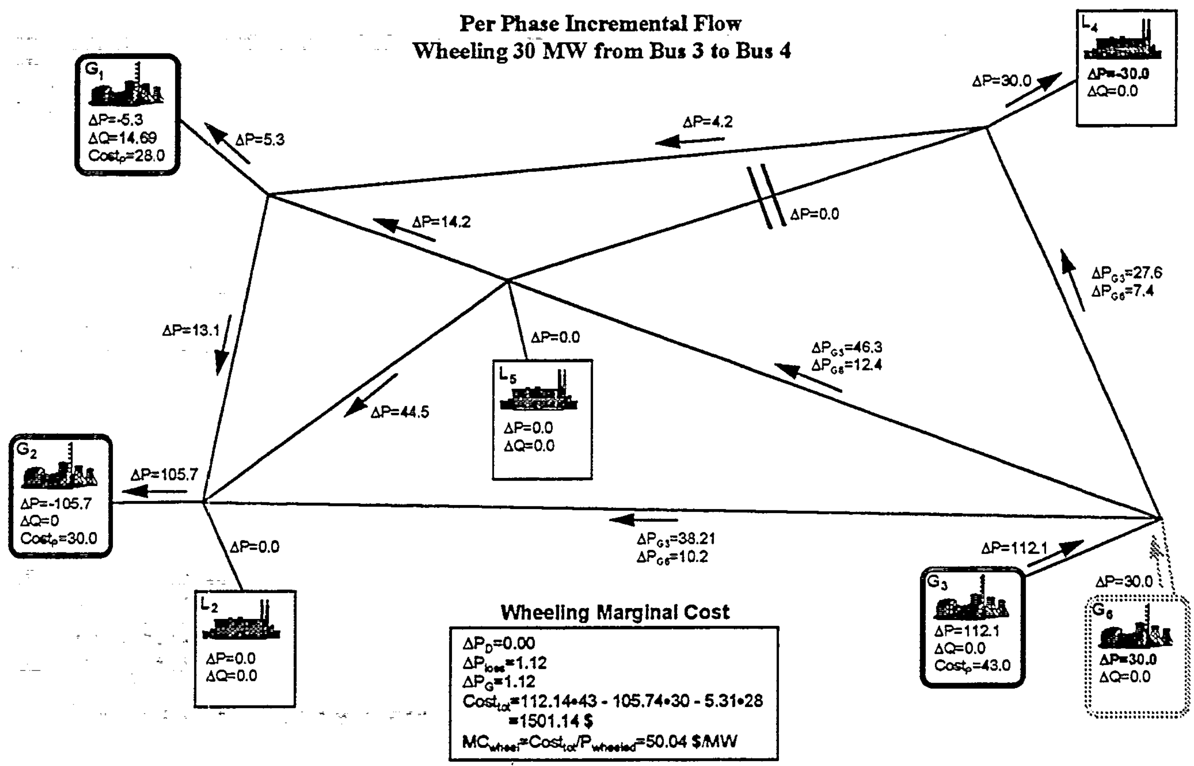
Marginal Cost on Constraints

Variable	λ ($/Mwh)	Variable	λ ($/Mwh)
P_{G1} ||	30.1	P_{L2}	30.0
P_{G2}	30.0	P_{L4}	30.8
P_{G3}	43.0	P_{L5}	30.5
Q_{G1}	0.1	Q_{L2}	0.0
Q_{G2}	0.0	Q_{L4}	0.5
Q_{G3} ||	0.1	Q_{L5}	0.3

Wheeling Cases

Examining the utilization and costs of the base case network, there are at least two locations where a wheeling transaction might appear very tempting, especially under *contract path* protocols for selling and pricing of wheeling rights. This is the widespread industry convention of contracting for wheeling loads as if they were to be transmitted over just one currently underutilized line. (Actual loop flows are ignored, unless they become so onerous on a neighbor that some amends must be negotiated or imposed by regulators.) One such opportunity is from Bus 3 to Bus 4. This line is flowing slightly fewer MW than its surge impedance loading (112 MW vs. its 139.6 MW SIL) and considerably less than its stability limit of 338.3 MW. A second is from Bus 3 to Bus 2. This line connects two generators and one load, so it is very lightly used. The utility owning this line would have a strong incentive to incrcase its use of this line, if some wheeling revenues could be garnered. The wheeling cases evaluated here assume an incremental 30 MW flow on each of these paths (occurring one at a time). 30 MW of additional power is assumed to be costlessly injected at Bus 3, while demand is incrcased by 30 MW at either Bus 2 or 4.

The figure on the following page shows the incremental loop flows and optimal dispatch adjustments associated with a 30 MW wheel is from Bus 3 to Bus 4, assuming all other loads are unchanged. First note that the path from 3 to 4 does not simply carry an additional 30 MW. Instead, it actually carries 35 MW (=27.6 from G_3 + 7.4 from G_6) more than in the base case, of which only 7.4 MW are from the third-party injecting 30 MW into the system at Bus 3. In order to honor Kirchhoff's Laws, stay within the system constraints, and maintain economic loading of the plants, it was necessary to alter the relative utilization of other plants. G_3 now produces an additional 112.1 MW while G_2 and G_1 have been reduced by 105.7 + 5.3 = 111.0 MW. The missing 1.1 MW is attributable to incrcased line losses, so the marginal loss rate on the wheeling load is 1.1/30 = 3.67% (vs. 1.01% on average before the wheel). Note that this wheel also requires a 14.7 MVAR incrcase in reactive power supply from G_1. The incrcased current flowing due to the wheel increases the I^2X losses on the system, which would cause unacceptable voltage drops absent the reactive power support.



The total cost of these dispatch adjustments is $50.04 per MW, derived in the box below the figure (denoted MC_{wheel}). This high cost may be an unexpected result to those who are accustomed to wheeling prices of a few mills/kWh, but there is a very intuitive explanation: *Short run marginal costs for wheeling will equal the difference in the marginal costs of delivered electricity at the wheeling customer's delivery and receipt* (*injection and withdrawal*) *points*, because a wheel is physically and mathematically equivalent to a shift of load from the injection to the withdrawal point. Roughly speaking, the wheeling power injected by the third party will be used to serve immediately adjacent loads, so the prevailing marginal cost at the injection bus is avoided. This requires that the wheeling power diverted to local loads be supplied by other units at the receipt end of the deal, so the marginal cost at the withdrawal bus is added. In the base case, Bus 4 had a marginal cost of $91.3/MWh while Bus 3 was at $43.0, a difference of $48.3/MWh. Adding the incremental wheeling losses of 3.67% brings this to $50.04 per MWh, the same value that was calculated in terms of incremental generator costs in the figure on the previous page.

This mathematical explanation for why transmission marginal costs can be so high may seem unsatisfying, given that the average cost for recovering the invested capital in transmission facilities is typically a few mills per kWh. Two clarifications are helpful. First, OPF shadow prices are marginal costs, not average or embedded, costs. High marginal costs should prevail only under certain short-lived circumstances rather than for all or even much of the time. Second, a transmission system is properly understood to be much more than a passive conduit for MW flows. On the margin. the transmission system can and should be adjusted efficiently to substitute for (in effect, to compete with) MW generation. For instance, reactive power compensation via switched shunt capacitors will incrcase transfer capacity and reduce real power line losses, thereby effectively supplying kWh. As another example, it is obvious that transmission line expansion between market and supply centers is a substitute for building more local generation, and vice versa. Thus, the occasionally high marginal value of transmission services must be recognized.

Because of the MW flow (stability) constraint on Line 5 to 4, it is infeasible to simply absorb the wheeled power without redispatching. Forcing the output of G_2 and G_3 to stay the same as before the wheel would require that an additional 13.27 MW flow from Bus 5 to Bus 4. However, if this were feasible, the marginal cost of the wheel would be much less: $0.904 per MWh. Thus there is a very large opportunity cost to this wheeling service, enough to raise the question of whether the system could be expanded between Buses 5 and 4 for less than the marginal wheeling cost. Of course, an expansion analysis must take into account many different potential modes of operation, since the system flow pattern will not always be the same as that analyzed in any given OPF snapshot. If the system is not constrained under many other operating scenarios, then it may be most economic to simply redispatch and charge an occasionally high time-of-use cost for the wheel.[17]

The principle that wheeling marginal costs are equal to the difference in bus marginal costs of course implies that the marginal cost of a wheel could be negative, if the injection bus had a higher cost than the withdrawal bus. For instance, if the wheel were from Bus 3 to Bus 2, then the optimal dispatch would be to reduce G_3 by 30 MW and incrcase the output at G_2 by 30 MW. This solution involves no change in power flows

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[17] That is, efficient transmission prices may exceed cost recovery prices for system expansion or reinforcement, as long as such premiums do not persist for very long periods of time. It will not generally be economic to expand at the first instance when short run costs equal or exceed replacement costs. Letting prices briefly climb above replacement costs will help ration demand and will allow more cost-effective subsequent expansion of the grid.

whatsoever from the base case, so zero MW-miles are used by this wheel. G_3 costs \$43/MWh to operate, while G_2 costs \$30, so the marginal cost of this wheel is a -\$13. The host utility is better off for carrying the wheel. In this instance it would be feasible for the dispatch of the host utility to be held constant before and after the wheel, but by so doing the \$13/MWh savings would be foregone.

If both wheels were served simultaneously, the system incremental cost would be the sum of the marginal costs (calculated as bus marginal cost differences). Obviously, this sum would not be equal to either of the parts. In this instance, a wheel of 30 MW from 3 to 4 and 30 MW from 3 to 2 would have a net marginal cost per wheeled MW of \$18.52 = (30·50·04 - 30-13)/60. It would be inefficient for those two wheels to have the same price, even though they are concurrent, identical in size, and use the same injection point.

The above examples demonstrate that bus marginal costs can span a range much broader than might be expected from looking at generation operating costs. The interaction between constrained resources causes the marginal adjustment in production and flows to be widely dispersed, not only across lines due to Kirchhoff's Laws but also across generators, due to the P-Q production tradeoffs and the need to maintain voltages and respect line capacity limits throughout the system. Nonetheless, the ultimate reason for marginal cost variation can be readily understood with incremental demand OPF simulations.

Pricing and planning Applications

Several applications of OPF have been identified in the above discussion. Among the most current of many possible applications are pricing of transmission services, appraising the benefits from transmission expansion, and assessing some of the benefits of "distributed utility" resources.

It is well known that allocationally efficient prices in a perfectly competitive economy are equal to marginal costs but that this theoretical ideal does not apply to industries that exhibit substantial economies of scale, scope, or of coordination. It is increasingly suspected that the generation side of electric service need no longer be constrained by such concerns, but it is generally agreed that the transmission and distribution functions are legitimate natural monopolies for which regulatory oversight of some kind should continue.[18] Nonetheless, experience with other restructured industries

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18 There is a vast literature on economic principles for pricing electric utility services, and there are many ways of reflecting marginal costs in prices, depending on how much latitude is available to use multi-part tariffs. See Crew and Kleindorfer, 1986, for derivations of several "second-best" efficient pricing alternatives for peak-load pricing subject to a profit constraint.
Most of the economic theory on utility pricing was not developed with an eye to transmission access,

strongly indicates that efficient pricing of electric services, i.e. prices reflecting spatial and temporal differences in marginal costs, is likely to become more important as customers demand a greater degree of transmission access and a larger menu of types of services. Such prices would help "preserve law and order" in an open access environment by providing information about current and prospective system conditions to transmission users who will effectively be making an increasing number of the power flow decisions in pursuit of their own interests. Efficient prices will also help eliminate cross-subsidies (between customers, and between different services concurrently and over time) that invite inefficient consumption of utility services (e.g., uneconomic bypass, sometimes called "cherry picking" or "cream-skimming") that could waste societal resources and harm the financial health of the utility industry.[19]

OPF shadow prices are a good place to start when trying to move towards a more efficient pricing foundation. For instance, industry discussion about transmission access has generally acknowledged that wheeling "opportunity costs" may include line losses, out-of-merit dispatch, reactive power or voltage support, and foregone opportunities to pursue power purchases or sales on behalf of existing full-service customers that would be foreclosed by wheeling commitments. The first three of these are closely related, and the shadow prices from an OPF analysis reflect all three. (With a formulation that included optional power sales, OPF could even capture opportunities foregone due to obligatory wheels.) OPF shadow prices also reflect all of the operating costs of loop flows that occur within the boundaries of the system that is evaluated.

However, it must be recognized that OPF marginal costs are static snapshots of the system, conditional on one assumed set of supply and demand constraints. For them to be informative about multiple time periods (even for a few hours), each time frame and potential operating scenario must be evaluated separately and given due weight.[20]

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but the basic concepts of how to recognize marginal costs still apply, with two important exceptions: First, as has been explained in this chapters discussion of capacity constraints, the notion of transmission capacity is state-dependent, hence intrinsically far less well-defined than the notion of production capacity. Thus identifying precisely which resource is constrained and how much that costs the system is a very dynamic, conditional problem for transmission while it is a relatively more static one for production capacity when network considerations are ignored. This means that transmission marginal costs will change frequently by location and time of use in a manner that production capacity costs to be recovered in peak lead prices do not. Consequently, efficient transmission service prices must be reassessed and posted frequently, putting a significant administrative burden on the utility and on potential shippers.
Second, much of the literature addresses pricing in the context of a single service, albeit to different classes of customers. In contrast, transmission services may be very numerous. For instance, wheelings between different pairs of buses constitute different services. There may be cross-elasticities of demand between alternative pairs, so setting prices that result in stable patterns of consumption can be somewhat tricky.

[19] See Craves and Carpenter, 1991, for a discussion elbow such issues arose in the "unbundling" (transmission access) of the US natural gas pipeline industry.

[20] One way around this difficulty would be to calculate OPF shadow prices very often, adjusting

Analyzing multiple scenarios can be analytically burdensome, because the security constraint boundaries for different scenarios might be different, requiring separate contingency analysis in addition to OPF. Also, OPF obviously cannot impute a cost to foregone power deals that are not identified ex ante for inclusion in the OPF simulation as available resources or markets. More generally, no out-of-system effects can be captured by OPF (or any model). What constraints are binding may depend critically on how large a system is evaluated. This is particularly important when loop flows are anticipated to be significant.

OPF modeling can play a role in transmission expansion planning, by revealing what shadow prices on constrained transmission lines or buses behind bottlenecks could be avoided if various kinds of alternative expansions were considered. However, economies of scale and scope in expansion make the transmission expansion problem a difficult one, somewhat beyond the power of OPF to resolve. For instance, because of Kirchhoff's Laws, many kinds of grid expansions or enhancement could relieve a given constraint. Identifying expansion trunk or line reinforcement that would most benefit the rest of the system is a very elaborate task, since there will be reliability and security benefits to additional lines as well as direct economic benefits. OPF by itself will not quantify the indirect benefits. On the other hand, regulatory or environmental considerations may limit the number of practical alternatives to a small number. OPF could be used to see how much each alternative tends to reduce the total system costs for the existing system while meeting the incrcased demand for power or wheeling services.[21]

It has been demonstrated in this chapter that network marginal costs are specific to distinct locations on the network and that they can differ significantly from each other and from the variable cost of the highest cost generator dispatched on the system. This variation in marginal costs is part of the basis for interest in the *distributed utility* concept. OPF simulation of peak vs. off-peak usage would reveal how network

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transmission access and/or power purchase prices in almost real-time (e.g. every half-hour or so), thereby creating a spot market in electric power services. A few utilities and regulatory commissions (especially in California) are considering restructuring along these lines. See, e.g., the filed comments of William Hogan or Dick Tabors in the 1993 FERC NOTC on transmission pricing, Docket No. 93-19400. Another approach to handling multiple time periods would be to include network constraints in planning analyses that have a longer horizon, and then to produce a schedule of expected bus marginal costs over that time frame. For instance, the unit commitment problem involves the optimal scheduling of plant availability over the course of a few days, recognizing start-up and shutdown costs as well as plant adjustment limits, in addition to operating costs. Generally this problem is not solved with network constraints, but research is in process (e.g. by Paul Davis and Kevin Clements at Worcester Polytechnic Institute) on an algorithm for doing so.

[21] The formulation of OPF discussed throughout this chapter minimizes production costs subject to fixed demands. Accordingly, it simply becomes infeasible in the event that demand exceeds supply. A formulation in which revenues were maximized subject to customer willingness-to-pay would produce shadow prices that included "scarcity rents" (premiums above marginal operating costs) sufficient for market-clearing prices whereby available supply equaled demand.

marginal costs vary over time. These differences are the potential energy benefits of storage devices that could redistribute loads over time to make them more even and predictable over time, and the associated capacity benefits, from more efficient sizing and utilization of generation and transmission facilities, could be even more substantial. Locally distributed power supplies might also alleviate certain voltage problems and improve network security, by reducing the need to transmit large amounts of real or reactive power.

In sum, though OPF has its limitations (as does every modeling technique), OPF or some similar modeling scheme that reflects network constraints must play a part in the design of efficient, unbundled prices for transmission services.[22] Regulators, utility planners, and energy policy economists must understand the engineering basis for these analyses, while the transmission engineers and system operations managers must appreciate the economic pressures and objectives facing those seeking (or obligated to provide) access, so that policy solutions and service design will preserve system efficiencies while creating new market opportunities.

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[22] Another means of obtaining system marginal cost information would be to rely on operating information obtained from the transmission control process itself. Research by Dr. Marija Ilic at M.I.T. suggests that an extension of the AGC framework could allow both better security and stability of the network while performing the accounting for the marginal and total costs of actual (not just forecasted) wheeling transactions and loop flows. This accounting could be posted over very short intervals of time, hence providing the basis for spot prices, or it could be accumulated over any conveniently longer time frame (such as a month or season) for more conventional billing of transmission operating costs. Attractive features of this approach include that it captures some dynamic considerations not included in OPF, especially frequency control, and it can be implemented hierarchically, i.e. with transmission management responsibilities distributed across several utility control areas, as is currently practiced See Ilic, Eidson, and others, 1993.

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